QFI ADV Model Solutions
Spring 2014

1. Learning Objectives:

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Farmland and Timber

Learning Outcomes:

(6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.

(6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.

(6d) Demonstrate an understanding of the due diligence process for alternative investments.

Sources:

QFIA-111-13: Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8

QFIA-112-13: Commercial Real Estate Analysis & Investment, Chapter 12

Commentary on Question:

This question contrasts different assets and tests candidates understanding of the due diligence process and the selection of appropriate investments to meet the objectives of liquidity, inflation hedging and risk diversification.

Solution:

(a) Outline a due diligence process to evaluate new investment classes.

1. **Continued**

Commentary on Question:
The candidates performed relatively poorly on this section. In general candidates were not able to recognize the main feature s of the due diligence process and provide a satisfactory list. Points were given for explanations closely related to the characteristics.

The list below represents a good process to capture the information needed. To obtain full credit, four (4) items were needed from the list:

- Market opportunity : Identify market inefficiencies
- Investment process: identify best practices and competitive advantages
- Organization: stable and well-staffed for risk management, research of investments, compensation and turnover among staff
- People: Do we trust people? meet principals, review experience, integrity etc.
- Term and structure: the ownership structure, details by market, asset and strategy.
- Service providers: Who supports the hedge fund, auditor, lawyers, brokers, lenders etc.
- Documentation: read prospectus and private memorandum
- Formally documents the due diligence process. Prior to making a decision ensure a write up is done to summarize the findings

(b) Evaluate the main characteristics of hedge funds relative to the stated investment objectives.

Commentary on Question:
The candidates performed relatively well on two items, liquidity and diversification, but most candidates did not recognize the hedge fund as an inflation hedge. Also, for liquidity, many candidates did not mention that poor liquidity can be associated with the private trading aspect and loosely regulated entities.

A hedge fund (HF) is relatively illiquid since a private investment with loosely regulated entities. HF usually contains a lock-up period that limits the liquidity of this investment for a period of time. Also, HF can be investments with short and long positions using leverage aggressively and this can increase the risk of default and the risk of failing to return money to investors.

As an inflation hedge, historically the Hedge Fund Composite Index (CISDM) is positively correlated with unexpected inflation according to US studies (1990 – 2004).

1. Continued

HFs use different investment strategies and styles so this creates diversity in risk and a range of investment choices. HFs that are less correlated to stocks and bonds is another reason offered to include HF in a well-diversified portfolio for an institutional investor.

(c) Describe the main considerations relative to risk and cost of capital in determining the price of the property for the pension fund as the buyer.

Commentary on Question:
The candidates performed poorly on this section. Most candidates were able to mention that risk resides in the assets to purchase but unable to specify that the cost of capital of the pension plan can't be used to compute the purchase price of the property.

The strategy is to fix the price as the present value of the cash flow of the new property and discount at an IRR that reflects the risk of the property. The cost of capital of the pension plan's portfolio of 9% cannot be used as the discount rate since it reflects the average risk of all its assets which may not be the same as the risk of property to be purchased. The risk resides in the assets to be purchased, not in the investor. Using 9% increases the price of the property.

(d) Assess both real estate strategies relative to the objectives of plan and recommend which of the two is more appropriate for the pension plan.

Commentary on Question:
The candidates performed well on this section, recognizing the main features for each objective.
Candidates who were able to get full credit for all objectives generally were also able to recommend the REIT investment for the pension plan. Some candidates were not able to comment that a small pension plan generally does not have sufficient experience with ownership of commercial property and thus should not invest in a single property.

Direct investment in Real Estate:
Ability to meet liquidity objective:
- Relatively illiquid since not publicly traded on the market.
- High transaction costs which reduce net income and the value at time of resale for an owner without experience.

Ability to meet inflation (hedging) objective:
- Overall may be able to provide an inflation hedge.
- Office, retail and industrial sector may include an inflation component.

1. **Continued**

Ability to meet diversification objective:

- Low correlations with stock and bond provide diversification for the portfolio.
- Direct ownership in a specific area may not provide geographical diversification.

Indirect investment in Real Estates through REIT:

Ability to meet liquidity objective:

- Securitizes illiquid assets through public markets with small outlay.
- Lower transaction costs than direct investment so better resale value.

Ability to meet inflation (hedging) objective:

- Analyzing U.S. REIT's found some long-run but no short-run inflation-hedging ability.
- Lower transaction costs improve chance to increase returns and reach inflation objective.

Ability to meet diversification objective:

- Fund with many geographical locations reduces exposure to catastrophic risks.
- REIT may provide less diversification when added to a portfolio of stocks and bond than direct investment since REIT has higher historical correlation with S&P500.

<u>Recommendation</u>

REIT does not require specific knowledge as property manager especially for a small pension plan mainly invested in stocks and bond assets.
REIT is more liquid than direct ownership.
REIT has lower transaction costs which increase chance to have better return and reach inflation target.
REIT offers a lower diversification comparatively with current stocks / bond assets but a great diversification for geographical location.
Then REIT should be a better selection.

2. Learning Objectives:

7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

Learning Outcomes:

(7a) Identify and evaluate the impact of embedded options in liabilities, specifically variable annuities guaranteed riders (GMAB, GMDB, GMWB and GMIB).

(7b) Demonstrate understanding of risks associated with guarantee riders including: market, insurance, policyholder behavior, basis, credit, regulatory and accounting.

(7c) Demonstrate understanding risk management and dynamic hedging for existing GMXB and it embedded options – including:
- (i) Hedgeable components including equity, interest rate, volatility and cross Greeks
- (ii) Partially Hedgeable or Unhedgeable components include policyholder behavior, mortality and lapse, basis risk, counterparty exposure, foreign bonds and equities, correlation and opration failures
- (iii) Static vs. dynamic hedging

Sources:

Stochastic Modeling: Theory and Reality from an Actuarial Perspective, Section IV

The Impact of Stochastic Volatility on Pricing, Hedging, and Hedge Efficiency of Withdrawal Benefit Guarantees in Variable Annuities

Commentary on Question:

The question tested the candidates' basic understanding of using stochastic modeling for the risk management of variable annuity guaranteed benefits.

Solution:

(a) Describe the features of GMDB and GMIB riders which create embedded options.

Commentary on Question:

The candidates performed relatively well on this section. Candidates were generally able to identify that the riders resemble put options. A common mistake was that some candidates merely defined the rider without describing how that rider creates an embedded put option and thus did not receive full credit.

GMDB Rider – Guarantees minimum level of death benefit to beneficiary regardless of the account value at the time of death. The liability is therefore higher at times when the account value is low.

GMIB Rider - Guarantees a minimum annuity conversion rate regardless of whether the account value is sufficient to support that rate.

2. Continued

The guaranteed riders are effectively embedded put options. Policy owners are protected from the downside of financial risks.

(b) List and describe three types of risks (other than equity risk and interest rate risk) which are associated with the GMIB rider.

Commentary on Question:
The candidates performed relatively well on this section. Candidates were generally able to identify at least one of the risk, however most did not identify Model Risk. A common mistake was to identify risks generally associated with reinsurance that were outside the scope of the question.

1) Longevity Risk – Higher liability for guaranteed payments with mortality rates lower than assumed.
2) Policyholder Behavior Risk – Unexpected random policyholder behavior may increase the liability. For example, higher lapse rates at times when the guarantees are in-the-money exacerbates the impact of equity risks
3) Model Risk – Simulations are mostly dependent on stochastic models of equity returns, interest rates, and other variables. Inappropriate assumptions in these models may lead to incorrect assessments of the risks.

(c) Identify and explain three potential problems with Mr. Chen's analysis.

Commentary on Question:
The candidates performed relatively well on this section. Candidates were generally able to identify at least one of the potential problems with the analysis. A common mistake was to generally state that there should be more sensitivity analysis without specifying what variables should be sensitivity tested.

Potential Problems:

- The pricing of guaranteed products should be done using risk-neutral scenarios. Mr. Chen used the real-world AAA scenarios.
- The convergence test appears to indicate that a significant deviation of results occur when the number of scenarios drops from 10,000 to 1,000. However, Mr. Chen used the 1,000 scenarios to draw conclusions.
- The weighted average cost under the 10,000 scenario assumption is 71.85 basis points which exceeds the assumed average cost of 60. Under the 1,000 scenario assumption, the weighted average cost is 49.31 basis points, which is below the assumed average cost. Both assumptions offer contradicting results on the adequacy of charges, which raises red flags on the credibility of the reduced scenario simulation.
- There is no consideration of the dynamic policyholder behavior, which may underestimate the overall liability.

