November 2013 MLC Solutions

1. Key: B

$$A_{x:\overline{n}|}^{\;\;\;1} = {}_nE_x$$

$$A_x = A_{x:\overline{n}|}^{1} + {}_nE_x A_{x+n}$$

$$0.3 = A_{x:\overline{n}|}^{1} + (0.35)(0.4) \Rightarrow A_{x:\overline{n}|}^{1} = 0.16$$

$$A_{x:\overline{n}|} = A_{x:\overline{n}|}^{1} + {}_nE_x = 0.16 + 0.35 = 0.51$$

$$\ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d} = \frac{1 - 0.51}{(0.05/1.05)} = 10.29$$

$$a_{x:\overline{n}|} = \ddot{a}_{x:\overline{n}|} - 1 + {}_nE_x = 10.29 - 0.65 = 9.64$$

2. Key: C

$$\bar{a}_{xy} = \bar{a}_x + \bar{a}_y - \bar{a}_{\overline{xy}} = 10.06 + 11.95 - 12.59 = 9.42$$

$$\bar{a}_{xy} = \frac{1 - \bar{A}_{xy}}{\delta}$$

$$9.42 = \frac{1 - \bar{A}_{xy}}{0.07} \Rightarrow \bar{A}_{xy} = 0.34$$

$$\bar{A}_{xy} = \bar{A}_{\overset{1}{x}y} + \bar{A}_{x\overset{1}{y}}$$

$$0.34 = \bar{A}_{\overset{1}{x}y} + 0.09 \Rightarrow \bar{A}_{\overset{1}{x}y} = 0.25$$

3. Key: E

$${}_{2.2}q_{[51]+0.5} = \frac{l_{[51]+0.5} - l_{53.7}}{l_{[51]+0.5}}$$

$$l_{[51]+0.5} = 0.5 l_{[51]} + 0.5 l_{[51]+1} = 0.5(97{,}000) + 0.5(93{,}000) = 95{,}000$$

$$l_{53.7} = 0.3 l_{53} + 0.7 l_{54} = 0.3(89{,}000) + 0.7(83{,}000) = 84{,}800$$

$${}_{2.2}q_{[51]+0.5} = \frac{95{,}000 - 84{,}800}{95{,}000} = 0.1074$$

$$10{,}000\,{}_{2.2}q_{[51]+0.5} = 1{,}074$$

4. Key: B

$$\text{Prob}(H \to D \text{ in 2 months}) = (0.75\ \ 0.2\ \ 0.05)\begin{pmatrix} 0.05 \\ 0.20 \\ 1 \end{pmatrix} = 0.1275$$

You could do more extensive matrix multiplication and also obtain the probability that it is *H* after 2 or it is *S* after 2, but those aren't needed.

Let *D* be the number of deaths within 2 years out of 10 lives

Then *D*~binomial with $n = 10, \quad p = 0.1275$

$$P(D = 4) = \binom{10}{4}(0.1275)^4(1 - 0.1275)^6 = 0.0245$$

5. Key: A

$\mathring{e}_{40} = \dfrac{1}{\mu} = 50$ So receive *K* for 50 years guaranteed and for life thereafter.

$$10{,}000 = K\left[\bar{a}_{\overline{50|}} + {}_{50|}\bar{a}_{40}\right]$$

$$\bar{a}_{\overline{50|}} = \int_0^{50} e^{-\delta t} = \frac{1 - e^{-50\delta}}{\delta} = \frac{1 - e^{-50(0.01)}}{0.01} = 39.35$$

$${}_{50|}\bar{a}_{40} = {}_{50}E_{40}\bar{a}_{40+50} = e^{-(\delta+\mu)50}\frac{1}{\mu + \delta} = e^{-1.5}\frac{1}{0.03} = 7.44$$

$$K = \frac{10{,}000}{39.35 + 7.44} = 213.7$$

6. Key: C

Simplest solution is retrospective:

