SOCIETY OF ACTUARIES
Advanced Topics in General Insurance

Exam GIADV

Date: Thursday, May 1, 2014
Time: 2:00 p.m. – 4:15 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 40 points.

   This exam consists of 8 questions, numbered 1 through 8.

   The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets because they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam GIADV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

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Exam GIADV-Front Cover

****BEGINNING OF EXAMINATION****

1. (*4 points*) Property R Us Insurance Company is a small insurer that insures commercial buildings of all sizes. It has purchased surplus share reinsurance with a retained line of 100,000.

(a) (*0.5 point*) Explain why Property R Us may have purchased surplus share reinsurance instead of quota share reinsurance.

The surplus share treaty has a sliding scale commission with terms as follows:

Minimum Commission	20% at a 70% loss ratio
Sliding 1:1 to	30% at a 60% loss ratio
Sliding 0.5:1 to a Maximum	40% at a 40% loss ratio

There is a 20% probability of a loss ratio below 40%, a 20% probability of a loss ratio above 70%, and the loss ratio is uniformly distributed in the range from 40% to 70%.

(b) (*1.5 points*) Calculate the expected commission.

(c) (*1 point*) State whether the expected commission will increase or decrease as Property R Us grows its business and writes more risks. Support your conclusion.

Property R Us wishes to purchase a 50,000 excess of 50,000 property per risk excess treaty. The surplus share reinsurance will inure to the benefit of the property per risk treaty.

(d) (*1 point*) Explain how an exposure curve can be used to price risks with an insured value of 1,000,000 for the property per risk treaty.

2. (*4 points*) You are setting the premium for a one-year policy using the following assumptions:

- The premium will be collected when the policy becomes effective.
- Expenses of 20 will be paid when the policy becomes effective.
- Losses are expected to be 80 and will be paid at policy expiration.
- The tax rate on all income is 35% and taxes will be paid at policy expiration.
- Equity of 50 supports the policy.
- The risk-free rate is 1%.
- The risk-adjusted rate for losses is –2%.

(a) (*2.5 points*) Calculate the premium for this policy using the Risk Adjusted Discount Technique.

Your actuarial student Rocky has proposed using the target total rate of return model with an underwriting profit margin of $(S/P)\left[R_f + RP - (IA/S)(IR)\right]$, where:

S = shareholders' equity
P = premium
R_f = risk-free rate
RP = insurer's risk premium
IA = investable assets
IR = investment return

Rocky has pointed out that the required underwriting profit margin could be decreased by paying an immediate dividend to shareholders, thus reducing S. He has suggested that this would enhance your company's competitive position.

(b) (*1 point*) Evaluate Rocky's suggestion.

(c) (*0.5 point*) Explain the purpose of the funds generating coefficient in the Capital Asset Pricing Model applied to insurance.

3. (*9 points*) You are interested in determining the variability of reserve estimates. The triangle of data you are working with is presented below (AY = accident year). The shaded cells have been completed using the standard chain ladder method. It is assumed that all claims are fully developed after ten years.

	Development Year									
AY	1	2	3	4	5	6	7	8	9	10
1	358	1,125	1,735	2,218	2,746	3,320	3,466	3,606	3,834	3,901
2	352	1,236	2,170	3,353	3,799	4,120	4,648	4,914	5,339	5,432
3	291	1,292	2,219	3,235	3,986	4,133	4,629	4,909	5,285	5,378
4	311	1,419	2,195	3,757	4,030	4,382	4,588	4,835	5,206	5,297
5	443	1,136	2,128	2,898	3,403	3,873	4,207	4,433	4,773	4,857
6	396	1,333	2,181	2,986	3,692	4,075	4,427	4,665	5,022	5,110
7	441	1,288	2,420	3,483	4,089	4,513	4,902	5,166	5,562	5,659
8	359	1,421	2,864	4,174	4,900	5,408	5,875	6,191	6,665	6,782
9	377	1,363	2,382	3,471	4,075	4,498	4,886	5,149	5,543	5,640
10	344	1,200	2,098	3,057	3,589	3,961	4,303	4,534	4,882	4,967

In his paper "Measuring the Variability of Chain Ladder Reserve Estimates," Mack states that there are three statistical assumptions that are implicit in the chain ladder method.