2. **Continued**

- Mr. Chen used a combination of AAA pre-packaged scenarios and the company's internal model, which may cause inconsistency, thereby increasing model risk.

(d) Recommend which hedging strategy to use.

Commentary on Question:
The candidates performed relatively well on this section. Candidates generally provided a recommendation with sufficient justification to merit at least some credit. A common mistake was recommending delta-vega hedging without demonstrating that the candidate had reviewed the data accordingly.

Although delta hedging was the desired answer, full credit was possible for a delta-vega hedging with sufficient justification.

After reviewing the simulation results, delta hedging appears to be the most appropriate strategy.

Delta hedging is more appropriate than a no hedge strategy due to the following:

- With no hedge in place, higher volatility values lead to larger hedging errors
- The delta-hedging results show very little sensitivity with respect to the volatility parameter
- The delta-hedging strategy reduces the risk measures from ~10% to ~3%

Delta hedging is more appropriate than a delta-vega hedging due to the following:

- There appears to be very little reduction in risk measures from using delta-vega hedging.
- Delta-vega hedging would be more expensive than delta hedging due to the costs of the extra derivatives needed. The increased costs do not appeared to be justified from the simulation results.

3. **Learning Objectives:**

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.

(2f) Demonstrate an understanding of modeling approaches for correlated defaults.

Sources:

Introduction to Credit Risk Modeling, 2nd Edition, Bluhm, Chapter 1

Commentary on Question:

This question tests the candidate's basic understanding of Unexpected Loss (UL) and Default Correlations.

Solution:

(a) Calculate the unexpected loss due to default of the portfolio.

Commentary on Question:

The candidates performed relatively well on this section. However, distribution was barbelled where the candidates either understood this question and gained most of the points or answered poorly.

Let X_{ABC} and X_{XYZ} be Bernoulli variables that represent default. (e.g. X can be either 0 for no default or 1 if company defaults.)

Let L represent the loss of the portfolio, then

$L = X_{ABC} \times EAD_{ABC} \times LGD_{ABC} + X_{XYZ} \times EAD_{XYZ} \times LGD_{XYZ}$

where EAD is the exposure at default and LGD is the loss given default.

The Unexpected Loss (UL) is then the standard deviation of L:

$\text{VAR}(L) = \text{VAR}(X_{ABC} \times EAD_{ABC} \times LGD_{ABC} + X_{XYZ} \times EAD_{XYZ} \times LGD_{XYZ})$

$= \text{VAR}(X_{ABC} \times 100 \times 1 + X_{XYZ} \times 100 \times 1)$

$= 10{,}000\ \text{VAR}(X_{ABC} + X_{XYZ})$

$= 10{,}000(\text{VAR}(X_{ABC}) + \text{VAR}(X_{XYZ}) + 2\times\text{COR}(X_{ABC}, X_{XYZ})\times\text{STD}(X_{ABC})\times\text{STD}(X_{XYZ}))$

$= 10{,}000((0.1)(0.9) + (0.05)(0.95) + 2(0.5)\text{SQRT}((0.1)(0.9)(0.95)(0.05))$

$= 10{,}000 \times 0.2029$

$= 2029$

3. **Continued**

STD(L) = UL = 45

(b) Evaluate and recommend the best company to add to each investor's portfolio based on the situations described above.

Commentary on Question:
The candidates performed relatively well on this section.
Candidates received marks for making a recommendation and supporting that recommendation. Full credit was given to candidates who clearly provided a recommendation and supported that recommendation with facts from the question. No marks were given for candidates who only commented that there was not enough information to answer the question.
The recommendations given in the solutions below are not the only recommendations that received marks. In particular Investor 2 we also accepted a recommendation of Alpha and Investor 3 we accepted a recommendation of Gamma as long as an acceptable explanation was provided.
There were a number of candidates that stated a portfolio with a correlation of -1 is "well-diversified." This is an incorrect statement. A well-diversified portfolio is one that has a correlation close to 0, a portfolio with a correlation of -1 would be considered a hedged portfolio.

Investor 1: Recommend Beta.
Since the investor has such a large confidence that Mamda will not default and the correlation between Mamda and Beta is relatively large, we can infer that the investor would think Gamma would not have a large default probability. The investor would be wise to select Beta so that they can earn the returns of both Mamda and Beta.

Investor 2: Recommend Gamma.
Gamma is pretty much independent of Mamda which would provide very good diversification. Since the investor has no additional information, they would not want to select Beta because that would provide too much concentration risk.

Investor 3: Recommend Alpha.
Alpha is almost a natural hedge to Mamda since is negatively correlated to the portfolio. By investing in Alpha the investor is almost guaranteed to have at least one asset still in their portfolio at the end of the 12 months

Investor 4: Recommend Gamma.
Would not recommend Alpha to the plan as that would add to the concentration risk. Would not add Beta to the plan since it is so highly correlated to Mamda that would increase the default risk. Adding Gamma would be the best solution since it is relatively independent to Mamda.

4. **Learning Objectives:**

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.

(2d) Demonstrate an understanding of Merton asset value models in the context of credit risk.

Sources:

Sec 2 - Bluhm, An Introduction to Credit Risk Modeling, 2nd Ed, Ch3

Commentary on Question:

This question tests candidates understanding of Merton Asset Value Model in the context of credit risk analysis.

Solution:

(a) Calculate each analyst's implied estimate of DeF's stock price volatility using the Merton Asset Value Model.

Commentary on Question:

Generally, candidates performed poorly on this section with only about 10% of candidates scoring well. The key to this question is to understand the relationship between asset value volatility and stock price volatility implied by Merton Asset Model. Partial credit was given for any calculations completed correctly.

$$\sigma_E = \sigma_A \frac{E}{A} N(d_1)$$

where: σ_A = Each analyst's assumed assert volatility
= 8.7% from analyst one
= 7.0% from analyst two
= 10.0% from analyst three

E = Equity value = \$19/share * 1 million share = \$19 million

A = Asset value
= Equity value + Estimated market value of zero-coupon bond
= 19 + 79.7 = 98.7 for analyst one
= 19 + 81.5 = 100.5 for analyst two
= 19 + 75.0 = 94.0 for analyst three

4. Continued

$$d_1 = \frac{\ln\frac{A}{F} + \left(r + \frac{\sigma_A^2}{2}\right)T}{\sigma_A\sqrt{T}}$$, F = 100, r = 2%, T = 9, σ_A varies by analyst's estimate

$d_1 = 0.77$, $N(d_1) = 0.78$ for analyst one
$d_1 = 0.99$, $N(d_1) = 0.84$ for analyst two
$d_1 = 0.54$, $N(d_1) = 0.71$ for analyst three

σ_E = 35.2% for analyst one
σ_E = 31.0% for analyst two
σ_E = 35.1% for analyst three

(b) Calculate each analyst's implied estimate of DeF's equity value using the Merton Asset Value Model.

Commentary on Question:
Generally, candidates performed poorly on this section with only about 10% of candidates scoring well. This section is a straightforward extension of Part (a) wherein the firm's equity is a call option under Merton Asset Value Model. Partial credit is given for each calculation completed correctly.

Implied estimate of DeF's equity value = $C = AN(d_1) - e^{-rT}FN(d_2)$
where all variables are defined/calculated in Part (a), except d_2
$d_2 = d_1 - \sigma_A\sqrt{T}$
$d_2 = 0.51$, $N(d_2) = 0.69$
$d_2 = 0.78$, $N(d_2) = 0.78$
$d_2 = 0.24$, $N(d_2) = 0.60$

Implied equity value = 18.9 for analyst one
= 19.0 for analyst two
= 16.6 for analyst three

(c) Determine which analyst provided the best estimated market value of the zero-coupon bond that is consistent with the Merton Asset Value Model based on your calculations in (a) and (b).

Commentary on Question:
Generally, candidates performed poorly on this section with only about 10% of candidates scoring well. This is an extension of Part (a) and (b). Some candidates correctly described the criteria for best fulfilling the consistency requirement in the Merton Asset Value Model and thus got partial credit if they had not done parts (a) and (b) successfully.

4. **Continued**

Analyst One provided the best estimate as it nearly fulfills consistency requirement in Merton's Asset Value Model by having both its implied equity volatility (35.2%) and equity value ($18.9 million) closely matching actual equity volatility (35%) and equity value ($19 million), respectively. Analyst Two's implied equity volatility of 31% is lower than the market quote of 35%; Analyst Three's implied equity value of $16.6 million is lower than the market value of $19 million.