$$q_{[55]} = (0.7)(0.00896) = 0.006272$$

$$_1V = \frac{(24.453)(1.06) - (1000)(0.006272)}{1 - 0.006272} = 19.77$$

Prospectively, $q_{[55]+1} = (0.8)(0.00975) = 0.0078; \quad q_{[55]+2} = (0.9)(0.01062) = 0.009558$

$$A_{[55]+1} = (1000)(0.0078)v + (1000)(1 - 0.0078)(0.009558)v^2$$
$$+ (1 - 0.0078)(1 - 0.009558)(342.65)v^2 = 315.49$$
$$\ddot{a}_{[55]+1} = \left(1 - A_{[55]+1}\right) / d = (1 - 0.31549) / (0.06 / 1.06) = 12.093$$
$$_1V = 315.49 - (12.093)(24.453) = 19.78$$

7. Key: A

Let P = 0.00258 be the monthly benefit premium per 1 of insurance.

$$_{10}V = 100,000\left[\frac{i}{\delta} A^{1}_{55:\overline{10}|} + A_{55:\overline{10}|}^{\ \ 1} - 12P\ddot{a}^{(12)}_{55:\overline{10}|}\right]$$
$$= 100,000\left[1.02971(0.09102) + 0.48686 - (12 \times 0.00258)(7.21928)\right]$$
$$\approx 35,700$$

Where

$$A^{1}_{55:\overline{10}|} = A_{55} - {}_{10}E_{55} A_{65} = 0.30514 - (0.48686)(0.43980) = 0.09102$$
$$A_{55:\overline{10}|}^{\ \ 1} = {}_{10}E_{55} = 0.48686$$
$$\ddot{a}_{55:\overline{10}|} = \ddot{a}_{55} - {}_{10}E_{55}\, \ddot{a}_{65} = 12.2758 - (0.48686)(9.8969) = 7.45740$$
$$\ddot{a}^{(12)}_{55:\overline{10}|} = \alpha(12)\ddot{a}_{55:\overline{10}|} - \beta(12)\left[1 - {}_{10}E_{55}\right]$$
$$= 1.00028(7.45740) - 0.46812(1 - 0.48686) = 7.21928$$

8. Key: C

Use superscript g for gross premiums and gross premium reserves.
Use superscript n (representing "net") for benefit premiums and benefit reserves.
Use superscript e for expense premiums and expense reserves.

$P^g = 1,605.72$ (given)

$$P^e = \frac{0.58P^g + 450 + \left(0.02P^g + 50\right)\ddot{a}_{45}}{\ddot{a}_{45}}$$

$$= \frac{0.58(1,605.72) + 450 + [0.02(1,605.72) + 50]14.1121}{14.1121} = 180.00$$

Alternatively,

$$P^n = \frac{100,000A_{45}}{\ddot{a}_{45}} = 1425.73 \qquad P^e = P^g - P^n = 180$$

$${}_5V^e = \left(0.02P^g + 50\right)\ddot{a}_{50} - P^e\ddot{a}_{50} = [0.02(1,605.72) + 50](13.2668) - 180(13.2668) = -1,298.63$$

Alternatively,

$${}_5V^n = 100,000A_{50} - P^n\ddot{a}_{50}$$

$$= 100,000(0.24905) - 1,425.73(13.2668) = 5,990.13$$

$${}_5V^g = 100,000A_{50} + \left(50 + 0.02P^g - P^g\right)\ddot{a}_{50}$$

$$= 100,000(0.24905) + [50 + 0.02(1,605.72) - 1,605.72](13.2668) = 4,691.63$$

$${}_5V^e = {}_5V^g - {}_5V^n = -1298.51$$

9. Key: D

$$\ddot{a}_{50:\overline{10}|} = \ddot{a}_{50} - {}_{10}E_{50}\,\ddot{a}_{60} = 13.2668 - 0.51081(11.1454) = 7.5736$$

$$A^{1}_{50:\overline{20}|} = A_{50} - {}_{20}E_{50}A_{70} = 0.24905 - 0.23047(0.51495) = 0.13037$$

$$\ddot{a}_{50:\overline{20}|} = \ddot{a}_{50} - {}_{20}E_{50}\ddot{a}_{70} = 13.2668 - 0.23047(8.5693) = 11.2918$$

