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**SOCIETY OF ACTUARIES**  
**Enterprise Risk Management – Investment Extension**

**Exam ERM-INV**

**Date:** Wednesday, October 29, 2014

**Time:** 8:30 a.m. – 12:45 p.m.

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**INSTRUCTIONS TO CANDIDATES**

**General Instructions**

1. This examination has a total of 80 points.

This exam consists of 9 questions, numbered 1 through 9.

The points for each question are indicated at the beginning of the question. Questions 8 and 9 pertain to the extension readings and/or the Case Study, which is enclosed inside the front cover of this exam booklet.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

**Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam ERM-INV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.



## **CASE STUDY INSTRUCTIONS**

**The case study will be used as a basis for some examination questions. Be sure to answer the question asked by referring to the case study. For example, when asked for advantages of a particular plan design to a company referenced in the case study, your response should be limited to that company. Other advantages should not be listed, as they are extraneous to the question and will result in no additional credit. Further, if they conflict with the applicable advantages, no credit will be given.**



**\*\*BEGINNING OF EXAMINATION\*\***

**1.** (5 points) Humber is a small life insurance company based in the U.S. It has two primary lines of business:

- I. Group health benefits
- II. Individual term life insurance

Humber has been slow to adopt current ERM best practices due to its lack of expertise and resources in this practice area. It currently does not have a centralized risk function or an economic capital model. It employs ERM at a line of business level, which includes performing scenario testing and stress testing of business-specific risks. It does not fully aggregate risks or perform scenario or stress testing at a company-wide level.

Humber's corporate actuarial department is assessing what the company needs to do to implement the new Own Risk and Solvency Assessment (ORSA) requirements.

- (a) (2 points) Explain how Humber will need to adjust its current ERM practices in order to comply with the new ORSA requirements.
- (b) (1 point) Describe how complying with the ORSA requirements could benefit Humber's risk management.

Humber's corporate actuarial department contacts Horton Consulting about ORSA. Horton offers an expensive but comprehensive set of services that could help Humber develop ORSA capabilities that would be consistent with the capabilities of its larger competitors.

- (c) (2 points) Provide arguments for why Humber may not wish to purchase Horton's services.

2. (10 points) Simcoe’s ERM team is reviewing the company’s risk budgeting and capital management practices. The CRO would like to analyze Simcoe’s current capital allocation methodology and has asked for your help.

The company currently uses the Discrete Marginal Contribution method in determining the risk budget for each line of business. You are provided with the following:

Line of Business	Risk Budget (\$ millions)	Standalone VaR (\$ millions)
A	47.8	50.0
B	24.5	30.0
C	40.9	60.0
<b>Total</b>	<b>113.2</b>	

Correlations between Simcoe’s lines of business:

- $\rho_{A,B} = 0.9$
- $\rho_{A,C} = 0.4$
- $\rho_{B,C} = 0.2$

- (a) (1 point) Describe how diversification helps Simcoe’s risk management.

The CRO wants you to evaluate the following two alternative approaches for allocating the company’s capital:

- I. Pro-Rata
- II. Shapley Value

You have been given the information below:

Line of Business ( $X_i$ )	Total VaR Excluding Business $X_i$ (\$ millions)	Unscaled Discrete Marginal Contribution (\$ millions)
A	72.3	41.0
B	92.2	21.0
C	78.1	35.1
<b>Total</b>		<b>97.1</b>

## 2. Continued

(b) (5 points)

- (i) Explain what the Shapley Value allocation method tries to accomplish in risk allocation and the assumptions underlying the method.
- (ii) Calculate all missing table values below using the Shapley Value Method:

**Shapley Value VaR**

<b>Line of Business</b>	<b>1<sup>st</sup> In Contribution</b>	<b>2<sup>nd</sup> In Calculations</b>		<b>Last In Contribution</b>	<b>Average Shapley Value</b>
<b>A</b>					
<b>B</b>					
<b>C</b>					
<b>Total</b>	<b>113.2</b>			<b>97.1</b>	

Show your work.