(d) Calculate DeF's LGD (as percent of EAD) based on Analyst One's estimated market value of the zero-coupon bond.

Commentary on Question:
Generally, candidates performed poorly on this section with only about 10% of candidates scoring well. This section tested candidates' knowledge of applying Merton Asset Value Model in credit risk analysis. Partial credit was given for any calculations completed correctly. Surprisingly many candidates did not calculate the asset value correctly which was a simple sum of two values provided in the table.

The risk neutral probability of default, *PD*, is given by
$PD = N(-d2) = 1 - N(d_1) = 1 - 0.77 = 0.23$

$\text{Market Value of Zero-Coupon Bond} = (1 - PD * LGD) * F * e^{-rT}$

$79.7 = (1 - 0.23*LGD) * 100 * e^{-2\%*9}$

$LGD = 21.6\%$

5. **Learning Objectives:**

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2i) Demonstrate an understanding of mortgage default models in the valuation of MBS.

Sources:

Portfolio Models including structured credit- Caouette ch. 24

Commentary on Question:

This question tests candidates understanding of securitization, how the process works, parties involved and the financial impact on the balance sheet

Solution:

(a) Explain the economic rationale for securitization as it applies to JPS.

Commentary on Question:

Candidates did relatively well on this section. Some candidates only provided a list however the question asked for an explanation and thus only got some credit.

Increased liquidity: JPS is able to convert illiquid cash flow to liquid assets. In this case, it would be converting the mortgage receivables into cash.
More efficient use of capital: JPS can create loans with the intention of selling them in order to raise additional cash flow to further create more loans. They are able to do so without increasing their leverage or debt-equity ratio.
Regulatory capital arbitrage: JPS will able to lower their capital charges by removing these illiquid assets off their balance to independent entities such as a special purpose vehicle.

(b) Calculate the maximum amount that JPS can raise through traditional debt funding after considering the sale of its Mortgages.

Commentary on Question:

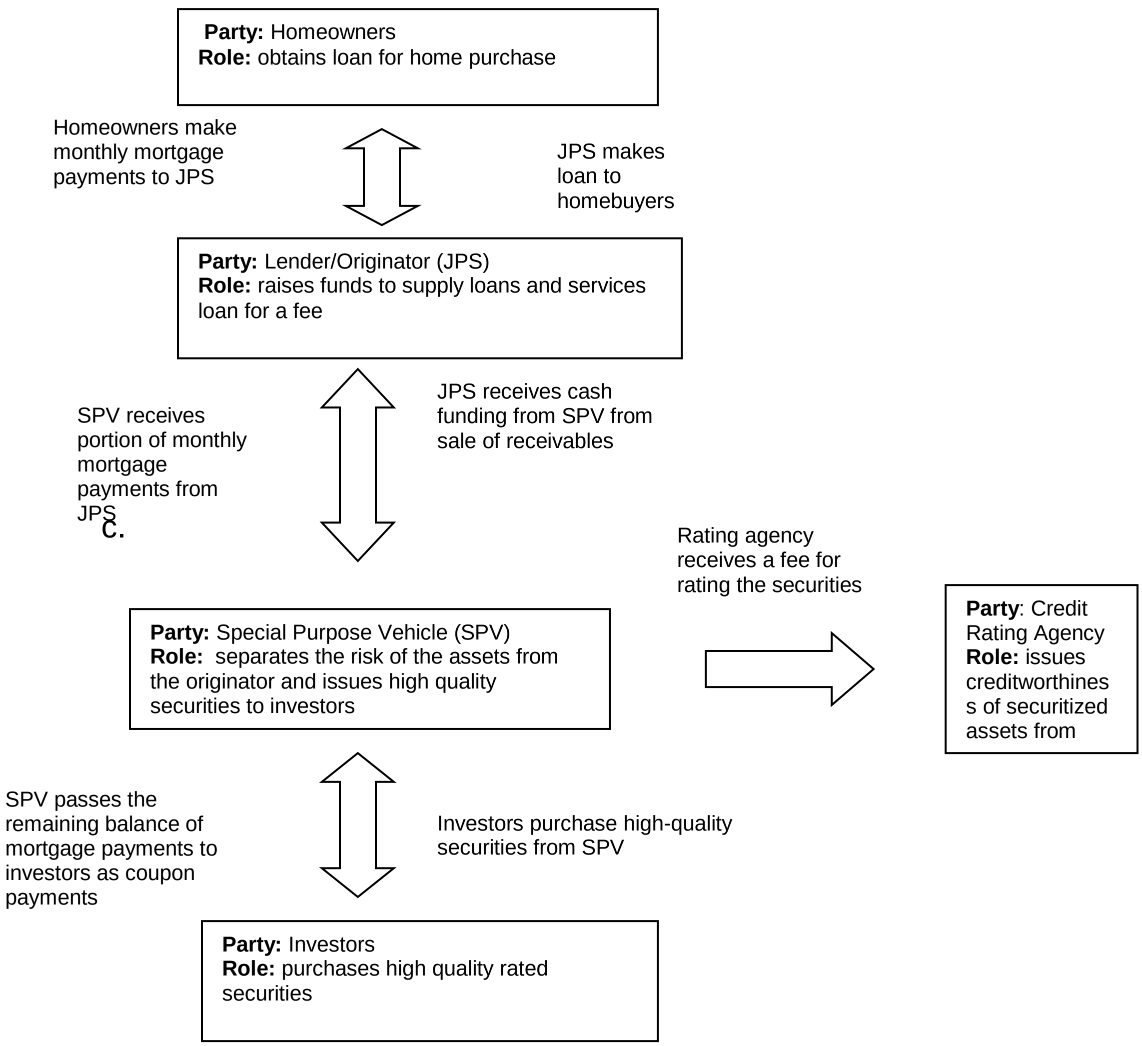
Candidates did relatively well on this section. The most common mistake was not realizing that the shareholder equity increases to $220 after the sale of the mortgages.

- Upon securitization shareholder equity increases to $220,liabilities remain at 25
- Debt can be issued such that the debt equity ratio does not exceed 0.5
- Issuing 85 raises liabilities to 110 and equity to 220 hitting the debt equity maximum
- Increase cash to $220M

5. Continued

(c) Construct a flow chart describing the mortgage securitization process showing each party involved, their roles, and the flow of the mortgage payments between them.

Commentary on Question:
Candidates did relatively well on this question. Most candidates were able to name the parties; however, some did not name the role. Most candidates were also able to identify the cash flow. If candidates answered Swap Counterparty or Servicer instead of Credit Rating Agency, full marks were awarded.



6. **Learning Objectives:**

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

(3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.

(3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.

Sources:

Quantitative Credit Portfolio Management, Chapter 5

Commentary on Question:

This question tested the candidates' knowledge of the nature and measurement of liquidity risk in assets and in the market.

Solution:

(a) Calculate the minimum bid price such that you do not violate your company's investment policy.

Commentary on Question:

The candidates performed well on this section. Most candidates were able to correctly calculate the LCS using the spread quoted formula and then able to use that result in the price quoted formula to arrive at the final answer. A common mistake was incorrectly remembering the price quoted formula.

LCS = (Bid-Ask Spread) * Option-Adjusted Spread Duration, if bond is spread quoted.

LCS = (Ask Price – Bid Price) / Bid Price, if bond is price quoted.

LCS of given spread bond = 0.2 * 6.5 = 1.3%

Plugging this result into the price quoted formula results in:

0.013 = (200 – Bid) / Bid => Bid = 200/1.013 = 197.43

(b) Describe how LCS could be applied to non-quoted bonds.

6. Continued

Commentary on Question:
The candidates performed relatively poorly on this section. Most candidates were able to identify that an adjustment from the LCS values of a quoted bonds was needed to account for the relatively illiquidity of non-quoted bonds. However, many candidates did not correctly describe that the process of determining the LCS for a non-quoted bond was a regression model based upon certain attributes.

Use LCS values for quoted bonds to estimate an LCS model for non-quoted bonds

Attributes used as inputs to model:

- Trading volume (LCS is negatively related to trading volume)
- Amount outstanding (LCS is lower for larger size issues)
- Age (LCS for seasoned bonds higher)
- DTS or OAS (greater excess return volatility have higher LCS)

Running a multiple regression summarizes the relationship between a quoted bond's LCS and above attributes.

Adjustments to model are made to account for the incremental illiquidity of non-quoted because quoted bonds have an inherent liquidity advantage.

(c) Explain how a portfolio manager would use LCS to compare two or more asset classes, and to quantify macro changes in market liquidity over time.