APV of Premiums = APV Death Benefit + APV Commission and Taxes + APV Maintenance

$$G\ddot{a}_{50:\overline{10}|} = 100,000A^{1}_{50:\overline{20}|} + 0.12G\ddot{a}_{50:\overline{10}|} + 0.3G + 25\ddot{a}_{50:\overline{20}|} + 50$$

$$7.5736G = 13,037 + 1.2088G + 332.30$$

$$6.3648G = 13,369$$

$$\Rightarrow G = 2,100$$

10. Key: E

Let A denote Alive, which is equivalent to not Dead. It is also equivalent to Healthy or Disabled. Let H denote Healthy. The conditional probability is:

$$P(H|A) = \frac{P(H \text{ and } A)}{P(A)} = \frac{P(H)}{P(H) + P(\text{Disabled})},$$

Where

$$P(H) = {}_{10}p^{00} = e^{-\int_0^{10}(\mu^{01}+\mu^{02})ds} = e^{-\int_0^{10}(0.05)ds} = e^{-0.5} = 0.607$$

And

$$P(\text{Disabled}) = {}_{10}p^{01} = \int_0^{10} e^{-\int_0^u(\mu^{01}+\mu^{02})ds}\mu^{01}e^{-\int_u^{10}\mu^{12}ds}du$$

$$= \int_0^{10} e^{-0.05u}(0.02)e^{0.05u-0.5}du$$

$$= \int_0^{10} (0.02)e^{-0.5}du$$

$$= (0.02)e^{-0.5} = 0.121$$

Then

$$P(H|A) = \frac{P(H)}{P(H) + P(\text{Disabled})} = \frac{0.607}{0.607 + 0.121} = 0.83$$

11. Key: A

We need to adjust the cash flows at time 4 and time 5.

$$\ddot{a}_{80:\overline{5}|} = 4.3868 + \frac{870}{1,000} \times \left[\frac{1}{1.05^4} - \frac{1}{1.04^4}\right] = 4.3589$$

$$A^1_{80:\overline{5}|} = 0.1655 + \frac{50}{1000} \times \left[\frac{1}{1.05^4} - \frac{1}{1.04^4}\right] + \frac{60}{1000} \times \left[\frac{1}{1.06^5} - \frac{1}{1.04^5}\right] = 0.1594$$

Benefit Premium $= 100,000 \times \frac{0.1594}{4.3589} = 3657$

12. Key: B

The probability that the endowment payment will be made for a given contract is:

$$
\begin{aligned}
{}_{15}p_x &= \exp\left(-\int_0^{15} 0.02t\,dt\right) \\
&= \exp\left(-0.01t^2\Big|_0^{15}\right) \\
&= \exp\left(-0.01(15)^2\right) \\
&= 0.1054
\end{aligned}
$$

Because the premium is set by the equivalence principle, we have $E\left[{}_0L\right] = 0$. Further,

$$
\begin{aligned}
\text{Var}\left({}_0L\right) &= 500\left[\left(10{,}000v^{15}\right)^2\left({}_{15}p_x\right)\left(1-{}_{15}p_x\right)\right] \\
&= 1{,}942{,}329{,}000
\end{aligned}
$$

Then, using the normal approximation, the approximate probability that the aggregate losses exceed 50,000 is

$$
P\left({}_0L > 50{,}000\right) = P\left(Z > \frac{50{,}000-0}{\sqrt{1{,}942{,}329{,}000}}\right) = P(Z > 1.13) = 0.13
$$

13. Key: B

Time	Age	q_x^{ILT}	Improvement factor	q_x
0	70	0.03318	100.00%	0.03318
1	71	0.03626	95.00%	0.03445
2	72	0.03962	90.25%	0.03576

$v = 1/1.06 = 0.943396$

$$
\begin{aligned}
EPV &= 1{,}000[0.03318v + 0.96682(0.03445)v^2 + 0.96682(0.96555)(0.03576)v^3] \\
&= 88.97
\end{aligned}
$$