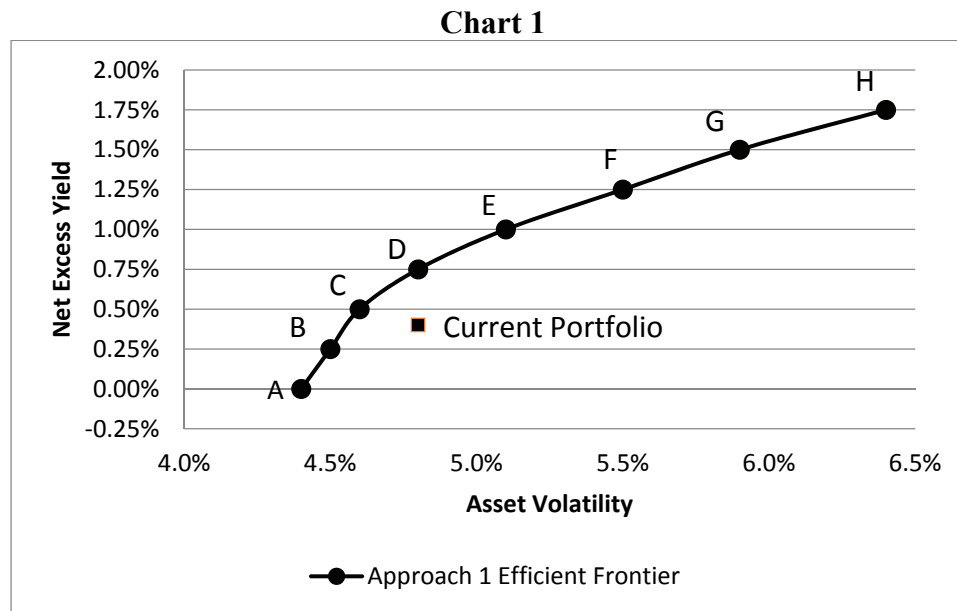
(c) (3 points)

- (i) Summarize the results of your calculations for the two alternative capital allocation methods, I and II, for the CRO.
  - (ii) Explain how each line of business leader may react to the potential change in the risk budgeting approach.
- (d) (1 point) Recommend one of the three approaches to set risk budgets for Simcoe. Justify your answer.

3. (9 points) Yorkton Life is seeking to increase the excess spread earned in its Group Pension product line without increasing its current level of risk. Yorkton plans to do this by expanding the asset universe backing the product to include equity, asset-backed securities (ABS) and high yield bonds.

The investment team performs a bottom-up strategic asset allocation (SAA) analysis incorporating the expanded asset universe. Their bottom-up SAA analysis, Approach 1, minimizes the asset portfolio volatility for a given level of net excess yield. The net excess yield is defined as the portfolio asset yield less the average Group Pension liability pricing rate.

Their analysis produces an efficient frontier of portfolios A through H. They are plotted along with the current portfolio in Chart 1.



- (a) (1 point) Explain why the Approach 1 Efficient Frontier is above and to the left of the current portfolio.

You have been asked to recommend a new efficient portfolio asset mix that is compliant with Yorkton’s risk management constraints. You decide to perform another SAA analysis on a holistic basis – Approach 2.

- (b) (1 point) Contrast the “bottom-up” approach used by the investment team in their SAA analysis to your holistic approach.