Commentary on Question:
The candidates performed relatively well on this section. The candidates were able to correctly identify that a higher LCS meant that the asset class was comparatively less liquid. Some candidates were not able to identify the impact of macro changes on the overall liquidity.

The absolute difference in LCS is what is relevant to portfolio managers when comparing two asset classes. A higher LCS denotes an asset that is less liquid.

Aggregate LCD time series (using market value weights) is used to quantify macro changes in market liquidity over time.

(d) Describe 3 other uses of LCS, in addition to being a measure of liquidity of a bond.

Commentary on Question:
The candidates performed relatively well on this section. Candidates were generally able to identify at least some of the other uses. A common mistake was to state that the LCS would be useful in risk management without any explanation as to how.

6. **Continued**

- Construction of “liquid” tracking portfolios
- Identification of the liquidity cost embedded in credit spreads
- Execution strategies for buying or selling bonds
- Creation of liquid credit benchmarks

7. **Learning Objectives:**

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Farmland and Timber

Learning Outcomes:

(6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.

Sources:

V-C192-11: Commercial Real Estate Analysis & Investments by Geltner, Miller, Clayton and Eichholtz, Chapter 12, Market Value and Investment Value

Commentary on Question:

This question tests the understanding of considerations in the valuation of real estate property and an application of the theory in a transaction setting.

Solution:

(a) Define investment value with respect to real estate by differentiating it from market value.

Commentary on Question:

The candidates performed relatively well on this section. Most candidates did well to distinguish market value and investment value. Some candidates could have provided more details for investment value to receive full marks.

MV = expected price that asset can be sold in current market
MV is same for a given asset for all investors
IV = value to a particular owner who would hold it for a long time
IV is unique to each investor and investors differ in ability to generate and use cash flows from asset <second part of the statement generates full knowledge>

(b) Explain the key considerations that make the real estate asset market informationally inefficient.

7. **Continued**

Commentary on Question:
The candidates performed relatively poorly on this section. Many candidates were able to note real estate is an asset with infrequent transactions and high transaction costs but did not regularly comment on importance of appraisals. Very few commented on the lack of predictability of the value of real estate.

1. Random noise:
Property markets have infrequent, privately negotiated deals
Difficult to know at any given time the precise MV of any given asset, needs to be estimated
Can lead to risks for prospective parties:
- sometimes parties make mistakes in their valuation
- one party may have better information
Opportunity: reap benefits for research efforts more easily than more efficient markets
2. Predictability:
Prices move more slowly in response to news, OR Prices partially adjust in short or medium term in response to news
Thus future market movements are less predictable
Opportunity:
profit through market timing
Risks:
Markets not completely predictable
Transaction costs high
Random noise may offset any market timing gains

(c) Recommend and justify an appropriate market value for each property that will be distributed by the pricing service

Commentary on Question:
The candidates performed relatively poorly on this section. Many candidates identified the second most motivated buyer would establish the MV. However many of those did not apply the concept correctly by incorrectly including the sellers opinion as one of the two highest buyers. Those that didn't use the second most motivated buyer theory often averaged values to get a MV.

Second most motivated buyer used to establish MV
Maximum price this buyer would be willing to pay (the IV for the buyer) is taken to be the MV (TPP for both properties)
$12 million for Gardiner and $13 million for Clark

7. Continued

(d) Evaluate each possible transaction.

Commentary on Question:
The candidates performed relatively poorly on this section. Most candidates correctly identified that Kessel and Burke could transact on the Gardiner property. However many candidates discussed possible transactions that could occur but often contradicted themselves by suggesting that deals could occur on the same property at different prices. Almost all candidates failed to incorporate Kessel's $13m debt service needs which was key to eliminating a transaction with Lupul and TPP.

Many candidates did correctly recognize that a transaction on the Clark property would not occur.

when IV>MV, best to hold as a seller and buy as a buyer
When IV<MV, best to sell as a seller and not buy as a buyer
If the market value is taken as the answer to (c), then the following possible transaction types are possible:
None of the Clark transactions can occur since all of the IVs are below Kessel's IV. From the pricing service, Kessel knows the other IVs even though it does not know their source firms. Hence, Kessel would not sell this property and would instead concentrate on the Gardiner property
Both Lupul and TPP have IVs that exceed Kessel's IV for Gardiner but since they do not meet the target that Kessel has for its debt service (13 million) and since Kessel knows that there is an IV that does, 15 million, Kessel would not transact with these firms.
A deal can conceivably be struck between Kessel and Burke for the following reasons: • The market value is known to be 12 million (from the pricing service) and 13 million is needed to meet the debt service needs of Kessel • Because both of these exceed Kessel's IV for Gardiner, it would be willing to sell • Burke's IV for the property is in excess of both 12 million and 13 million and so it would be willing to purchase the property. • Under a conservative philosophy, no deal would be struck since Burke would not pay more than 12 million (the MV and therefore the only optimal deal) but Kessel would not sell since it couldn't make the debt service need • But under a liberal philosophy, a deal can be struck at 13 million since Kessel can meet it debt service need and Burke gets a good deal even though it is not the optimal deal.

8. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.

(1b) Understand and explain the terms Time Homogeneous Models, Affine Term Structure Models and Affine Coefficient models and explain their significance in the context of short rate interest models.

(1d) Explain the features of the Black-Karasinski model.

Sources:

Brigo, Ch. 3.1 – Introduction (One-factor short-rate models)

Brigo, Ch. 3.2 – Classical Time-Homogeneous Short-Rate Models

Brigo, Ch. 3.5 – The Black-Karasinski model

Commentary on Question:

This question tests the understanding of the development of short rate stochastic models and of the characteristics of individual short rate models. It also tests the ability of the candidate to discuss the practical use of short rate models.

Solution:

(a) Explain the characteristics of the time homogenous short-rate model and its major drawbacks.

Commentary on Question:

The candidates performed excellently on this section. The concept of time homogeneity was well understood and communicated by most candidates. Most candidates also correctly identified the main drawback. A few candidates did not mention a drawback or did not clearly explain the concepts above.

A model is time homogenous when the short rate dynamics depend only on constant (not a function of time) coefficients.

The major drawback of time homogenous models is that they produce an endogenous term structure of interest rates (they cannot reproduce satisfactorily the initial yield curve).

8. Continued

(b) Compare and contrast the Cox, Ingersoll and Ross (CIR) model and Vasicek model.

Commentary on Question:
The candidates performed well on this section. Most candidates described a sufficient amount of key similarities and differences between the two models. The candidates did not need to cover all characteristics to get full credit. Candidates that did not perform well missed key characteristics or did not provide clear description of them.

Both models are time homogenous
The Vasicek model can produce negative rates while the CIR model cannot
They both are mean reverting towards a long term average rate
They both are both one factor diffusion models of the short rate.
They both produce an affine term structure
They both are analytically tractable and can produce closed form solutions for bond prices
The diffusion coefficient of the CIR model is a function of the square root of the instantaneous short rate while it is constant in the Vasicek model.

(c) Calculate the variance of the short rate two years from now with the CIR model.

Commentary on Question:
The candidates performed well on this section. Most candidates were able to recognize the right formulas to be used to calculate the expected value under Vasicek and variance under CIR and to recognize that the parameter k needed to calculate the variance could be derived from the expected value under the Vasicek model. Common mistakes were 1) to confuse the time parameters for both calculations, 2) to incorrectly input the volatility and 3) computational mistakes.

Under the initial Vasicek model:
$E\{r(t) \mid F_s\} = r(s)e^{-k(t-s)} + \theta(1- e^{-k(t-s)})$
$E\{r(1) \mid r(0) = 0.03\} = 0.032, \theta = 0.04$
$k = -\ln(0.008 / 0.01)$
$k = 0.22314$

Under the Cox-Ingersoll-Ross model, using the same parameters
$Var\{r(t) \mid F_s\} = r(s)\sigma^2/k(e^{-k(t-s)} - e^{-2k(t-s)}) + \theta\sigma^2/2k(1- e^{-k(t-s)})^2$
$k = 0.22314, \theta = 0.04, \sigma^2 = 0.25$
$Var\{r(2) \mid r(0) = 0.03\} = 0.010648$

(d) Assess the suitability of the Black Karasinski model for the sensitivity testing and recommend whether it should be used instead of the CIR model.

8. **Continued**

Commentary on Question:
The candidates performed relatively well on this section. Most candidates identified a few advantages and disadvantages of the BK model. Both models could be recommended to receive full credit as long as the recommendation was well supported by the advantages and disadvantages. While most candidates did come up with a recommendation, it was often not well supported or connected to the specific characteristics of the BK model and even less to the needs of testing the movements of interest rates. A minority of candidates were heavily penalized for not giving a recommendation.