14. Key: D

Under the Equivalence Principle

$$P\ddot{a}_{62:\overline{10}|} = 50,000\left(\ddot{a}_{62} - \ddot{a}_{62:\overline{10}|}\right) + P\left((IA)^{\,1}_{62:\overline{10}|}\right)$$

where $(IA)^{\,1}_{62:\overline{10}|} = 11A^{\,1}_{62:\overline{10}|} - \sum_{k=1}^{10} A^{\,1}_{62:\overline{k}|} = 11(0.091) - 0.4891 = 0.5119$

So $$P = \frac{50,000\left(\ddot{a}_{62} - \ddot{a}_{62:\overline{10}|}\right)}{\ddot{a}_{62:\overline{10}|} - (IA)^{\,1}_{62:\overline{10}|}} = \frac{50,000(12.2758 - 7.4574)}{7.4574 - 0.5119} = 34,687$$

15. Key: D

$$\ddot{a}_{x:\overline{3}|} = \frac{\text{Actuarial PV of the benefit}}{\text{Level Annual Premium}} = \frac{152.85}{56.05} = 2.727$$

$$\ddot{a}_{x:\overline{3}|} = 1 + \frac{0.975}{1.06} + \frac{0.975\left(p_{x+1}\right)}{(1.06)^2} = 2.727$$

$\Rightarrow p_{x+1} = 0.93$

Actuarial PV of the benefit =

$$152.85 = 1,000\left[\frac{0.025}{1.06} + \frac{0.975(1-0.93)}{(1.06)^2} + \frac{0.975(0.93)\left(q_{x+2}\right)}{(1.06)^3}\right]$$

$\Rightarrow q_{x+2} = 0.09 \Rightarrow p_{x+2} = 0.91$

16. Key: C

For calculating P

$A_{50} = vq_{50} + vp_{50}A_{51} = v(0.0048) + v(1-0.0048)(0.39788) = 0.38536$

$\ddot{a}_{50} = \left(1 - A_{50}\right)/d = 15.981$

$P = A_{50}/\ddot{a}_{50} = 0.02411$

For this particular life,

$A'_{50} = vq'_{50} + vp'_{50}A_{51} = v(0.048) + (1-0.048)(0.39788) = 0.41037$

$\ddot{a}'_{50} = \left(1 - A'_{50}\right)/d = 15.330$

Expected PV of loss = $A'_{50} - P\ddot{a}'_{50} = 0.41037 - 0.02411(15.330) = 0.0408$

17. Key: B

$$L = 10{,}000v^{K_{45}+1} - P\ddot{a}_{\overline{K_{45}+1}|} = 10{,}000v^{11} - P\ddot{a}_{\overline{11}|}$$
$$4450 = 10{,}000(0.58468) - 8.7217P$$
$$P = (5{,}846.8 - 4{,}450)/8.7217 = 160.15$$
$$A_{55} = 1 - d\ddot{a}_{55} = 1 - (0.05/1.05)(13.4205) = 0.36093$$
$${}_{10}V = 10{,}000A_{55} - P\ddot{a}_{55} = (10{,}000)(0.36093) - (160.15)(13.4205) = 1{,}460$$

18. Key: E

1,020 in the solution is the 1,000 death benefit plus the 20 death benefit claim expense.

$$A_x = 1 - d\ddot{a}_x = 1 - d(12.0) = 0.320755$$
$$G\ddot{a}_x = 1{,}020A_x + 0.65G + 0.10G\ddot{a}_x + 8 + 2\ddot{a}_x$$
$$G = \frac{1{,}020A_x + 8 + 2\ddot{a}_x}{\ddot{a}_x - 0.65 - 0.10\ddot{a}_x} = \frac{1{,}020(0.320755) + 8 + 2(12.0)}{12.0 - 0.65 - 0.10(12.0)} = 35.38622$$