### 3. Continued

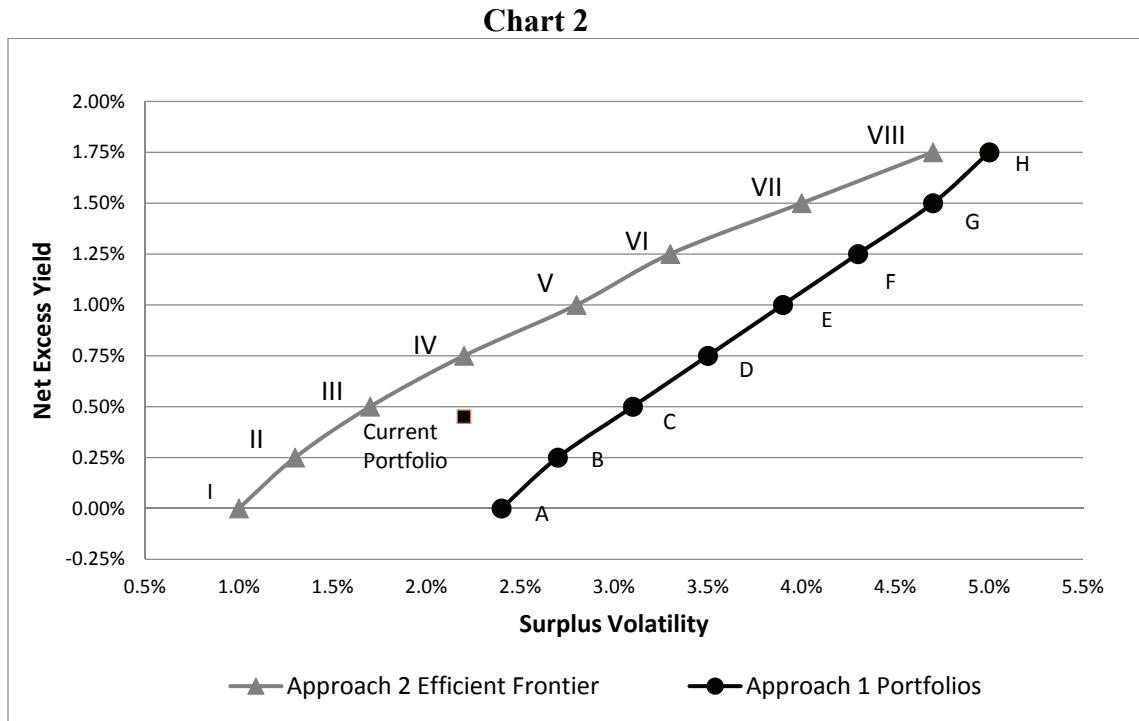
To bring the liabilities into your SAA analysis, you have determined the following key rate duration profile of the Group Pension liabilities:

Term (Years)	1	3	5	10	30
Duration	0.2	0.6	1.5	4.0	1.5

- (c) (2 points) Construct a portfolio of zero coupon bonds that will replicate a \$100 million Group Pension liability. Show your work.

Your Approach 2 SAA analysis produces another efficient frontier, consisting of portfolios I through VIII that minimize surplus volatility for a given level of net excess yield.

You also calculate the surplus volatility for each of the Approach 1 portfolios A through H. You plot these results alongside your Approach 2 efficient frontier and the current portfolio in Chart 2.



Question 3 continued on next page

### 3. Continued

- (d) (2 points) Sketch approximately where you would expect the Approach 2 portfolios to fall on Chart 1 provided at the beginning of the question. Explain your sketch.
- (e) (1.5 points) Recommend a new efficient portfolio to Yorkton management for each of (i) and (ii):
  - (i) Approach 1 Efficient Frontier, portfolios A through H
  - (ii) Approach 2 Efficient Frontier, portfolios I through VIII

Justify your recommendations.

- (f) (1.5 points) Indicate how likely it is that each of Approaches 1 and 2 will meet risk management goals with respect to:
  - (i) Portfolio Risk
  - (ii) Surplus Risk
  - (iii) Economic Capital Requirement
  - (iv) Market Risk Diversification

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4. (9 points) You are employed by a firm that is currently evaluating a potential investment in a windmill farm. You are concerned with the exposure this investment has to natural disasters. You determine that the natural disasters that would completely ruin the windmill farm have occurred two times in the past 160 years.

While the firm typically accounts for risk implicitly when determining its hurdle rate for a project, you decide to model the frequency of natural disasters as a Poisson process.

You are given the Poisson distribution:

$$P(N(t) = n) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}$$

The cash flows and metrics relating to this investment are as follows:

Year ( $t$ )	Cash Flow
0	(500)
1	-
2	-
3	-
4	-
5	1,650
IRR	26.97%
Hurdle Rate	25.00%

- (a) (2 points)
- (i) Demonstrate that the Maximum Likelihood Estimator (MLE) for the  $\lambda$  parameter of the Poisson distribution is equal to the sample mean.
  - (ii) Calculate the MLE for the  $\lambda$  parameter relating to natural disasters.

#### 4. Continued

- (b) (3 points) You have confirmed that the risk premium assumed in the hurdle rate did not explicitly consider the risk of natural disaster.
- (i) Assess whether the potential for natural disaster alters the decision on whether to accept this investment. Show your work.
  - (ii) Identify the shortcomings of using a Poisson approach for modeling the risk of natural disaster for this investment.