Positive features of the Black-Karasinski model:

- Exogenous term structure of interest rates (can reproduce satisfactorily the initial yield curve)
- Time varying parameter (non-time-homogenous)
- Implies a lognormal distribution of the short-rate process at each time
- Can be fitted to the term structure of spot or forward-rate volatilities
- No negative rates
- Good fitting quality to market data

Negative features of the Black-Karasinski model:

- No affine term structure (no analytical formulas for bonds are available)
- Infinite expectation (explosion problem, future value of a money market account is infinite)
- Not analytically tractable
- Calibration is burdensome

Both options can be recommended as long as it is appropriately supported by accurate features of the Black Karasinski model and that it considers those features in the context of testing interest rate sensitivity.

9. **Learning Objectives:**

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1c) Explain the dynamics of and motivation for the Hull-White extension of the Vasicek model.

(1f) Explain how deterministic shifts can be used to fit any given interest rate term structure and demonstrate an understanding of the CIR++ model.

(1p) Describe and contrast several approaches for modeling smiles, including: Stochastic Volatility, local-volatility, jump-diffusions, variance-gamma and mixture models.

Sources:

Brigo, D and Mecurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Chapters 3.8 , 3.9

Rebonato Ch. 8.1, 8.2

Kling, A., Ruez, F, and Russ, Rochen, The Impact of Stochastic Volatility on Pricing, Hedging and Hedge Efficiency of Withdrawal Benefit Guarantees in Variable Annuities, ASTIN Bulletin 41(2), 511-545, 2011

Commentary on Question:

Part (a) asks the candidate to define the Jump Diffusion CIR model and demonstrate the understanding of affine model. Part (b) tests the candidate's understanding of the CIR ++ model. Part (c) asks the candidate to apply their knowledge of Jump Diffusion Heston model, Extended Vasicek model and the CIR++ to pick the appropriate models for three different jobs.

Solution:

(a) Define a Jump Diffusion CIR model and identify whether this is an affine model.

Commentary on Question:

This is relatively simple textbook recall work. Candidates generally did well on this part.

$dr_t = k(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t + dJt,$

Where J is a jump process with jump arrival rate α >0 and jump size distribution Π on R+, mean reversion to Θ at rate k, volatility term σ

9. Continued

This model is an affine model, in that the bond price formula maintains the familiar log-affine shape.

(b) Critique your colleague's conclusion.

Commentary on Question:
Candidates generally did relatively poorly on this part. Most candidates did not understand the concept of monotonic decreasing vs. monotonic increasing function.

h = $\sqrt{(k^2 + 2\sigma^2}$ = 0.234521
X(0) = 0.03
ϴ h/ k = 2.3452
X(0) < ϴ so it cannot be the 3rd case => f is not monotonically decreasing and supremum not equal to x(0)
X(0) < ϴ h/ k => belongs to the first case => f is monotonically increasing with supremum = 2kϴ/(h+k) = 0.59787

(c) Recommend the best model for each of the following tasks and explain your choice.
(i) pricing equity options
(ii) modeling short term interest rates with negative interest rates allowed
(iii) modeling swaptions

Commentary on Question:
Candidates generally did relatively well on this part. Most candidates understood the important features of interest rate models involved in this question. An error made by some candidates was to mix up i) and iii).

(i) Equity Option modeling – Heston model with jump diffusion model
- Capture smile for both short and intermediate maturities, jump diffusion captures the short and the Heston model captures the intermediate maturities.
- Heston model is a CIR model
- It is a CIR model reverting to long run mean
- Heston model always produce positive rates
- Exact fit term structure

9. Continued

(ii) Model short term interest with negative interest allowed – use Extended Vasicek

- Both CIR ++ and Heston do not allow negative interest rate (rejected)
- Extended Vasicek incorporates mean reversion, arbitrage free

(iii) Model Swaptions – use CIR ++

- Exact fit of any observed term structure
- Analytical formulas for bond prices, bond-option prices, swaptions and caps prices
- The distribution of the instantaneous spot rate has tails that are fatter than in the Gaussian case and, through restriction on the parameters,
- Allows modeling of imperfect correlation between rates of different maturities
- Possible to model humped volatility surface
- It is always possible to guarantee positive rates without worsening the volatility
- Calibration in most situations.

10. **Learning Objectives:**

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

(3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.

(3f) Apply liquidity scenario analysis with various time horizons.

Sources:

Hyun Song Shin, Reflections on Modern Bank Runs: A Case Study of Northern Rock

Commentary on Question:

This question aims to test the candidate's understanding of liquidity risk in the context of the Northern Rock case study, the understanding of the various stress-testing methods available for liquidity risk stress testing, and the recommendation of a suitable method.

Solution:

(a) Critique your colleague's position.

Commentary on Question:

The candidates performed well on this section. Most candidates were able to identify that securitization was not the major factor, and the fact that Northern Rock had heavily relied on institution investors for funding, and eventually went into trouble when these funding sources dried up. Sometimes the answers below were provided by the candidate in part (b) to which we gave credit for.

- Colleague is incorrect. Securitization was not a major factor
- High Leverage coupled with reliance on institutional investors
- Deleveraging of credit market shrunk number of institutional investors or pressures on creditors led to crisis
- Institutional investors were short and medium term liabilities (very short-term funding < 1yr) (pg 12) and thus even more susceptible to non-renewal than traditional branch based deposits which tend to be sticky

(b) Explain the market conditions and balance sheet positions that contributed to the "run on the bank" at Northern Rock.

Commentary on Question:

The candidates performed relatively well on this section. For the part on "Market Conditions", most candidates were able to identify the liquidity crisis in 2007, and that the short-term funding from institutional investors dried up because of their investment constraints.

10. Continued

Most candidates were not able to address the fact that prior to the crisis, the investors had high leverage, but low value-at-risk. For the part on "Balance Sheet Positions", some candidates were able to note the fact that Northern Rock had a mismatch of illiquid long-term assets to short-term liabilities. Relatively few candidates were able to identify the other balance-sheet positions unique to Northern Rock.

Market Conditions:

- Credit Crisis in 2007
    - Short-term funding and interbank lending froze
    - General rising reliance of banks to use asset-backed paper to fund longer liabilities
- Prior to crisis, leverage of institutional investors was high and balance sheets were large and value at risk is low

Balance Sheet Position:

- Northern Rock was highly leveraged in 2007 and held long term illiquid assets funded by short-term liabilities
- Retail deposits a small proportion of total liabilities (23%)
    - Small proportion of retail deposits are branch based (traditional), most were postal and telephone accounts
- Significant growth in Securitized notes and other longer term liabilities.
- Securitized notes of medium to long term (over 1 yr)
    - SPE were kept on balance sheet in the UK, versus in the US where they went off balance sheet
- Used similar funding methods as SIVs and conduits aimed at institutional investors (much less than 1 yr)

(c) Compare and contrast the three methods above in the context of liquidity risk stress testing.

Commentary on Question:
The candidates performed relatively poorly on this section. Most candidates were able to list some features for each stress-testing method. However, most candidates did not directly address each method's appropriateness for liquidity stress testing.

Historical Value-at-Risk

- Simple and easy to apply and explain
- Liquidity events often not normally distributed, but follow a more fat-tailed distribution
- Black Swan problem – extreme scenarios are often not experienced in history

10. Continued

- Poor tool for liquidity risk - historical events are not approximately reflective of future events

Deterministic Modeling

- Will test shocks that have never occurred or did not occur with enough frequency or severity in historical data
- Single scenarios evaluated at multiple stress levels
- Provides no information as to probability of loss; provides only severity
- Advantage - helps identify most important vulnerabilities
- Disadvantage - inherently subjective
- Not appropriate for liquidity testing, but probably most appropriate of the 3 methods

Monte Carlo

- Provides both information as to probability of loss and severity
- Requires a starting state and parameterization (mean reversion and volatility), but no effective means to obtain them – introduces Black Swan problem again
- Not appropriate for Liquidity testing

(d) Assess each method's ability to measure Northern Rock's liquidity risk.

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were able identify that historical VAR is the least appropriate method. However, most candidates could not identify that deterministic scenarios would have been more appropriate than stochastic ones, as they can be subjectively manipulated to include extreme stress scenarios.