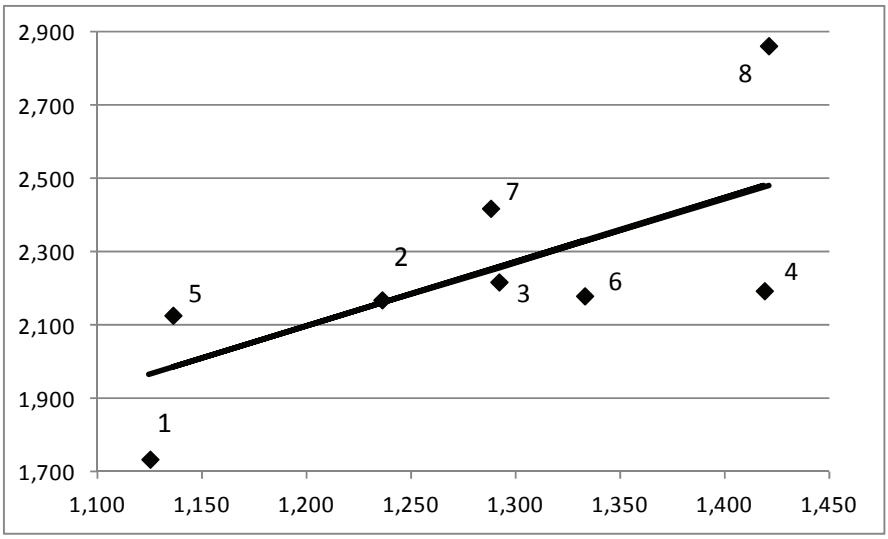
One of the assumptions is that $E(C_{i,k+1} \mid C_{i,1}, \ldots, C_{i,k}) = C_{i,k} f_k$ for all i and k.

(a) (*0.5 point*) Describe this assumption in words.

(b) (*0.5 point*) Describe a reserving situation in which this assumption may not hold.

Mack suggests a regression test to evaluate this assumption. A plot is constructed for each lag. For the above triangle, the following plot was made to assess the development from lag 2 to lag 3. The chain ladder estimate of f_2 is 1.75.

3. Continued



Plot of $C_{i,3}$ (vertical axis) against $C_{i,2}$ (horizontal axis) with the line $y = 1.75x$ added. The numbers indicate the accident year for that plotted value.

(c) (*1 point*) Determine if this plot provides evidence that this assumption holds. Support your answer.

(d) (*1.5 points*) Describe, using words and/or formulas as appropriate, the other two statistical assumptions identified by Mack.

You now wish to continue checking the same chain ladder assumption you tested with the scatterplot and regression line, this time using ideas from "Testing the Assumptions of Age-to-Age Factors" by Venter. Your assistant has analyzed four models that describe the development pattern. For each model he determined the parameter estimates along with the estimated ultimate values (both not shown) and the sum of squared errors. The notation used in the models is:

$c(w,k)$ = cumulative claims for accident year w at development year k

$q(w,k)$ = incremental claims for accident year w at development year k

The results are in the table on the next page.

3. Continued

Model Number	Model Description	Sum of Squared Errors
1	$q(w,k+1)=c(w,k)f(k)$	1,869,591
2	$q(w,k+1)=c(w,k)f(k)+g(k)$	1,120,615
3	$q(w,k+1)=g(k)$	1,696,523
4	$q(w,k+1)=f(k+1)h(w)$	1,029,484

(e) (*2 points*) Rank the four models from best fitting to worst fitting using one of the three methods Venter suggests for accounting for the number of estimated parameters when comparing sums of squared errors. Indicate if your results support Mack's assumption.

(f) (*1.5 points*) Describe two other tests Venter recommends for determining the viability of using the chain ladder method.

