
SOCIETY OF ACTUARIES
Quantitative Finance and Investment Advanced Exam

Exam QFIADV

MORNING SESSION

Date: Friday, May 2, 2014
Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
 - a) The morning session consists of 10 questions numbered 1 through 10.
 - b) The afternoon session consists of 6 questions numbered 11 through 16.The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFIADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION****
Morning Session

- 1.** (5 points) You are the consultant for a small defined benefit pension plan which currently invests in bonds and equities. Currently, the long term expectation is that low interest rates will not be sufficient to secure the accrued retirement benefit payments, all of which are indexed annually to a cost of living index.

The pension plan investment committee wants to explore REIT investments and hedge funds as potential new investment classes.

- (a) (1 point) Outline a due diligence process to evaluate new investment classes.

The key investment objectives of the pension plan are liquidity, inflation hedging, and diversification.

- (b) (1 point) Evaluate the main characteristics of hedge funds relative to the stated investment objectives.

The pension plan is considering the purchase of a new commercial property, located in an area which attracts many new commercial developments. Recent comparable properties have produced an ROI of 14 – 16%. The pension plan sponsor has an average cost of capital of 9% and its capital structure contains no debt.

- (c) (1 point) Describe the main considerations relative to risk and cost of capital in determining the price of the property for the pension fund as the buyer.

The pension fund also has the option of investing in a REIT.

- (d) (2 points) Assess both real estate strategies relative to the objectives of plan and recommend which of the two is more appropriate for the pension plan.

2. (6 points) The QFI Life Inc. is issuing new variable annuity (VA) products with a variety of guaranteed benefits.

- (a) (1 point) Describe the features of GMDB and GMIB riders which create embedded options.
- (b) (1 point) List and describe three types of risks (other than equity risk and interest rate risk) which are associated with the GMIB rider.

QFI Life’s GLWB product generates revenue from monthly asset-based fees and declining surrender charges. The sales distribution is given as follows:

Issue age	5-year Deferral	10-year Deferral
55	30%	20%
65	40%	10%

QFI Life hired an actuarial intern, Mr. Chen, to run simulations for the testing of the adequacy of charges. In order to reduce the run time, Mr. Chen has conducted a convergence test to determine the desirable number of scenarios. In the test, it is assumed that the expected charge level is 0.6% of the benefit base per annum. Mr. Chen used a combination of 1,000 pre-packaged real-world scenarios developed by the American Academy of Actuaries (AAA) and an additional 9,000 real-world scenarios based on QFI Life’s own internal equity return model which meets the AAA calibration criteria. The costs of the GLWB rider are summarized in the following tables for various scenarios. All results are expressed as basis points.

10,000 Combined Scenarios			1,000 AAA Scenarios		
	Deferral Period			Deferral Period	
Age	5	10	Age	5	10
55	77.4	63.6	55	59.2	47.6
65	76.7	52.3	65	44.5	42.3

After reading the results, Mr. Chen finds that the set of 1,000 scenarios can significantly reduce the run time and hence is the optimal set to perform adequacy test on. Under the assumption that policies lapse at a fixed rate of 3%, the majority of cases show that the GLWB costs are covered under the assumed rider charges. With the feedback from the marketing and sales teams, Mr. Chen decides to make the recommendation of 0.6% rider charge to the management.

2. Continued

- (c) (2 points) Identify and explain three potential problems with Mr. Chen's analysis.

Now Mr. Chen proceeds to assess the efficiency of hedging strategies. Two measures – Expected Loss and CTE90 of the GLWB liabilities were simulated to compare the no-hedge, Delta-hedge and Delta-Vega-hedge strategies. The following results are obtained under two assumptions of the volatility coefficients. All results are expressed in terms of percentages of policyholders' purchase payments.

Volatility Coefficient	σ	22%	25%
No Hedge	Expected Loss	0.0	0.1
	CTE90	25.7	29.3
Delta Hedge	Expected Loss	1.0	1.3
	CTE90	2.4	2.5
Delta-Vega-Hedge	Expected Loss	1.4	1.6
	CTE90	2.3	2.4

- (d) (2 points) Recommend which hedging strategy to use.

3. (5 points) Consider the following portfolio of loans:

Obligor	Probability of Default (PD)	Exposure at Default (EAD)	Loss Given Default (LGD)
Company ABC	0.1	100	100%
Company XYZ	0.05	100	100%

Default correlation between Company ABC and Company XYZ = 0.50

(a) (2 points) Calculate the unexpected loss due to default of the portfolio.