A colleague recognizes the similarity between your proposed adjustment for the risk of ruin related to natural disaster and the estimation of default risk relating to fixed income securities.

- (c) (1 point) Explain the analogy between your estimation of ruin relating to natural disaster and credit default risk assessment.
- (d) (3 points) Compare and contrast your use of the Poisson distribution to the following default risk assessment approaches:
- (i) Credit migration models
  - (ii) Structural credit risk models
  - (iii) Altman Z-score

- 5.** (11 points) Poutine, Inc.'s business is removing impurities in silver. The refining process is very energy-intensive, so energy costs are a large proportion of its total costs.

Customers provide raw silver to Poutine, which Poutine then refines for a fixed fee. Poutine uses coal as its only source of energy. Its fixed costs are extremely stable. The cost of coal is the only variable cost Poutine incurs.

Poutine's coal costs currently exceed the refining fee it charges. This situation has occurred several times in recent history.

- (a) (2 points) The CEO asks you to apply the PESTEL framework to identify the general environmental risks Poutine faces.
- (i) Identify each of the risks considered in the PESTEL framework.
  - (ii) Provide an example for two of the risks identified above which are specific to Poutine.

Poutine's CEO has proposed temporarily stopping the refining of silver until its coal costs are less than the fixed refining fee.

- (b) (1 point) Identify strategic risks Poutine faces if it implements the CEO's proposal.
- (c) (2 points) Explain how the CEO's strategy can be described as a financial derivative.

Poutine's CFO has concerns about the CEO's plan and asks you to look at implementing a hedging strategy in order to prevent temporary stoppages in refining.

- (d) (3 points)
- (i) Describe how Poutine could hedge its risk exposure to changes in the price of coal with forward contracts.
  - (ii) Describe how Poutine could hedge its risk exposure to changes in the price of coal with futures contracts.
  - (iii) Explain whether you would recommend using futures or forward contracts as a hedging strategy. Justify your answer.
- (e) (3 points) You are asked to assess the CEO's shutdown strategy versus the CFO's hedging strategy.
- (i) Identify the factors you would consider in choosing between the two.
  - (ii) Explain how these factors will inform your decision.

6. (5 points) Yonge Life is a U.S.-based life insurance company with the following characteristics:
- Yonge currently offers 10- and 20-year term life products with face amounts up to \$50 million.
  - It has the second largest share of annual sales volume for the 10- and 20-year U.S. term life insurance market.
  - Recent mortality experience has been higher than expected.
  - Yonge reinsures all policies that have a face amount in excess of \$5 million with a U.S.-based reinsurer.
  - Yonge mostly invests in high-yield U.S. corporate bonds.
  - Over the past several years, management and staff turnover has been low.
  - Yonge has followed a consistent corporate strategy for several years.
  - External auditors and regulators have not found any major issues with the company's management or processes.
  - Yonge's systems use state-of-the-art technology.

The ERM department created the following list of ten risk categories for classifying company risks:

- Market and economic risk
- Interest rate risk
- Foreign exchange risk
- Credit risk
- Liquidity risk
- Systemic risk
- Demographic risk
- Non-life insurance risk
- Operational risk
- Strategic risk

- (a) (3 points) Classify each of the ten risk categories as High, Medium, or Low Importance for Yonge Life. Justify your choices.
- (b) (2 points) Yonge Life is considering expanding its presence by acquiring a Chinese company that sells 5-, 10- and 20-year term policies with face amounts up to the equivalent of U.S. \$1 million. There is currently no reinsurance on this book of business and the company's investments are in Chinese investment grade corporate bonds.

Identify which risk categories would be of High Importance if Yonge Life makes the acquisition. Justify your choices.