Let $Z = v^{K_x+1}$ denote the present value random variable for a whole life insurance of 1 on (x).
Let $Y = \ddot{a}_{\overline{K_x+1}|}$ denote the present value random variable for a life annuity-due of 1 on (x).

$$\begin{aligned} L &= 1{,}020Z + 0.65G + 0.10GY + 8 + 2Y - GY \\ &= 1{,}020Z + (2 - 0.9G)Y + 0.65G + 8 \\ &= 1{,}020v^{K_x+1} + (2 - 0.9G)\frac{1 - v^{K_x+1}}{d} + 0.65G + 8 \\ &= \left(1{,}020 + \frac{0.9G - 2}{d}\right)v^{K_x+1} + \frac{2 - 0.9G}{d} + 0.65G + 8 \end{aligned}$$

$$\begin{aligned} Var(L) &= \left[{}^2A_x - (A_x)^2\right]\left(1{,}020 + \frac{0.9G - 2}{d}\right)^2 \\ &= (0.14 - 0.320755^2)\left(1{,}020 + \frac{0.9(35.38622) - 2}{d}\right)^2 \\ &= 0.037116(2{,}394{,}161) \\ &= 88{,}861 \end{aligned}$$

19. Key: D

If $T_{45} = 10.5$, then $K_{45} = 10$ and $K_{45} + 1 = 11$.

$${}_0L = 10{,}000 v^{K_{45}+1} - G(1-0.10)\ddot{a}_{\overline{K_{45}+1|}} + G(0.80-0.10) = 10{,}000 v^{11} - 0.9G\ddot{a}_{\overline{11|}} + 0.7G$$

$$3{,}767 = 10{,}000(0.52679) - 0.9G(8.3601) + 0.7G$$

$$G = (5{,}267.9 - 3767)/(6.8241) = 219.94$$

$$E({}_0L) = 10{,}000 A_{45} - (1-0.1)G\ddot{a}_{45} + (0.8-0.1)G$$

$$= (10{,}000)(0.20120) - (0.9)(219.94)(14.1121) + (0.7)(219.94)$$

$$E({}_0L) = -627.48$$

20. Key: E

Defined Benefit:

$0.015 \times \text{Final Average Earnings} \times \text{Years of Service}$

$$= 0.015 \times \left(50{,}000 \times \left(1.05^{19} + 1.05^{18} + 1.05^{17}\right)/3\right) \times 20 = 36{,}128 \text{ per year}$$

APV at 65 of Defined Benefit $= 36{,}128\ddot{a}_{65} = 36{,}1268(10.0) = 361{,}280.$

Defined Contribution accumulated value at65:

$$X\% \times 50{,}000 \times 1.05^{20} + X\% \times (50{,}000 \times 1.05) \times 1.05^{19} + \ldots + X\% \times \left(50{,}000 \times 1.05^{19}\right) \times 1.05$$

$$= X\% \times 50{,}000 \times 1.05^{20} \times 20 = X\%(2{,}653{,}298)$$

Therefore,

$$361{,}280 = X\%(2{,}653{,}298)$$

$$X\% = 0.136$$

$$X = 13.6$$

21. Key: A

$$ {}_t p_x^{00} = \exp\left[-\int_0^t \left(\mu_{x+s}^{01} + \mu_{x+s}^{02} \right) ds \right] = \exp\left[-\int_0^t \left(0.20 + 0.10s + 0.05 + 0.05s \right) ds \right] $$

$$ = \exp\left[-\left(0.25s + 0.075s^2 \right) \Big|_0^t \right] = \exp\left(-0.25t + 0.075t^2 \right) $$

$$ {}_3 p_x^{00} = \exp\left(-0.25 \times 3 + 0.075 \times 9 \right) = \exp(-1.425) = 0.2405 $$

$$ EPV = \int_0^n g(t)dt, \text{ where } g(t) = 10,000 \times \left({}_t p_x^{00} \mu_{x+t}^{02} + {}_t p_x^{01} \mu_{x+t}^{12} \right) e^{-\delta t} dt $$

$$ g(3) = 10,000 \times \left[0.2405 \times \left(0.05 + 0.05 \times 3 \right) + 0.4174 \times \left(0.15 + 0.01 \times 3^2 \right) \right] \times e^{-3 \times 0.02} = 1,400 $$