You are given the following default correlations between Company Mamda and Companies Alpha, Beta and Gamma:

	Alpha	Beta	Gamma
Mamda	-0.8	0.7	-0.1

Four different investors have chosen to invest in Mamda and are looking to add only one of Alpha, Beta, or Gamma to their portfolio. The investors have set their investment horizon at 12 months.

Investor 1 has done some research on Mamda and is confident that they will not default over the next year. He wants to maximize his return while minimizing risk.

Investor 2 is an astute investment manager trying to maximize her return while minimizing her default risk. She has no additional information about Mamda.

Investor 3 is a highly risk adverse manager who does not like to take any risks and wants to ensure that there is money in the account at the end of the investment horizon.

Investor 4 is creating a portfolio to support the pension plan for Obligor Alpha.

(b) (3 points) Evaluate and recommend the best company to add to each investor's portfolio based on the situations described above.

4. (6 points) ABC Life purchased a zero-coupon bond that was privately issued by DeF, a publicly traded stock company. On a given valuation date, the Chief Investment Officer of ABC Life asked three of his analysts to independently estimate the fair market value of this zero-coupon bond. Below are the data and formula the three analysts used to estimate the fair market value of the zero-coupon bond on the same valuation date:

Data and formula that is common to all analysts:

- This zero-coupon bond is the only liability of DeF. The maturity value of this zero-coupon bond is \$100 million and the remaining time-to-maturity is 9 years.
- DeF has 1 million outstanding shares and is operating business as usual.
- DeF's stock price is \$19 per share.
- DeF's stock price volatility is 35%.
- The continuously compounded risk-free interest rate is 2.0% for all maturities.

Assumptions and the estimates that vary by analysts:

	Assume DeF's Asset Volatility	Estimated Market Value of the Zero-Coupon Bond
Analyst One:	8.7%	\$79.7 million
Analyst Two:	7.0%	\$81.5 million
Analyst Three:	10.0%	\$75.0 million

Taking DeF's total asset value to be the sum of the market value of its equity and the analysts' estimated market value of the zero-coupon bond, and using the analysts' assumed asset volatilities:

- (2 points) Calculate each analyst's implied estimate of DeF's stock price volatility using the Merton Asset Value Model.
- (2 points) Calculate each analyst's implied estimate of DeF's equity value using the Merton Asset Value Model.
- (1 point) Determine which analyst provided the best estimated market value of the zero-coupon bond that is consistent with the Merton Asset Value Model based on your calculations in (a) and (b).
- (1 point) Calculate DeF's LGD (as percent of EAD) based on Analyst One's estimated market value of the zero-coupon bond.

5. (6 points) JPS Financial Bank is experiencing a rapidly growing business. Below are items from JPS balance sheet and financial reporting as of 12/31/2013.

Assets		Liabilities: \$25M
Cash:	\$20M	
Mortgage:	\$180M	
Other Assets:	\$25M	
Total Assets	\$225M	Shareholder's Equity: \$200M

To support its business objectives JPS plans to sell its Mortgage for a cash settlement of \$200M. JPS has decided to use a pay-through security in order to achieve this. JPS internal guidelines will not allow a Debt-to-equity ratio greater than 0.5.

- (a) (1 point) Explain the economic rationale for securitization as it applies to JPS.
- (b) (1 point) Calculate the maximum amount that JPS can raise through traditional debt funding after considering the sale of its Mortgages.

Assume JPS does not require external credit enhancement.

- (c) (4 points) Construct a flow chart describing the mortgage securitization process showing each party involved, their roles, and the flow of the mortgage payments between them.

- 6.** (5 points) You are given that a spread quoted bond's option-adjusted spread duration is 6.5 and the current Bid-Ask spread is 0.2%. The resulting LCS for this spread quoted bond falls at the maximum level your company's investment policy will accept for either a spread quoted or price quoted bond. You are considering purchase of a bond on a price quoted basis, which currently has an Ask price of 200.
- (a) (2 points) Calculate the minimum bid price such that you do not violate your company's investment policy.
 - (b) (1 point) Describe how LCS could be applied to non-quoted bonds.
 - (c) (1 point) Explain how a portfolio manager would use LCS to compare two or more asset classes, and to quantify macro changes in market liquidity over time.
 - (d) (1 point) Describe 3 other uses of LCS, in addition to being a measure of liquidity of a bond.

