GI ADV Model Solutions
Spring 2014

1.

Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4a) Calculate the price for a proportional treaty.

(4b) Calculate the price for a property per risk excess treaty.

Sources:

Basics of Reinsurance Pricing, D. R. Clark

Solution:

(a) Explain why Property R Us may have purchased surplus share reinsurance instead of quota share reinsurance.

A key difference between the two types of reinsurance is that surplus share reinsurance limits the retention per risk while quota share, being proportional, has no limit. Property R Us is a small company and so will likely prefer surplus share as it limits its exposure and allows larger amounts of coverage to be sold.

(b) Calculate the expected commission.

There are four terms to be calculated and added. They are:
0.2 probability x 0.2 commission = 0.04

(1/3)x0.6 probability x 0.25 commission = 0.05 (This represents the 1:1 sliding scale from 20% to 30% where the average commission is 0.25 and the probability of being between 20% and 30% is 1/3 of the 60% probability of the loss ratio interval of 40-70%.)

(2/3)x0.6 probability x 0.35 commission = 0.14 (Similarly, this represents the 0.5:1 sliding scale from 30% to 40% where the average is 0.35 and the probability of being between 30% and 40% is 2/3 of the 60% probability of the loss ratio interval of 40-70%.)
0.2 probability x 0.4 commission = 0.08

The sum is the expected commission of 0.31.

1. **Continued**

(c) State whether the expected commission will increase or decrease as Property R Us grows its business and writes more risks. Support your conclusion.

Commentary on Question:
There are two components to a full answer. The first is the effect on the distribution and the second is how the change in the distribution affects the commission.

As more business is written the mean will not change but the variance will decrease. Hence the loss ratios will be more concentrated around the mean of a 55% loss ratio. In the extreme, at this loss ratio, the commission is 32.5%, which is an increase from the original 31%.

(d) Explain how an exposure curve can be used to price risks with an insured value of 1,000,000 for the property per risk treaty.

An exposure factor corresponding to 50-100% of insured value on the exposure curve is applied to the subject premium net of the surplus share treaty.

2. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5a) Calculate an underwriting profit margin using the target total rate of return model.

(5b) Calculate an underwriting profit margin using the capital asset pricing model.

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Sources:

Ratemaking: A Financial Economics Approach, S. P. D'Arcy and M. A. Dyer

Solution:

(a) Calculate the premium for this policy using the Risk Adjusted Discount Technique.

The equation is

$$P = \frac{80}{0.98} + 20 + \frac{(P-20)(0.35)}{1.01} - \frac{80(0.35)}{0.98} + \frac{(50+P-20)(0.01)(0.35)}{1.01}.$$

The solution is P = 101.90.

(b) Evaluate Rocky's suggestion.

Rocky's assertion is false. Underwriting profit margin will not be decreased by paying an immediate dividend to shareholders. The change will increase leverage and thus increase the risk premium.

(c) Explain the purpose of the funds generating coefficient in the Capital Asset Pricing Model applied to insurance.

The funds generating coefficient measures the average time the insurer holds premiums. It provides an offset in the formula for the underwriting profit margin.

3. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1a) Identify the assumptions underlying the chain ladder estimation method.

(1b) Test for the validity of these assumptions.

(1c) Identify alternative models that should be considered depending on the results of the tests.

(1d) Estimate the standard deviation of a chain ladder estimator of unpaid claims.

Sources:

Measuring the Variability of Chain Ladder Reserve Estimates, T. Mack and Testing the Assumptions of Age-to-Age Factors, G. G. Venter

Solution:

(a) Describe this assumption in words.

The expected cumulative claims at lag k+1 are proportional to those at lag k. The constant of proportionality depends only on the lag, and not on the accident year.

(b) Describe a reserving situation in which this assumption may not hold.

Commentary on Question:

The solution below lists two possible responses. Only one is needed for full credit.

A change in underwriting or marketing practices in one accident year can change the rate at which claims develop and so the factor will not be the same over all accident years.

A change in the claims handling process can have a calendar year effect with the factor for a given lag depending on how the accident year plus lag relates to the year of the change.

(c) Determine if this plot provides evidence that this assumption holds. Support your answer.

