SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

AFTERNOON SESSION

Date: Friday, May 2, 2014
Time: 1:30 p.m. – 3:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 6 questions numbered 11 through 16 for a total of 40 points. The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

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Exam QFIADV-Front Cover

****BEGINNING OF EXAMINATION****
Afternoon Session
Beginning with Question 11

11. (*5 points*) You are reviewing models for pricing and hedging equity options.

(a) (*1 point*) Describe a fully stochastic volatility model and a local-volatility model.

(b) (*1 point*) List the desirable features of a local volatility model.

You are thinking of using either a fully stochastic volatility model or a local-volatility model. However, your colleague suggests using a mixed model that combines a Jump-Diffusion model with a fully stochastic volatility model.

(c) (*1 point*) List the pros and cons of Jump-Diffusion models with a large but finite number of jump amplitudes.

(d) (*2 points*) Recommend which of the above models is most appropriate to use and support your recommendation.

12. (*5 points*) You have been asked to perform a performance attribution for a US dollar denominated fixed income portfolio.

(a) (*1 point*) Describe the three requirements for successful performance attribution.

(b) (*2 points*) Describe the general properties of each of the following performance attribution models:

(i) Total Return Model

(ii) Excess Return Model (versus yield curve and volatility)

(iii) Fully Analytical Model (based on top level and sector level)

You are given the following additional information about the fixed income portfolio:

- It is a high yield portfolio seeking to maximize returns.
- The portfolio manager allocates capital based on her views of sector and security returns.

(c) (*1 point*) Recommend and justify which performance attribution model is most appropriate for this portfolio.

Suppose instead that the portfolio contains multiple asset classes, and capital is allocated based on market risk factors.

(d) (*1 point*) Recommend and justify which performance attribution model is most appropriate, given this information.

13. (*6 points*) You are implementing the following GARCH-M model to use for Monte Carlo simulation of embedded investment guarantees:

$$r_t = \mu + c\sigma_t^2 + \sigma_t \varepsilon_t$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 (\sigma_{t-1}\epsilon_{t-1})^2 + \beta_1 \sigma_{t-1}^2$$

where:
r_t is the log-return of the asset from time $t-1$ to t,
$\{\varepsilon_t\}$ is a sequence of independent and identically distributed standard normal random variables, and
μ, c, α_0, α_1 and β_1 are the GARCH-M model parameters.

(a) (*1 point*) Interpret each of the parameters above.

Your colleague gives you the following parameters to use for your simulation:

μ	c	α_0	α_1	β_1
0.0013	0	0.0022	0.129	0.459

(b) (*1 point*) Calculate the long-term stationary volatility.

(c) (*2 points*) Calculate the covariance between σ_t^2 and r_{t-1}.

(d) (*1 point*) Identify a potential limitation of your simulation, based on the answer to (c).

(e) (*1 point*) Recommend a way to avoid the limitation in (d).

14. (*7 points*) The Neymar Company is considering the following vector AR "Model M" for the log prices of two assets.

Model M:

Suppose V_t is as follows:

$$V_t = \begin{bmatrix} 1.0 & 0.5 \\ -0.3 & 1.8 \end{bmatrix} V_{t-1} - \begin{bmatrix} 0.2 & 0.3 \\ -0.6 & 1.1 \end{bmatrix} V_{t-2} + a_t$$

$\{a_t\}$ is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix Σ.

The AR polynomial matrix $\phi(B) = \begin{bmatrix} 1-\phi_{11}(B) & -\phi_{12}(B) \\ -\phi_{21}(B) & 1-\phi_{22}(B) \end{bmatrix}$

where B is the backward shift operator

(a) (*1 point*) Write down the AR polynomial matrix $\phi(B)$ for model M and calculate the values for $\phi(1)$.

(b) (*2 points*) Show that V_{1t} and V_{2t} are unit-root non-stationary under Model M.

(c) (*2 points*) Write down an Error-Correction Model under Model M.

In addition, the company is considering the following bivariate Error-Correction Model **("Model K")** for predicting the dynamic behavior of the log dividend-price ratio.

Model K:

Let s_t denote the log of stock prices and

d_t denote the log of dividend prices

$$\Delta s_t = c_s + 0.75\left(d_{t-1} - s_{t-1} - \mu\right) + \varepsilon_{st}$$
$$\Delta d_t = c_d + \varepsilon_{st}$$

where $c_s > 0,\ c_d > 0,\ \varepsilon_{st} \sim iid\ N(0,1)$

$Y_t = \left(s_t,\ d_t\right)$ is at most an integrated process of order 1.

14. Continued

The long-run equilibrium is $d_t = s_t + \mu + \mu_t$ where μ is the mean of the log dividend-price ratio, μ_t is an I(0) random process representing the dynamic behavior of the log dividend-price ratio (disequilibrium error).

(d) (*1 point*) Determine $E[\Delta s_t \mid Y_{t-1}]$ and $E[\Delta d_t \mid Y_{t-1}]$ under Model K

(e) (*1 point*) Explain what the Model K predicts under the following 2 situations:

Case 1: $d_{t-1} - s_{t-1} - \mu = 0$
Case 2: $d_{t-1} - s_{t-1} - \mu > 0$

15. (*7 points*) Given the following information for a simple world with only three possible credit ratings:

Year End Rating	Probability Weighted of State	New Bond Value plus Coupon
A	1.0%	105
B	98.5%	100
C	0.5%	95

(a) (*2 points*) Compare and contrast marking to market for market risk to marking to market for credit risk.

(b) (*2 points*) Calculate for each possible year end rating.

(i) The probability weighted value

(ii) The difference of value from mean

(iii) The probability weighted difference squared

(c) (*1 point*) Identify and compare the two approaches used by CreditMetrics to describe the volatility of an asset.

(d) (*2 points*) Determine the 1 percent value based on the information calculated in part (b).

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16. (*10 points*) You are an investment analyst of Investment Bank ABC that participates in option markets that are exposed to changes in implied volatilities for the S&P 500 Index.

(a) (*1 point*) List four stylized facts about the volatility smile for the S&P 500 Index.

(b) (*2 points*) Explain the following two volatility smile behaviours:

(i) Floating smile

(ii) Sticky smile

You are given the following implied volatility formulas:

- Range regime: $\sigma_{impl}^{range}(K,T;S) = a - b(T)(K - S_0)$
- Trending regime: $\sigma_{impl}^{trending}(K,T;S) = a - b(T)(K - S)$
- Jumpy regime: $\sigma_{impl}^{jumpy}(K,T;S) = a - b(T)(K + S) + 2b(T)S_0$

(c) (*1 point*) Interpret the relationship between the values of the underlying index and the volatilities under these three regimes.

(d) (*2 points*) Derive an expression for the following, under each of the three regimes:

(i) ATM spot volatility

(ii) Volatility Skew

You are given the following observations from a volatility surface.

K	S	T(months)	Volatility
1425.00	1500	3	20.00%
1350.00	1500	6	29.00%
1275.00	1500	9	37.00%

S = current S&P 500 Index, K = strike price, and T = time to maturity.

16. Continued

(e) (*2 points*) Calculate the moneyness of each of the observed points, using the following two measures of moneyness:

(i) $h = \ln(K / S)$

(ii) $h = \dfrac{\ln \dfrac{K}{S}}{\sigma\sqrt{T}}$

(f) (*2 points*) Interpret the measure of moneyness in (ii) above and explain why this measure might be used.

****END OF EXAMINATION****
Afternoon Session

USE THIS PAGE FOR YOUR SCRATCH WORK