7. (11 points) You are the Chief Actuary for Hamsik Re, a newly formed reinsurance company domiciled in the U.S. The company is in discussions with its first two potential clients. You have the following information regarding these two potential reinsurance transactions:

**Blocks Considered for Reinsurance by Hamsik Re**  
(\$ millions)

	Reinsured Block A	Reinsured Block B
Underlying Business	Universal Life	Payout Annuities
Statutory Reserve	1,095	1,694
Economic Reserve	429	1,400
Market Value of Assets		
• Private Placements	412	63
• Treasuries	84	424
• Corporates AAA	211	678
• Corporates AA	134	233
• Corporates A	99	144
• Corporates BBB	155	152
Standalone Capital Measurements		
• Company Action Level RBC	64	112
• S&P AA Level Capital	194	336
• Economic Capital	170	325

- (a) (1.5 points) Explain why it is important for Hamsik Re to consider each of the three standalone capital measurements in its assessment of these contemplated transactions.
- (b) (3 points) You are considering two methods of determining the aggregate amount of capital Hamsik Re would require assuming it reinsured both blocks:
- I. The standalone economic capital presented above and a correlation coefficient of  $-\rho$
  - II. Fixed diversification factor of  $\rho$
- (i) Determine the value of  $\rho$  which equates the aggregate capital required under methods I and II.
  - (ii) Explain why this same  $\rho$  might not equate the aggregate capital required under methods I and II if S&P AA level capital were used rather than economic capital.



## 7. Continued

Hamsik Re is contemplating reinsuring only one of these two potential blocks. The contemplated transaction would entail the underlying assets being transferred to and held by Hamsik Re. Capital is assumed to be invested using the same asset allocation as the reserves.

(c) (3 points)

- (i) Assess the liquidity risk profile associated with each block (i.e., Block A and Block B) from Hamsik Re's perspective.
- (ii) Identify the more favorable of the two blocks from a liquidity risk profile perspective. Justify your response.

Hamsik Re manages liquidity risk using a "liquidity risk ratio" or LRR. The LRR is defined as the ratio of  $\alpha$  to  $\beta$  where:

- $\alpha$  is the value that can be realized by selling the asset portfolio backing the reserve under a liquidity stress scenario (i.e., the market value of assets net of the "liquidity haircut")
- $\beta$  is the statutory reserve

You are provided the following information regarding the value at which certain securities could be sold in a liquidity stress scenario:

	Private Placements	Treasuries	Corporates AAA	Corporates AA	Corporates A	Corporates BBB
Liquidity Haircut	30%	0%	5%	10%	15%	25%

(d) (3.5 points)

- (i) Determine which of the two blocks would produce the more favorable LRR. Show your work.
- (ii) Explain the shortcomings of Hamsik Re's approach to liquidity risk management.
- (iii) Propose improvements to Hamsik Re's LRR approach.

**Questions 8 and 9 pertain to the Case Study and/or extension readings.  
Each question should be answered independently.**

- 8.** (11 points) SLIC's Chief Marketing Officer, Danielle Wolfe, is concerned about SLIC's continued marketing and sales of its SPIA product in the face of higher-than-expected mortality improvements. She has heard of the growing availability and use of indices to hedge longevity risk.

You have found two U.S.-based longevity indices available to use as a potential longevity hedging instrument for SPIA:

- I. LongLife Longevity Index (LL)
- II. VitaBrevis Longevity Index (VB)

Your first task will be to determine which of the two indices would be more suitable to hedge the longevity risk on the SPIA product. To perform your analysis, you decide that your hedge objective will be to remove the uncertainty in the SPIA annuity values over the next 10 years due to longevity risk. You will do this using a static 10-year swap based on either the LL or VB longevity index.