Commentary on Question:

There are two observations regarding the plot provided. Both are needed to receive full credit.

3. Continued

The first item to check is if the line adequately describes the points. The points appear to be distributed randomly around the line.

There should be no pattern of the deviations by accident year (which is why the points are labeled with the accident year). No pattern is evident.

These two observations are consistent with the assumption holding.

(d) Describe, using words and/or formulas as appropriate, the other two statistical assumptions identified by Mack.

Commentary on Question:
A formula for the second assumption is also acceptable.

One assumption is the independence of the accident years. The other is that the variance of cumulative claims at lag k+1 is proportional to cumulative claims at lag k with the constant depending only on the lag.

(e) Rank the four models from best fitting to worst fitting using one of the three methods Venter suggests for accounting for the number of estimated parameters when comparing sums of squared errors. Indicate if your results support Mack's assumption.

Commentary on Question:
While Venter offered three methods, all candidates chose to work with the same method. It is the only one presented in this model solution.

The formula is to divide the sum of squared errors by the square of $n - p$. The four values are:
1: $1,869,591/(45-9)^2 = 1,443$
2: $1,120,615/(45-18)^2 = 1,537$*
3: $1,696,523/(45-9)^2 = 1,309$
4: $1,029,484/(45-18)^2 = 1,412$*

*p = 17 is also reasonable for models 2 and 4.

Small values are better, so Model 3 is best, followed by 4, 1, and 2.

Mack's assumption is represented by Model 1 and so this analysis does not support the assumption because two other models provide a better fit.

3. Continued

(f) Describe two other tests Venter recommends for determining the viability of using the chain ladder method.

Commentary on Question:
Venter recommends several tests as listed below. Any two can earn full credit.

- Residuals should appear random when plotting against cumulative losses.
- Residuals should be stable against accident year.
- Sample correlations of factors by accident year for adjacent lags should be near zero.
- A regression test can be used to test if diagonal dummy variables are significant.
- Factors should be significant after dividing by their standard deviations.

(g) Calculate the variance of the chain ladder estimate of the reserve for claims from accident year 3.

Commentary on Question:
The formula is on Page 116 of Mack.

$$C_{3,10}^2 \sum_{k=8}^{9} \frac{\alpha_k^2}{f_k^2} \left(\frac{1}{C_{i,k}} + \frac{1}{\sum_{j=1}^{10-k} C_{\mathrm{j},k}} \right)$$

$$= C_{3,10}^2 \left[\frac{\alpha_8^2}{f_8^2} \left(\frac{1}{C_{3,8}} + \frac{1}{C_{1,8} + C_{2,8}} \right) + \frac{\alpha_9^2}{f_9^2} \left(\frac{1}{C_{3,9}} + \frac{1}{C_{1,9}} \right) \right]$$

$$= 5{,}378^2 \left[\frac{1.13}{1.077^2} \left(\frac{1}{4{,}909} + \frac{1}{3{,}606 + 4{,}914} \right) + \frac{0.44}{1.017^2} \left(\frac{1}{5{,}285} + \frac{1}{3{,}834} \right) \right]$$

$$= 14{,}584$$

4. Learning Objectives:

2. The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:

(2a) Describe a risk margin analysis framework.

(2b) Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c) Describe methods to assess this uncertainty.

Sources:

A Framework for Assessing Risk Margins, K. Marshall, S. Collings, M. Hodson, C. O'Dowd

Solution:

(a) Describe the following sources of uncertainty:

(i) Independent Risk
(ii) Internal Systemic Risk
(ii) External Systemic Risk

Independent Risk: Randomness inherent in the insurance process.

Internal Systemic Risk: Uncertainty arising from the model being an imperfect reflection of reality.

External Systemic Risk: Risks that are outside the modeling process.

(b) Identify the source of uncertainty in part (a) to which each of the following belongs:

(i) Random Claim Fluctuations
(ii) Unexpected Future Legal Changes
(iii) Parameter Selection Error

Commentary on Question:
No explanation is required, correct matching is sufficient.

Random claim fluctuations – Independent risk
Unexpected future legal changes – External systemic risk
Parameter selection error – Internal systemic risk

4. Continued

(c) Calculate the combined coefficient of variation for all sources of uncertainty.