The following table provides the development factors (f_k in Mack's paper) and the variance estimates (α_k^2 in Mack's paper, using one of the methods from the paper for the final value).

k	1	2	3	4	5	6	7	8	9
f_k	3.489	1.748	1.457	1.174	1.104	1.086	1.054	1.077	1.017
α_k^2	159.63	37.79	41.90	15.18	13.69	8.21	0.44	1.13	0.44

(g) (*2 points*) Calculate the variance of the chain ladder estimate of the reserve for claims from accident year 3.

4. (*4 points*) You are calculating a risk margin for claim liabilities using the methodology set out in "A Framework for Assessing Risk Margins."

The risk margin is to be calculated at the 75% adequacy level and is to be based on the following sources of uncertainty, which are considered independent of one another:

	Source of Uncertainty		
	Independent Risk	**Internal Systemic Risk**	**External Systemic Risk**
Coefficient of Variation	5%	8%	15%

The central estimate of claim liabilities is 100,000,000.
Claims are assumed to be normally distributed.
The z-value of the 75th percentile of the normal distribution is 0.674.

(a) (*1.5 points*) Describe the following sources of uncertainty:

(i) Independent Risk
(ii) Internal Systemic Risk
(iii) External Systemic Risk

(b) (*0.5 point*) Identify the source of uncertainty in part (a) to which each of the following belongs:

(i) Random Claim Fluctuations
(ii) Unexpected Future Legal Changes
(iii) Parameter Selection Error

(c) (*0.5 point*) Calculate the combined coefficient of variation for all sources of uncertainty.

(d) (*0.5 point*) Calculate the amount of the risk margin.

(e) (*1 point*) Describe two areas of additional analysis that you may conduct to provide further comfort regarding the outcomes from the deployment of this framework.

5. (*6 points*) You are given the following triangle of cumulative paid losses:

	Months of Development		
Accident Year	12	24	36
2011	4,000	7,000	8,000
2012	5,000	7,000	
2013	6,000		

The function $\sum_i c_i \ln(\mu_i) - \mu_i$ must be maximized to obtain maximum likelihood estimates of the parameters needed to apply Clark's stochastic reserving model.

(a) (*1 point*) Provide the term within this function corresponding to 0-12 months of development in accident year 2011, using Clark's LDF method and an exponential distribution with cumulative distribution function $G(x) = 1 - e^{-\frac{x}{\theta}}$.

The fitted triangle of cumulative paid losses is:

	Months of Development		
Accident Year	12	24	36
2011	4,343	7,340	8,000
2012	4,142	7,000	
2013	6,000		

(b) (*1 point*) Provide the two terms associated with accident year 2012 in the estimate of the scale factor, σ^2.

(c) (*0.5 point*) Identify the number of degrees of freedom associated with the estimate of σ^2.

The maximum likelihood estimate of θ is 7.94 and the estimate of σ^2 is 318.

(d) (*1 point*) Calculate the maximum likelihood estimate of accident year 2011 ultimate losses, ULT_{2011}.

(e) (*1 point*) Estimate the process standard deviation of the accident year 2011 reserve.

5. Continued

The covariance matrix of the estimates of ULT_{2011}, ULT_{2012}, ULT_{2013}, and θ, respectively, is:

$$\begin{pmatrix} 2,694,151 & 75,960 & 281,183 & 295 \\ 75,960 & 2,987,786 & 795,678 & 834 \\ 281,183 & 795,678 & 9,732,843 & 3,088 \\ 295 & 834 & 3,088 & 3.24 \end{pmatrix}$$

(f) (*1 point*) Provide an expression for the estimate of the parameter variance of the 2011 reserve using matrix notation. (Do not compute the result.)

(g) (*0.5 point*) Compare Clark's stochastic reserving model to the chain ladder model with respect to the assumption of independence of incremental losses within an accident year.

6. (*4 points*) You are an actuary at Orange Rock Insurance Company (ORCo). One of ORCo's products is sold in 32 jurisdictions and ratemaking has been done using seven years of experience. ORCo's current methodology is to estimate next year's pure premium for a given jurisdiction by taking a credibility-weighted average of the observed pure premium for that jurisdiction and the observed pure premium averaged over all jurisdictions. Bühlmann-Straub empirical Bayes credibility has been used to determine the credibility factors.