- Historical VAR, which needs historical experience, only looks at the past 8 years because prior to that, Northern Rock was primarily relying on retail deposits. Regardless, there were no significant credit events in that period, thus Historical VAR could not have anticipated the liquidity problem.
- Deterministic scenarios could have been used to identify the magnitude of the liquidity event only if the stress chosen to remove the ability to renew short term funding. This is better than nothing; however this would still have missed the probability of the occurrence of that event.
- Stochastic modeling would not be as good as deterministic because the historical experience would likely have seeded the parameters and therefore, a total collapse of short-term funding likely wouldn't have been identified as a risk.

11. **Learning Objectives:**

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1p) Describe and contrast several approaches for modeling smiles, including: Stochastic Volatility, local-volatility, jump-diffusions, variance-gamma and mixture models.

Sources:

Rebonato, R., Volatility Correlation – The Perfect Hedger and the Fox, Second Edition, 2004, Sections 8.1-8.5

Commentary on Question:

This question compares different models used to model volatility smiles for equity prices.

Solution:

(a) Describe a fully stochastic volatility model and a local-volatility model.

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were able to identify the main differences between the two models. Common mistakes were to write down the model equations without any explanation and others gave examples that were wrong.

In a fully stochastic volatility model, the stock price has two sources of randomness; one associated with the stock price, and one associated with the volatility term, which is itself stochastic. In a local volatility model, the volatility term is a deterministic function of time and of the stock price, and thus there is only one source of randomness (the one associated with the stock price).

(b) List the desirable features of a local volatility model.

Commentary on Question:
The candidates performed relatively well on this section. The candidates who performed well recognized that the local volatility model allows for a complete market. Some candidates' answers were too general and did not pertain to the local volatility model only.

11. Continued

Under a local volatility model, the market is complete, so any traded asset can be replicated with the underlying and the bond price. Thus, any option can be replicated with such a portfolio, which allows for the recovery of a unique non-arbitrage price for any admissible option. This allows us to recover exactly the prices of an exogenous set of admissible options.

(c) List the pros and cons of Jump-Diffusion models with a large but finite number of jump amplitudes.

Commentary on Question:
The candidates performed relatively poorly on this section. The ones who did well mentioned that the model performs well in the short-term, but fails to recover the smile in the long term. Some candidates appeared to misunderstand that the jumps in the model applied to the stock price and not the volatility. Most candidates failed to answer the question specifically for a model with a finite number of jumps which was required for full credit.

Pros: The main advantage of having a model with a finite number of jump amplitudes is that the market can be completed using traded options (the same number of options as the number of jump amplitudes). This simplifies the pricing process. In addition, jump diffusion models can capture the volatility smile for short maturities.
Cons: Even if the finite number of jump amplitude allows for a complete market, many options are needed to price a new option, so the model does not add much explanatory power over the information provided by the market. Also, for medium and long-term horizons, jumps are not sufficient to model the smile, because it tends to decay too rapidly under jump-diffusion models.

(d) Recommend which of the above models is most appropriate to use and support your recommendation.

Commentary on Question:
The candidates performed relatively well on this section. The candidates who did well recognized that a mixed model combines the advantages of stochastic volatility and jumps. Some candidates did not explain their choice well, or mentioned that the models fixed the short-comings of each separate model, without identifying those short-comings. Some candidates recommended a different model and partial credit was given if they provided a good support for their recommendation..

I recommend using a mixed model that combines jumps and stochastic volatility. By itself, a stochastic volatility model fails to recover the smile for short maturities. The opposite is true for jump diffusion models; they can replicate the smile well for short maturities, but it decays too rapidly.

11. Continued

In addition, using a model with a large but finite number of jump amplitude has a low explanatory power. Mixing jumps and stochastic volatility allows to capture the volatility smile appropriately for all maturities. Such a model has a large number of parameters to calibrate, but this task can be made easier with pragmatic calibration.

12. Learning Objectives:

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

(4h) Describe and assess performance measurement methodologies for assets, liability and hedge portfolios.

(4i) Describe and assess techniques that can be used to select or build a benchmark for a given asset, portfolio.

(4l) Explain the limitations of attribution techniques.

Sources:

Fabozzi, Chapter 69

Fabozzi, Chapter 70

Commentary on Question:

This question tests the understanding of different performance attribution models and the applicability of those models under specific situations.

Solution:

(a) Describe the three requirements for successful performance attribution.

Commentary on Question:

The candidates performed excellently on this section. Most candidates were able to properly identify the three requirements and given reasonable descriptions of them.

Additivity – Sum of two or more agents equal the sum of the contributions from the agents

Completeness – sum of all outperformance is equal to the total portfolio outperformance

Fairness – Allocation of outperformance is performed in a way perceived to be fair by all agents.

(b) Describe the general properties of each of the following performance attribution models:

(i) Total Return Model

(ii) Excess Return Model (versus yield curve and volatility)

12. **Continued**

(iii) Fully Analytical Model (based on top level and sector level)

Commentary on Question:
The candidates performed excellently on this section. Most candidates were successful in distinguishing the key features of the 3 models.

(i) Total Return: Simplest model that compares porfolio return to benchamrk. Benchmark return based upon sector allocation weight and return for each sector. Outperformance versus benchmark is attributed to portfolio's securiy selection within each sector.

(ii) Excess Return Model: Calculates returns versus benchmark portfolio which has equal volatility and yield curve. Any deviation in return versus benchmark can be attributed to over/under performance. (i.e. excess return after normalizing for volatility and yield curve).

(iii) Fully Analytical Model: Most comprehensive model which allocates outperformance based upon different factors (not just market value weight by sector, but perhaps other factors such as volatility, duration, spread, etc). Can be used with a portfolio which has different asset classes.

(c) Recommend and justify which performance attribution model is most appropriate for this portfolio.

Commentary on Question:
The candidates performed well on this section. Most candidates were able to identify the total return model as the most appropriate model for this situation. Some candidates recommended the other two models but neither was appropriate for this particular situation given the constraints.

Total Return model is most appropriate since it measures return based upon sector selection and then security selection within each sector. The required model in this situation does not need to cosinder conditions such as yield curve and volatility.

(d) Recommend and justify which performance attribution model is most appropriate, given this information.

Commentary on Question:
The candidates performed well on this section. Most candidates were able to identify the fully analyitcal model since it best allows for multiple asset classes. Some candidates recommended the Excess Return Model which is inferior in this situation and therefore did not receive credit.

12. Continued

Fully Analytical Model is most appropriate since it is the best model to use when multiple asset classes need to be considered. Fully analytical model allows for performance to be measured based upon market risk factors as well as market weight factors.

13. **Learning Objectives:**

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

(4a) Demonstrate an understanding of the mathematical considerations for analyzing financial time series.

(4b) Understand and apply various techniques for analyzing conditional heteroscedastic models including ARCH and GARCH.

Sources:

Tsay, Analysis of Financial Time Series, 3rd edition: Chapter 3 "Conditional Heteroscedastic Models" (through 3.8)

Commentary on Question:

This question tests the candidate's understanding of the qualitative and quantitative properties of the GARCH-M model.

Solution:

(a) Interpret each of the parameters above.

Commentary on Question:

The candidates performed well on this section. A common mistake is stating that α_0 is the long run volatility/variance.

μ is the mean rate of return or the drift, c is the risk premium, α_0 is the constant intercept or the deterministic part of the variance, α_1 is the weight on the last innovation, and β_1 is the decay rate or the weight on the lagged variance.

(b) Calculate the long-term stationary volatility.

Commentary on Question:

The candidates performed relatively well on this section. However, many candidates calculated variance instead of volatility.

Unconditional variance is $\alpha_0/(1-\alpha_1-\beta_1)$ = 0.0022/(1- 0.129 – 0.459) = 0.00534.

Therefore, the long-run volatility is $\sqrt{0.00534} = 0.073$.

The unconditional variance formula is derived as follows:

$\mathrm{E}[\sigma_t^2] = \alpha_0 + \alpha_1\mathrm{E}[\sigma_{t-1}^2] + \beta_1\mathrm{E}[\sigma_{t-1}^2] = \alpha_0 + \alpha_1\mathrm{E}[\sigma_t^2] + \beta_1\mathrm{E}[\sigma_t^2]$

The last equality is because that the variance is unconditional. So $\mathrm{E}[\sigma_t^2] = \alpha_0/(1-\alpha_1-\beta_1)$.

13. Continued

(c) Calculate the covariance between σ_t^2 and r_{t-1}.

Commentary on Question:
The candidates did relatively poorly on this section. A number of candidates knew the answer but failed to provide the proof.