Solution is the square root of $0.05^2 + 0.08^2 + 0.15^2$ which is 0.177 or 17.7%.

(d) Calculate the amount of the risk margin.

The calculation is 0.177(0.674)(100,000,000) = 11,929,800.

(e) Describe two areas of additional analysis that you may conduct to provide further comfort regarding the outcomes from the deployment of this framework.

Commentary on Question:
Any two of the five listed below are sufficient for full credit.

- Sensitivity testing
- Scenario testing
- Internal benchmarking
- External benchmarking
- Hindsight analysis

5. Learning Objectives:

1. The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:

(1e) Apply a parametric model of loss development.

(1f) Estimate the standard deviation of a parametric estimator of unpaid claims.

Sources:

LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, D. R. Clark

Solution:

(a) Provide the term within this function corresponding to 0-12 months of development in accident year 2011, using Clark's LDF method and an exponential distribution with cumulative distribution function $G(x) = 1 - e^{-\frac{x}{\theta}}$.

The requested term is

$$4{,}000 \ln\left(ULT_{2011}\left(1 - e^{-\frac{6}{\theta}}\right)\right) - ULT_{2011}\left(1 - e^{-\frac{6}{\theta}}\right).$$

(b) Provide the two terms associated with accident year 2012 in the estimate of the scale factor, σ^2.

Commentary on Question:

The two terms can be presented separately and earn full credit.

The two terms are

$$\frac{(5{,}000 - 4{,}142)^2}{4{,}142} + \frac{(2{,}000 - 2{,}858)^2}{2{,}858}.$$

(c) Identify the number of degrees of freedom associated with the estimate of σ^2.

The degrees of freedom are the number of observations (6) less the number of parameters (4). Thus there are 2 degrees of freedom.

5. Continued

(d) Calculate the maximum likelihood estimate of accident year 2011 ultimate losses, ULT_{2011}.

The MLE is

$$\frac{8{,}000}{1-e^{-\frac{30}{7.94}}} = 8{,}187 .$$

(e) Estimate the process standard deviation of the accident year 2011 reserve.

The estimate is $(187 \text{ x } 318)^{0.5} = 244$.

(f) Provide an expression for the estimate of the parameter variance of the 2011 reserve using matrix notation. (Do not compute the result.)

The expression is

$$\begin{pmatrix} e^{-\frac{30}{7.94}} & 8187\cdot\frac{30}{7.94^2}e^{-\frac{30}{7.94}} \end{pmatrix}\begin{pmatrix} 2694151 & 295 \\ 295 & 3.24 \end{pmatrix}\begin{pmatrix} e^{-\frac{30}{7.94}} \\ 8187\cdot\frac{30}{7.94^2}e^{-\frac{30}{7.94}} \end{pmatrix} .$$

(g) Compare Clark's stochastic reserving model to the chain ladder model with respect to the assumption of independence of incremental losses within an accident year.

Incremental losses within an accident year are assumed to be independent in Clark's model. The chain ladder model does not assume independence.

6. Learning Objectives:

3. The candidate will understand how to use a credibility model with parameters that shift over time.

Learning Outcomes:

(3a) Identify the components of a credibility model with shifting risk parameters.

(3c) Estimate the parameters of the model.

(3d) Compare various models that might be used.

Sources:

Credibility with Shifting Risk Parameters, S. A. Klugman

Solution:

(a) Explain why the ARIMA(0,1,1) model cannot be extended to a Bühlmann-Straub credibility framework.

The 1 in the model's second term indicates that the model is for the differences of observations. With no constant, every jurisdiction has a mean difference of zero. Thus there is no ability to model the variance of the hypothetical means.

(b) Explain why this pattern of autocorrelations suggests that neither an MA(1) nor an AR(1) model is likely to be appropriate.

The AR(1) model has geometrically decreasing autocorrelations and the MA(1) model has a non-zero autocorrelation at lag 1 and zero elsewhere. The autocorrelation estimates do not follow either of these patterns.

(c) Set up, but do not solve, the matrix equation for the vector of credibility weights, $Z_1, \ldots, Z_7$, to apply to the seven annual observations where the goal is to forecast the pure premium two years ahead (year 9).