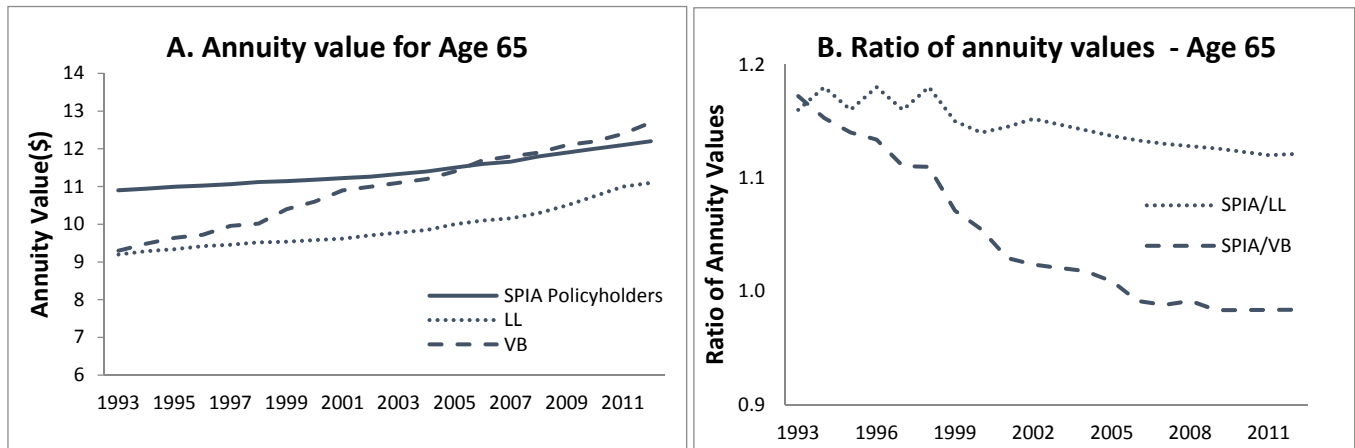
- (a) (3 points)
- (i) Define longevity basis risk with respect to implementing a longevity hedge.
  - (ii) Identify alternative metrics to the SPIA annuity values that could have been used for the hedge objective.
  - (iii) Explain whether the SPIA annuity values metric is better than the alternative metrics you identified in (ii).

To continue your analysis:

- You use 20 years of graduated historical mortality data to develop a base mortality table and projection scale for each index population for each year.
- You use SLIC's SPIA annuity pricing model to develop annuity values for \$1 of annual lifetime income based on the SPIA population mortality.
- You use the same pricing model to develop annuity values for \$1 of annual lifetime income based on the mortality underlying each of the two index populations, holding other assumptions constant.

## 8. Continued

You have plotted a representative sample of your results below. Chart A compares annuity values for males Age 65 based on the mortality for SLIC's SPIA and for each of the two index populations historically over time. Chart B shows the ratio of SPIA annuity values to each of the index annuity values through time for males Age 65.



- (b) (2.5 points) Explain to Danielle Wolfe which of the two available longevity indices would minimize the longevity basis risk in the hedge relative to the SPIA product by referencing key observations from Chart A and Chart B.

You proceed to conduct a retrospective hedge effectiveness assessment, deriving the annuity values of the SPIA population and of the selected longevity index population for each of 2,000 simulated mortality scenarios at the end of the 10-year hedge horizon.

The simulated annuity values with and without the calibrated longevity hedge exhibit the following histogram frequencies of the annuity dollar value averages for each bin at each of the tails of the distributions:

Unhedged SPIA	Left Tail			...	Right Tail		
Bin \$ Average	8.0	8.2	8.4	...	12.4	12.6	12.8
Frequency	25	25	50	1,725	75	80	20

Hedged SPIA	Left Tail			...	Right Tail		
Bin \$ Average	9.6	9.8	10.0	...	10.4	10.6	10.8
Frequency	10	90	600	640	560	85	15

Both distributions have a median annuity dollar value of 10.2. You use a VaR metric in ten years' time measured at a 95% confidence level relative to the median in your hedge effectiveness assessment.

**Question 8 continued on next page**

**Questions 8 and 9 pertain to the Case Study and/or extension readings.  
Each question should be answered independently.**

**8. Continued**

- (c) (2.5 points) Estimate the relative risk reduction of using the longevity hedge based on applying the 95% VaR risk metric to your histogram results.

You now decide to assess the natural hedge that exists between SLIC's annuity business and its life insurance business. As mortality risk is not currently assessed within SLIC's economic capital framework, you have obtained the following independent estimates of 99% VaR for the Life and Annuity lines of business:

Mortality Risk	VaR(99%)
Annuities	48.5
Life Insurance	134.7

- (d) (1 point) Calculate the diversification benefit that currently exists for SLIC's annuity and insurance mortality risk exposure, assuming a correlation of -60% between annuity and life insurance mortality risks.
- (e) (2 points) Recommend to Danielle Wolfe whether or not to pursue a static hedge based on a longevity index. Justify your recommendation.

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Each question should be answered independently.**

**9.** (9 points) The AHA Pension Committee has asked you to help them understand the surplus risk and return drivers of the defined benefit pension plan (the Plan). You perform an analysis using a Liability Driven Investment (LDI) Risk Factor framework.