Using properties of covariance and the fact that in the model σ_{t-1} and ε_{t-1} are independent. For example:

$$\text{cov}(r_{t-1}, \sigma_t^2) = \text{cov}(\sigma_{t-1}\varepsilon_{t-1}, \sigma_t^2) = \alpha_1 \text{cov}(\sigma_{t-1}\varepsilon_{t-1}, \sigma_{t-1}^2\varepsilon_{t-1}^2) + \beta_1 \text{cov}(\sigma_{t-1}\varepsilon_{t-1}, \sigma_{t-1}^2)$$

$$\text{cov}(\sigma_{t-1}\varepsilon_{t-1}, \sigma_{t-1}^2\varepsilon_{t-1}^2) = E[\sigma_{t-1}^3\varepsilon_{t-1}^3] - E[\sigma_{t-1}\varepsilon_{t-1}]E[\sigma_{t-1}^2\varepsilon_{t-1}^2]$$
$$= E[\varepsilon_{t-1}^3]E[\sigma_{t-1}^3] - E[\sigma_{t-1}]E[\varepsilon_{t-1}]E[\sigma_{t-1}^2\varepsilon_{t-1}^2] = 0$$
$$\text{cov}(\sigma_{t-1}\varepsilon_{t-1}, \sigma_{t-1}^2) = E[\varepsilon_{t-1}\sigma_{t-1}^3] - E[\sigma_{t-1}\varepsilon_{t-1}]E[\sigma_{t-1}^2] = E[\varepsilon_{t-1}](E[\sigma_{t-1}^3] - E[\sigma_{t-1}]E[\sigma_{t-1}^2]) = 0$$

(d) Identify a potential limitation of your simulation, based on the answer to (c).

Commentary on Question:
The candidates did relatively poorly on this section. Very few candidates compared the property of the model to the empirical results.

The zero covariance assumption is not realistic. Empirical evidence shows there is a negative correlation between return and volatility. Large negative shocks tend to increase volatility.

(e) Recommend a way to avoid the limitation in (d).

Commentary on Question:
The candidates did relatively poorly on this section as the answer depends on the candidate's understandings in the sections leading to this question.

EGARCH, NGARCH, GARCH-M (with c<>0) or any stochastic volatility model with non-zero correlation between volatility and asset returns.

14. **Learning Objectives:**

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

(4c) Understand and apply various techniques for analyzing multivariate time series.

(4d) Understand the concept of cross correlation in multivariate time series.

(4e) Understand various vector auto regressive models.

Sources:

Tsay, Analysis of Financial Time Series, 3rd edition

Ch 8, Multivariate Time Series Analysis and its Application

Commentary on Question:

The question tests the candidates knowledge of techniques for analyzing multivariate time series and vector auto regressive models. It tests the conditions for unit root nonstationary models in multivariate time series. In addition, the question tests the Error Correction Model and its interpretation under different scenarios.

Solution:

(a) Write down the AR polynomial matrix $\phi(B)$ for model M and calculate the values for $\phi(1)$.

Commentary on Question:

The candidates performed poorly on this section. Candidates were unable to determine the AR polynomial matrix. However, for those that did, they were able to substitute 1 for B and arrive at the correct answer.

The AR polynomial: $\begin{bmatrix} 1 - B + .2B^2 & -.5B + .3B^2 \\ 0.3B - .6B^2 & 1 - 1.8B + 1.1B^2 \end{bmatrix}$

Where B is the backward shift operator. Substitute B=1, and you arrive at:

$$\phi(1) = \begin{bmatrix} 0.2 & -0.2 \\ -0.3 & 0.3 \end{bmatrix}$$

(b) Show that V_{1t} and V_{2t} are unit-root non-stationary under Model M.

14. Continued

Commentary on Question:
The candidates performed poorly on this section. Candidates struggled to derive the determinant of the AR polynomial which was necessary to answer the question. In addition, candidates failed to recognize the condition necessary to be considered unit-root nonstationary.

To calculate the determinant:
(1-Φ11(B)) (1-Φ22(B)) - Φ12(B)*Φ21(B)
$= 1 - 2.8B + 3.25B^2 - 1.85B^3 + 0.4B^4$
Since $|\phi(1)| = 1-2.8+3.25-1.85+0.4 = 0$, the determinant contains the Factor (1-B).
Therefore, the marginal models of V_{1t} and V_{2t} are unit-root nonstationary.

(c) Write down an Error-Correction Model under Model M.

Commentary on Question:
The candidates performed poorly on this section. Some candidates were able to identify the formula for an error correction representation. Most candidates, however, were not able to plug the AR model correctly into the formula.

$$\Delta x_t = \alpha\beta' x_{t-1} + \sum_{i=1}^{p-1} \Phi_i^* \Delta x_{t-i} + a_t - \sum_{j=1}^{q} \Theta_j a_{t-j}, \qquad (8.33)$$

$$\Phi_j^* = -\sum_{i=j+1}^{p} \Phi_i, \qquad j = 1, \ldots, p-1,$$

$$\alpha\beta' = \Phi_p + \Phi_{p-1} + \cdots + \Phi_1 - I = -\Phi(1). \qquad (8.34)$$

$$\Delta V_t = \begin{bmatrix} -0.2 & 0.2 \\ 0.3 & -0.3 \end{bmatrix} V_{t-1} + \begin{bmatrix} 0.2 & 0.3 \\ -0.6 & 1.1 \end{bmatrix} \Delta V_{t-1} + a_t$$

(d) Determine $E[\Delta s_t \mid Y_{t-1}]$ and $E[\Delta d_t \mid Y_{t-1}]$ under Model K

Commentary on Question:
The candidates performed well on this section. Candidates were able to correctly substitute into the expected value equation and recognize that ϵ_{st} has an expected value of zero.

14. **Continued**

$E[\Delta s_t|Y_{t-1}] = c_s + 0.75(d_{t-1} - s_{t-1} - \mu)$
$E[\Delta d_t|Y_{t-1}] = c_d$

(e) Explain what the Model K predicts under the following 2 situations:

Case 1: $d_{t-1} - s_{t-1} - \mu = 0$
Case 2: $d_{t-1} - s_{t-1} - \mu > 0$

Commentary on Question:
The candidates performed relatively poorly on this section. Many candidates were able to recognize that case 1 was in equilibrium and so no adjustment was necessary. However, most candidates failed to mention that case 2 meant the stock price had to grow faster to restore equilibrium.

$d_{t-1} - s_{t-1} - \mu = 0$. Then $E[\Delta s_t | Y_{t-1}] = c_s$ and $E[\Delta d_t|Y_{t-1}] = c_d$ on average so that c_s and c_d represent that the growth rates of stock prices and dividends are in long-run equilibrium. There is no expected adjustment since the model was in long run equilibrium in the previous period.

$d_{t-1} - s_{t-1} - \mu > 0$. Then $E[\Delta s_t | Y_{t-1}] = c_s + 0.75(d_{t-1} - s_{t-1} - \mu) > c_s$ on average. Under this situation the dividend yield has increased above its long-run mean (positive disequilibrium error) and the ECM predicts that s_t will grow faster than its long-run rate to restore the dividend yield to its long run mean. Notice that the magnitude of the adjustment coefficient $\alpha_s = 0.75$ controls the speed at which s_t responds to the disequilibrium error.

15. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2c) Demonstrate an understanding of credit valuation models.

(2k) Understand and apply various approaches for managing credit risk in a portfolio setting.

Sources:

Managing Credit Risk by Caouette, chapter 20

Commentary on Question:

This question tests the candidate's understanding of Market and Credit Risk, as well as their ability to use the CreditMetrics model to quantify credit risk.

Solution:

(a) Compare and contrast marking to market for market risk to marking to market for credit risk.

Commentary on Question:

Candidates performed relatively poorly on this section. Many candidates were successful describing market risk and / or credit risk. However, to receive full marks on this question, candidates also needed to contrast between the two. Many candidates failed to include contrasts in their responses.

Market risk is the risk that the value of an asset will drop due to change in interest rates.
Example of market risk: when you are using short-term money to finance a long-term fixed rate mortgage and the funding rate goes up.
Credit risk is the risk that a borrower's ability to pay will diminish or disappear altogether.
Marking to market for market risk is more straightforward because the market prices are available every day.
Marking to market for credit risk is more difficult because it is very subjective.
Examples of subjectivity: it is hard to detect changes in credit quality and even harder to incorporate default correlations.

(b) Calculate for each possible year end rating.