Commentary on Question:

It is not necessary to show the sums to receive full credit, provided the entries are correct.

6. **Continued**

The matrix and vector for the linear equations are

$$\begin{bmatrix} 100+10 & 10 & 25+10 & 10 & 10 & 10 & 10 \\ 10 & 110 & 10 & 35 & 10 & 10 & 10 \\ 25+10 & 10 & 110 & 10 & 35 & 10 & 10 \\ 10 & 35 & 10 & 110 & 10 & 35 & 10 \\ 10 & 10 & 35 & 10 & 110 & 10 & 35 \\ 10 & 10 & 10 & 35 & 10 & 110 & 10 \\ 10 & 10 & 10 & 10 & 35 & 10 & 110 \end{bmatrix} \text{ and } \begin{bmatrix} 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 10 \\ 25+10 \end{bmatrix} \text{ respectively.}$$

(d) Explain why separate parameter estimates would now be required for σ^2 and δ_0.

The general formula for the diagonal term is $\sigma^2 / w_i + \delta_0 + \tau^2$. With varying weights there is no common value on the diagonal.

7. **Learning Objectives:**

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5d) Allocate an underwriting profit margin (risk load) among different accounts.

Sources:

An Application of Game Theory: Property Catastrophe Risk Load, D. F. Mango

Solution:

(a) Explain why neither the Marginal Variance nor Marginal Surplus methods for calculating risk load are renewal additive.

Renewal additivity for a risk load method requires that the renewal risk loads for individual accounts X and Y sum to the risk load for the combined account X+Y. The Marginal Surplus method allocates the risk load proportionally to the standard deviation, which is subadditive. The Marginal Variance method allocates the risk load proportionally to the variance, which is superadditive provided there is a positive covariance.

(b) Calculate the risk load for each account using the Shapley method.

The steps are as follows:
The variances of X and Y are 14,477,500 and 88,300 and the variance of X+Y is 16,822,800 (these are obtained by summing the last three columns, respectively).
The covariance is (16,822,800 – 14,477,500 – 88,300)/2 = 1,128,500.
Adding the covariance to each variance gives 15,606,000 and 1,216,800.
Multiplying each by 0.000025 gives the risk loads of 390.15 and 30.42.

(c) Explain how the Covariance Share method differs from the Shapley method.

The Shapley method allocates the covariance equally between the accounts while the Covariance Share method allocates the covariance in proportion to the loss size.

8. Learning Objectives:

4. The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcomes:

(4c) Calculate the price for a casualty per occurrence excess treaty.

(4d) Apply an aggregate distribution model to a reinsurance pricing scenario.

Sources:

Basics of Reinsurance Pricing, D. R. Clark

Solution:

(a) Calculate the expected losses in the layer using an exposure rating approach with an expected loss ratio of 60% and the following increased limits factors:

The first entry is below the layer and so makes no contribution. For the other four entries the contributions are:
5,000,000(1.2 – 1.0)/1.2 = 833,333
15,000,000(1.35 – 1.0)/1.35 = 3,888,889
10,000,000(1.35 – 1.0)/1.56 = 2,243,590
2,000,000(1.56 – 1.35)/(1.56 – 1.0) = 750,000
The total is 7,715,812. Multiplying by the expected loss ratio of 0.6 gives the answer, 4,629,487.

(b) Explain why applying a loading of 20% of layer losses to account for ALAE in the layer is problematical.

Applying 20% assumes that ALAE is a constant percentage of each loss. However, ALAE generally decreases as a percentage of loss as the size of loss increases. Thus, 20% is likely to be too high in the reinsurance layer.

(c) Explain one method for calculating probabilities when using a collective risk model approximation to the aggregate distribution to set the terms of the swing plan.

Commentary on Question:

Any one of the three methods listed is sufficient for full credit.

- Recursive calculation, such as the Panjer recursion formula
- Numerical methods, such as Heckman-Meyers inversion
- Simulation, generating the number of losses followed by the amount of an individual loss

8. Continued

(d) Recommend whether or not this ceding company should purchase any other casualty per occurrence excess coverage. Justify your answer.

Coverage for the layer 500,000 excess of 1,000,000 should be considered because it is exposed by the policies with a limit of 1,500,000. Clash coverage above this should also be considered.