7. (7 points) Kessel Holdings Corp. (“Kessel”) is a real estate development company that wants to raise 13 million in the near term to finance debt repayments. It has identified two properties that it can sell quickly: 1967 Gardiner Ave. and 1993 Clark Rd. Internally, Kessel has valued the properties at 11.0 million and 15.0 million, respectively.

Three firms have evaluated each of the two properties and have calculated the following investment values. Kessel and each of the three firms subscribe to a pricing service whereby they submit their investment values to a property/price database and are made aware of the market values and other investment values present in the database but they do not know which firm provided which values except for knowing their own submitted values. Kessel and the three firms are the only four subscribers to this service.

You are employed by the pricing service as the analyst assigned to these properties and need to prepare a statistical release to the subscribers regarding market values:

Firm	Gardiner	Clark
Kessel Holdings Corp.	11.0 million	15.0 million
Lupul Development LLC (“Lupul”)	11.5 million	8.5 million
Burke Condominiums Corp. (“Burke”)	15.0 million	14.0 million
Teacher’s Pension Plan (“TPP”)	12.0 million	13.0 million

- (a) (1 point) Define investment value with respect to real estate by differentiating it from market value.
- (b) (2 points) Explain the key considerations that make the real estate asset market informationally inefficient.
- (c) (1 point) Recommend and justify an appropriate market value for each property that will be distributed by the pricing service

There are six possible transactions to evaluate (three possible buyers for each of the two properties). Your manager has asked for an analysis on these transactions including their likelihood to occur and an appropriate price that might be struck.

- (d) (3 points) Evaluate each possible transaction.

8. (7 points) You are asked to propose a stochastic one-factor short-rate model to value insurance liabilities.

- (a) (1 point) Explain the characteristics of the time homogenous short-rate model and its major drawbacks.
- (b) (2 points) Compare and contrast the Cox, Ingersoll and Ross (CIR) model and Vasicek model.

You are given the following assumptions:

Current instantaneous spot rate	3%
Long term average instantaneous spot rates	4%
Volatility coefficient	0.5

The expected short rate under the Vasicek model one year from now is 3.2%. You decided to apply the same parameters to the CIR model.

- (c) (2 points) Calculate the variance of the short rate two years from now with the CIR model.

In order to test the sensitivity of the liabilities to a stochastic change in interest rates, you are reviewing the Black Karasinski model.

- (d) (2 points) Assess the suitability of the Black Karasinski model for the sensitivity testing and recommend whether it should be used instead of the CIR model.

9. (6 points) You have been asked to perform some interest rate modeling projects.

- (a) (1 point) Define a Jump Diffusion CIR model and identify whether this is an affine model.

Your colleague draws the following conclusion on the behavior the function $t \rightarrow f^{CIR}(0, t, \alpha)$, regarding to the CIR ++ model with

$$x_0 = 3\%$$

$$\theta = 1$$

$$K = 0.1$$

$$\sigma = 15\%$$

“ $f^{CIR}(0, t, \alpha)$ is monotonically decreasing and the supremum $f^*(\alpha)$ all the values of function f is 3%.”

- (b) (2 points) Critique your colleague’s conclusion.

You have the following models available:

Model 1: Extended Vasicek Model

Model 2: Extended CIR++ Model

Model 3: Heston Model with Jump Diffusion

- (c) (3 points) Recommend the best model for each of the following tasks and explain your choice.
- (i) Pricing equity options
 - (ii) Modeling short term interest rates with negative interest rates allowed
 - (iii) Modeling swaptions

10. (7 points) You have recently hired a new investment actuary in your risk management department. She has been studying various materials on liquidity risk, including the article on Northern Rock by Hyun Song Shin in her new role. She has suggested that a heavy reliance on securitization created liquidity issues for Northern Rock and contributed to the “run on the bank”.

- (a) (1 point) Critique your colleague’s position.
- (b) (2 points) Explain the market conditions and balance sheet positions that contributed to the “run on the bank” at Northern Rock.

The following stress testing methods are available:

- (i) Historical Value-at-Risk
 - (ii) Deterministic modeling
 - (iii) Monte Carlo modeling
- (a) (3 points) Compare and contrast the three methods above in the context of liquidity risk stress testing.
 - (b) (1 point) Assess each method’s ability to measure Northern Rock’s liquidity risk.

****END OF EXAMINATION****
Morning Session

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