(i) The probability weighted value

(ii) The difference of value from mean

(iii) The probability weighted difference squared

15. Continued

Commentary on Question:
Candidates performed well on this section. The most common omission was to not list the probability weighted value (for example) for each possible year end rating separately but instead just show the total for the three which is actually the mean. Candidates who did this were penalized; however the penalty was relatively minor.

Probability weighted value = Probability Weighted of State * New Bond Value plus Coupon
Probability weighted value A = 1% * 105
Probability weighted value A = 1.05
Probability weighted value B = 98.5% * 100
Probability weighted value B = 98.5
Probability weighted value C = 0.5% * 95
Probability weighted value C = 0.475

Mean = Probability weighted value A + Probability weighted value B + Probability weighted value C
Mean = 1.05 + 98.5 + 0.475
Mean = 100.025

Difference of value from mean A = 105 – 100.025
Difference of value from mean A = 4.975
Difference of value from mean B = 100 – 100.025
Difference of value from mean B = -0.025
Difference of value from mean C = 95 – 100.025
Difference of value from mean C = -5.025

Probability weighted difference squared A = 4.975^2 * 1%
Probability weighted difference squared A = 0.2475
Probability weighted difference squared B = 0.025^2 * 98.5%
Probability weighted difference squared B = 0.0006
Probability weighted difference squared B C = 5.025^2 * 0.5%
Probability weighted difference squared B C = 0.1263

(c) Identify and compare the two approaches used by CreditMetrics to describe the volatility of an asset.

Commentary on Question:
Candidates performed poorly on this section. The question asked about volatility of a single asset, however many candidates responses focused on correlation which is how CreditMetrics' handles defaults of two or more assets. As a result, no partial marks were given for comments related to correlation, even if the comments were valid.

15. **Continued**

The two methods CreditMetrics uses to describe the volatility of the value are the standard deviation and the "1 percent value".
Standard deviation is not very useful when the variable is asymmetrically distributed.
Standard deviation is simple and efficient to calculate.
The "1 percent value" is the lowest value that the portfolio will achieve 1% of the time.

(d) Determine the 1 percent value based on the information calculated in part (b).

Commentary on Question:
Candidates performed poorly on this section. Many candidates calculated the CTE99 value, assuming a normal distribution instead of the provided distribution. Partial marks were given for this approach.

Start at the worst possible outcome and work your way up.
There is only a 0.5% chance that the rating will be a C, so that is not the 1st percentile.
There is a 99% (=98.5% + 0.5%) chance the rating will be a B or below.
Since 99% > 1%, this is the 1st percentile.
This corresponds to a value of 100.
Therefore, the 1% value is the mean minus 100 = 100.025 – 100 = 0.025.

16. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1k) Define and explain the concept of volatility smile and some arguments for its existence.

(1m) Compare and contrast "floating" and "sticky smiles.

(1o) Identify several stylized empirical facts about smiles in a variety of options markets.

Sources:

Rebonato, R., Volatility Correlation – The Perfect Hedger and the Fox, 2nd edition, 2004 (Ch.6 and 7)

Commentary on Question:

This question tests the understanding of the empirical facts on volatility smiles, the behaviors of volatility smiles, and the reasonability of measure of moneyness with existence of volatility smiles.

Solution:

(a) List four stylized facts about the volatility smile for the S&P 500 Index.

Commentary on Question:

The candidates performed well on this section. This section required a straightforward memorization and most candidates accurately recalled 75%+ of the facts. A common mistake was the inaccuracy of the facts, which showed that candidates memorized certain phrases but did not fully understand the facts.

- Smiles have greatly increased in magnitude after the 1987 equity market crash.
- The magnitude of the smile as a function of a fixed money strike tends to decrease for increasing option expires: short maturities display pronounced smiles, and distant maturities give rise to shallow smiles.
- The magnitude of the smile as a function of the degree of out-of-moneyness is much more constant across different option expires.
- The smile is much more pronounced going from the ATM level towards out-of-the money puts than in the opposite direction. Going towards out-of-the money calls the smile either becomes less steep or is monotonically deceasing or sometimes it is even absent.
- The asymmetry in the smile tends to increase during periods of market turbulence.

16. **Continued**

(b) Explain the following two volatility smile behaviours:

(i) Floating smile

(ii) Sticky smile

Commentary on Question:
The candidates performed excellently on this section. Most candidates fully understood the distinction between floating smile and sticky smile.

(i) Floating smile: volatility depends on the underlying.

- When both the underlying and the strike move from S and K to S $(1+\Delta)$ and K $(1+\Delta)$ respectively, the ratio of the stock price to the strike would not change, the implied volatility would be the same, the price of a call would simply be multiplied by $(1 + \Delta)$.

(ii) Sticky smile: volatility does not depend on the underlying.

- K-strike option would always have the same implied volatility irrespective of what the underlying moved, i.e. irrespective of the degree of the in-the-moneyness of the option itself.

(c) Interpret the relationship between the values of the underlying index and the volatilities under these three regimes.

Commentary on Question:
The candidates performed relatively well on this section. The majority of the candidates were able to accurately describe the relationship between the underlying index and the volatilities, especially for range and trending regimes. Jumpy regime was more difficult for candidates. A common mistake was to not address the question asked.

Range Regime: Fixed strike volatility independent of the index level
$\text{Impl vol}(K, T; S) = a - b(T)(K - S_0)$

Trending Regime: Increase with the index level
$\text{Impl vol}(K, T; S) = a - b(T)(K - S)$

Jumpy Regime: Decrease when the index goes up and increase when the index goes down
$\text{impl vol}(K, T; S) = a - b(T)(K + S) + 2b(T)S_0$

16. Continued

(d) Derive an expression for the following, under each of the three regimes:

(i) ATM spot volatility

(ii) Volatility Skew

Commentary on Question:
The candidates performed poorly on this section. A fair amount did not score any points on the question; for those that did, most of the points came from part (i). Candidates that scored well in (i) realized that K = S and recognized the need to substitute K for S in the three regimes. Most candidates did not score in (ii). For those who did, the points mostly came from accurately writing out the formula for volatility skew. Almost no candidates derived any expression under the three regimes.

(i) Substitute K = S
Range: a – b(T)*(S – S0)
Trending: a
Jumpy: a −2b(T)(S – S0)

(ii) Volatility Skew is defined as (σ+ - σ-)/σ0
σ+ is implied volatility at strike K+, σ- is implied volatility at strike K-, σ0 = implied volatility for strike K0

Range: volatility skew = (a – b(T) (K+ - S0) – (a – b(T) (K- - S0)))/ σ0
= -b(T) (K+ - K-) / σ0

Trending: volatility skew = (a – b(T) (K+ -S) – (a – b(T) (K- - S)))/ σ0
= -b(T) (K+ - K-)/ σ0

Jumpy: volatility skew = (a – b(T) (K+ + S) + 2b(T) S0 – (a – b(T) (K- + S) + 2b(T) So))/ σ0
= -b(T) (K+ - K-)/ σ0

All 3 equal to the same expression – linear relationship

(e) Calculate the moneyness of each of the observed points, using the following two measures of moneyness:

(i) $h = \ln(K/S)$

(ii) $h = \dfrac{\ln \frac{K}{S}}{\sigma\sqrt{T}}$

16. Continued

Commentary on Question:
The candidates performed excellently on this section. Most candidates successfully completed part (i). The most common mistake in (ii) was when candidates did not recognize that the t given in the question needed to be converted from monthly to annually.

(i) ln(1425/1500) = - 0.05129
ln(1350/1500) = - 0.10536
ln(1275/1500) = - 0.1625

(ii) -0.05129 / (sqrt(3/12) * 20%) = - 0.5129
-0.10536 / (sqrt(6/12) * 29%) = - 0.5138
-0.16250/ (sqrt (9/12) * 37%) = - 0.5072

(f) Interpret the measure of moneyness in (ii) above and explain why this measure might be used.

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were able to identify that volatility and time to maturity are taken into consideration in the preferred measure. Successful candidates were also able to recognize that it produced similar results across maturities. A common mistake included missing the observation that the preferred measure calculated in part (e) provided stable results across different maturities.

- This definition of moneyness corresponds to the average of the h1 and h2 terms in the Black formula
- σ is the number of standard deviations that the log strike is away from the log forward price in a Black world. σ takes into consideration volatility.
- T takes into consideration the time to maturity.
- For a fixed maturity, a common choice for moneyness metric is to select points with the same degree of Black out-of-the moneyness. (i.e., points for which the ratio ln (K/S) has the same value. (i) is not appropriate because it yields different values across maturities. (ii) is appropriate because it yields similar values across maturities.