There is a belief within the pricing unit that because there are changes over time, more recent observations should receive more weight than older observations. Your manager recalled that there may be a methodology that leads to decreasing weights on each jurisdiction's past observations along with some remaining weight on the overall observed pure premium.

An ARIMA(0,1,1) model with no constant term produces weights that are geometrically decreasing when applied to each jurisdiction separately.

(a) (*1 point*) Explain why the ARIMA(0,1,1) model cannot be extended to a Bühlmann-Straub credibility framework.

Models with only AR or MA terms can be directly extended to a Bühlmann-Straub credibility framework. For ORCo's data, the following estimates of the autocorrelations at each of the first four lags were obtained:

Lag	**1**	**2**	**3**	**4**
Estimated autocorrelation	0.040	0.237	0.026	0.062

(b) (*1 point*) Explain why this pattern of autocorrelations suggests that neither an MA(1) nor an AR(1) model is likely to be appropriate.

6. Continued

You decide to use a random effects model as described in Klugman. You further decide that the covariance structure should have non-zero values only at lag two. REML estimates of the required parameters are $\widehat{\sigma^2 + \delta_0} = 100$, $\hat{\delta}_2 = 25$, and $\hat{\tau}^2 = 10$. Also assume that the exposures for all jurisdictions and all years are equal to 1.

(c) (*1.5 points*) Set up, but do not solve, the matrix equation for the vector of credibility weights, $Z_1, \ldots, Z_7$, to apply to the seven annual observations where the goal is to forecast the pure premium two years ahead (year 9).

Suppose the exposures differed by year.

(d) (*0.5 point*) Explain why separate parameter estimates would now be required for σ^2 and δ_0.

7. (*4 points*) Your company is renewing two accounts, X and Y, which are exposed to two possible independent claim events, 1 and 2. You are given the following information:

Event (*i*)		**Loss for Account (*L*)**			$L^2 p(1-p)$		
i	***p*(*i*)**	**X**	**Y**	**X + Y**	**X**	**Y**	**X + Y**
1	2%	25,000	2,000	27,000	12,250,000	78,400	14,288,400
2	1%	15,000	1,000	16,000	2,227,500	9,900	2,534,400

- $p(i)$ represents the probability of Event i.
- The risk load multiplier, λ, is 0.000025.

(a) (*1 point*) Explain why neither the Marginal Variance nor Marginal Surplus methods for calculating risk load are renewal additive.

(b) (*2.5 points*) Calculate the risk load for each account using the Shapley method.

(c) (*0.5 point*) Explain how the Covariance Share method differs from the Shapley method.

8. (*5 points*) Casualty R Us Reinsurance Company has been presented with an opportunity to write a casualty per occurrence excess treaty covering the layer 500,000 excess of 500,000 on a swing plan. The following limits profile has been provided:

Subject Premium	Underlying Limit	Policy Limit
5,000,000	0	500,000
5,000,000	0	750,000
15,000,000	0	1,000,000
10,000,000	0	1,500,000
2,000,000	500,000	1,000,000

(a) (*3 points*) Calculate the expected losses in the layer using an exposure rating approach with an expected loss ratio of 60% and the following increased limits factors:

Policy Limit	Increased Limits Factor
500,000	1.00
750,000	1.20
1,000,000	1.35
1,500,000	1.56

ALAE on the underlying business is expected to be 20% of losses, and ALAE is allocated to each layer in proportion to losses.

(b) (*0.5 point*) Explain why applying a loading of 20% of layer losses to account for ALAE in the layer is problematical.

(c) (*1 point*) Explain one method for calculating probabilities when using a collective risk model approximation to the aggregate distribution to set the terms of the swing plan.

(d) (*0.5 point*) Recommend whether or not this ceding company should purchase any other casualty per occurrence excess coverage. Justify your answer.

****END OF EXAMINATION****

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