(a) (2 points) You have selected the following set of four risk factors that you will use to model the Plan’s asset and liability exposures:

- I. Effect of economic growth on equities
- II. Effect of economic growth on credit
- III. Real interest rates
- IV. Inflation

For each of these four selected risk factors:

- (i) Describe the nature of the risk exposure in general.
  - (ii) Identify both the long position and the short position of the Factor Mimicking Portfolio (FMP) that would be used to model the risk.
- (b) (1 point) Explain how AHA’s Plan liabilities are or are not exposed to each of the four selected risk factors, I through IV.

You simplify the analysis to test the Plan’s basic equity/bond split by focusing on the following three primary risk factors:

- F1: Effect of economic growth on equities
- F2: Effect of economic growth on credit
- F3: Real interest rates and inflation

You map the asset portfolio sensitivity and the Plan liabilities’ sensitivity to these risk factors as follows:

	Passive Benchmark Weights	F1	F2	F3
Equity	65%	2	0	0
Bond	35%	0	0.6	0.6
Plan Liability		0	1.2	1.2

## 9. Continued

The three risk factors have the following expected returns, return volatilities and variance-covariance matrix:

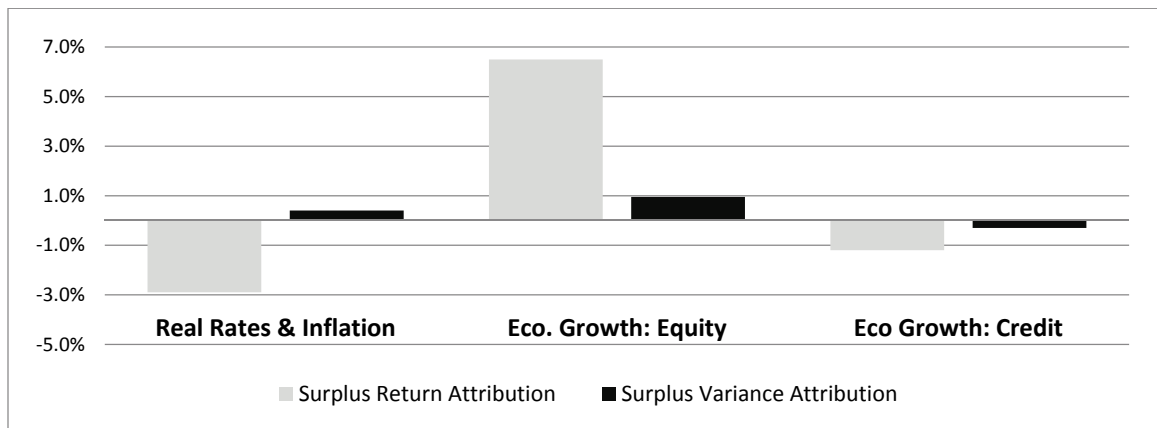
	Expected Return	Volatility	Variance-Covariance		
			F1	F2	F3
F1	5.0%	16%	2.6%	0.4%	0.5%
F2	1.0%	5%	0.4%	0.3%	-0.2%
F3	2.5%	14%	0.5%	-0.2%	2.0%

- (c) (2 points) You are interested in allocating the surplus portfolio expected dollar return to each of the three risk factors. You will use the Passive Benchmark Weights of the current Actuarial Value of Assets and Total Actuarial Accrued Liability as the basis for your allocation.

Calculate and allocate the surplus expected dollar return to each of the three risk factors to the nearest \$ million. Show your work.

- (d) (1 point) Calculate the volatility of the surplus portfolio expected dollar return. Show your work.

You perform an attribution of surplus return and surplus variance to the three primary risk factors as follows:



Question 9 continued on next page

**9. Continued**

- (e) (2 points)
  - (i) Explain the implications of positive and negative surplus returns and positive and negative surplus variances.
  - (ii) Assess the impact of each of the three key risk factors on the AHA Plan surplus.
- (f) (1 point) Recommend to the Pension Committee changes to the bond/equity mix that would improve the Plan's surplus risk/return profile. Justify your response.

**\*\*END OF EXAMINATION\*\***



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