22. Key: B

$$ AV_{11} = 1500 $$
$$ COI_{12} = \frac{10,000(0.003)}{1.004} = 29.88 $$
$$ AV_{12} = (1,500 + 100(1 - 0.15) - 10 - 29.88)(1.004) = 1,551.3 $$
$$ SV_{12} = 1551.3 - 400 = 1151.3 $$

Then

$$ 1,151.3 = X\left(\ddot{a}_{\overline{10|}} + {}_{10}E_{61}\, \ddot{a}_{71} \right) = X(7.8017 + 0.44231(8.2988)) = 11.4723X $$
$$ X = 100.35 $$

23. Key: E

$$AV_0 = 0$$

$$AV_1 = \left[3{,}000(1-0.7) - 75 - \frac{150{,}000(0.00122)}{1.04}\right](1.04) = 675$$

$$AV_2 = \left[675 + 3{,}000(1-0.1) - R - \frac{150{,}000(0.00127)}{1.04}\right](1.04)$$

$$= \left[(3375 - R) - 183.17\right](1.04)$$

$$= 3319.50 - R(1.04)$$

$$AV_3 = 6{,}028.95 = \left[3{,}319.50 - R(1.04) + 3{,}000(1-0.1) - R - \frac{150{,}000(0.00133)}{1.04}\right](1.04)$$

$$\Rightarrow \left[6{,}019.50 - 2.04R - 191.83\right](1.04)$$

$$6060.78 - 2.12R = 6028.95$$

$$\Rightarrow R = 15$$

24. Key: B

Let S denote the number of survivors.
This is a binomial random variable with $n = 4000$ and success probability $\frac{2,358,246}{9,565,017} = 0.24655$

$E(S) = 4,000(0.24655) = 986.2$

The variance is $Var(S) = (0.24655)(1-0.24655)(4,000) = 743.05$

$StdDev(S) = \sqrt{743.05} = 27.259$
The 90% percentile of the standard normal is 1.282

Let S^* denote the normal distribution with mean 986.2 and standard deviation 27.259.
Since S is discrete and integer-valued, for any integer s,

$$\Pr(S \geq s) = \Pr(S > s-0.5) \approx \Pr(S^* > 0.5)$$
$$= \left(\frac{S^* - 986.2}{27.259} > \frac{s - 0.5 - 986.2}{27.259} \right)$$

For 90%, $\frac{s - 0.5 - 986.2}{27.259} < -1.282$

$\Rightarrow s < 951.754$

So $s = 951$ is the largest integer that works
$s = 950$ is the largest from the list

25. Key: E

Using UDD

$$l_{63.4} = (0.6)66,666 + (0.4)(55,555) = 62,221.6$$
$$l_{65.9} = (0.1)(44,444) + (0.9)(33,333) = 34,444.1$$
$${}_{3.4|2.5}q_{60} = \frac{l_{63.4} - l_{65.9}}{l_{60}} = \frac{62,221.6 - 34,444.1}{99,999} = 0.277778 \qquad \text{(a)}$$

Using constant force

$$l_{63.4} = l_{63}\left(\frac{l_{64}}{l_{63}}\right)^{0.4} = l_{63}^{0.6}\, l_{64}^{0.4}$$
$$= \left(66,666^{0.6}\right)\left(55,555^{0.4}\right)$$
$$= 61,977.2$$
$$l_{65.9} = l_{65}^{0.1}\, l_{66}^{0.9} = \left(44,444^{0.1}\right)\left(33,333^{0.9}\right)$$
$$= 34,305.9$$

$${}_{3.4|2.5}q_{60} = \frac{61,977.2 - 34,305.9}{99.999}$$
$$= 0.276716 \qquad \text{(b)}$$

$$100,000(a-b) = 100,000(0.277778 - 0.276716) = 106$$