CFE FD Model Solutions
Fall 2013

1. **Learning Objectives:**
1. The candidate will understand how a business enterprise funds its activities with considerations for its business model, and the cost and constraints of the sources of capital.

2. The candidate will understand how an enterprise's structure and policies allow its management to prioritize and select among projects or business activities that are competing for scarce capital resources.

Learning Outcomes:
(1c) Interpret financial reinsurance and securitization as a form of capital funding.

(2d) Evaluate the capital efficiency of using reinsurance or securitizations for a given risk.

Sources:
F-100-13: Dynamic Financial Condition Analysis Handbook 8

F-102-13: Chapter 5 of Life, Health and Annuity Reinsurance, Tiller

Commentary on Question:
The question tests whether the candidate understands how reinsurance can be used as a financial planning tool and the different alternatives available to an insurer for dealing with surplus strain.
In general, candidates scored well on this question.

Solution:
(a) Evaluate the appropriateness of each of the four alternatives for Meigs. Support your evaluations.

Commentary on Question:
Cognitive Level: Analysis
To obtain full credit, the candidate must
- *State whether each alternative is appropriate or not in the context of the question*
- *Support the evaluation of each alternative with respect to surplus relief, time constraints and the ability to maintain control of assets*

1. Continued

Candidates who determined that a particular alternative is or is not appropriate when stated otherwise in the model solution below received full credit if supporting arguments were well presented.

Surplus Notes:
This is an appropriate alternative for Meigs.
Surplus notes can count as capital, equity or surplus. Surplus notes are a relatively fast, simple and straightforward source of capital.
Surplus notes allow the company to maintain full control of assets

YRT Reinsurance:
This is not an appropriate alternative for Meigs.
YRT reserves are relatively small. YRT does not provide much surplus relief.

Coinsurance:
This is not an appropriate alternative for Meigs.
Ceding company must transfer control of assets [equal to the reserves] to the reinsurer.
Transfer of assets is against the stated goal of maintaining control.

ModCo:
This is an appropriate alternative for Meigs.
ModCo provides surplus relief.
Ceding company maintains the reserves and assets [supporting the reserves].
Ceding company avoids liquidating or transferring ownership of the assets [to reinsurer].
This alternative satisfies the company's desire to maintain control of assets.

(b)   Calculate the mod-co adjustment as of 1/1/2015 and 12/31/2015. Show your work.

Commentary on Question:
Cognitive Level: Comprehension
This question focused on the calculation of the ModCo adjustment.
In order to receive full credit, the candidate must
- *Identify that the ceded reserve is 75% of reserves*
- *Incorporate interest on beginning reserve in calculating the ModCo adjustment at 12/31/2015*

Modco adjustment at 1/1/2015 = 75% x $40 = $30

Modco adjustment at 12/31/2015:
         ending reserve 75% of $60, or $45
         less beginning reserve $30

1.   Continued

       less interest on beginning reserve 10% * $30 = $3

       Modco adjustment at 12/31/2015 = $12.

(c)   Describe two advantages and two disadvantages of setting up a trust or escrow account to segment the relevant assets if using the mod-co alternative.

Commentary on Question:
Cognitive Level: Comprehension
The list of advantages and disadvantages given below is not exhaustive. Other reasonable advantages and disadvantages received full credit, provided the response was well explained.
A trust being a solution to the reserve credit problem was not accepted as an advantage because this does not apply to ModCo.
If less counterparty risk was listed as an advantage, the candidate had to clearly identify that the reinsurer was exposed to this risk in order to receive full credit.

Advantages:
- Investment income can be limited to the performance of specific assets
- A trust is a true transfer of ownership and is less suspect to both regulators and the IRS

Disadvantages:
- Use of a trust or escrow creates additional expenses
- Transfer of assets to a trust may necessitate recognition of capital gains and losses

2. Learning Objectives:

3. The candidate will understand how and when to apply various stochastic techniques to situations which have uncertain financial outcomes.

4. The candidate will understand how to critique the appropriateness of advanced risk assessment methods for a given situation.

Learning Outcomes:

(3f) Explain the differences and implications of the use of P-measure and Q-measure for risk assessment.

(4a) Apply and interpret the results of equilibrium pricing and no-arbitrage pricing theory to risk valuation.

Sources:
F-110-13: Chapters 4 and 5 of Financial Economics, Panjer, Harry, Et. Al.

Commentary on Question:
This question was testing basic understanding of two modern financial pricing tools - equilibrium and no arbitrage. Candidates needed to:

- *Explain and contrast the two methods,*
- *Understand the state price vector and apply to a one period model,*
- *Understand the difference between a P-Measure and Q-Measure,*
- *Apply the no-arbitrage principle to price a derivative and demonstrate basic knowledge about the replicating portfolios, and*
- *Apply the pricing approaches in business decision making.*

Candidates generally did well on part (b) through (e). Candidates surprisingly struggled with part (a), which was meant to be fairly straightforward. However quite a few candidates confused the pricing methods with discounting the cash flows with risk neutral and market rates. Part (f) was another part on which candidates struggled a bit. Most of them tried to distinguish the applications for non-arbitrage vs. equilibrium but failed to explain why non-arbitrage was not appropriate for this specific case.

Solution:

(a) Contrast no-arbitrage pricing and equilibrium pricing with regard to:

  (i) principles used in each methodology

  (ii) major applications in financial markets

  Commentary on Question:
  This part, which was a Comprehension cognitive level question, tested basic understanding of two modern financial pricing tools - equilibrium and no arbitrage.

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2. Continued

No Arbitrage Pricing:
We only gave credit if they said either two securities with the same payoff had the same price (or something equivalent to that) or pricing was such that no one could make a profit without risk.

For the examples, candidates had to specifically state derivatives. We did not accept hedging since technically equilibrium pricing could also be used.

Equilibrium pricing
For the description, the candidate needed to mention all parties unwilling to trade at current price because utility was maximized to get full credit.

For the examples, candidates had to give generic answers as was listed in answer key. We did not accept any specific answer (i.e. pricing, valuation, back testing, forecasting, etc.) since technically either approach could be used.

(i)   No-arbitrage pricing is based on the (no-arbitrage) principle that (in well-functioning financial markets) two securities that have the same payoffs must trade at the same price.
        Equilibrium pricing relates the price of securities to more fundamental economic concepts – no party has an incentive to trade at current price because utility has been maximized.

(ii)   No-arbitrage pricing is widely used in pricing derivative securities. The equilibrium approach provides a more general framework for analyzing markets and pricing securities.

(b)   Calculate the state price vector. Show your work

Commentary on Question:
This part, which was a Comprehension cognitive level, tested whether the candidate understood the state price vector and could apply the concept to a one period model.

Partial credit was given for each correct vector in the formula (excluding the starting vector) and for setting up the correct equations in the systems of equations, even if the candidate did not ultimately solve the system of equations correctly.

$S(0) = \psi^* S(1, \Omega)$

$$(1, 1, 1) = (\psi_1, \psi_2, \psi_3) \begin{pmatrix} 1 & 1.2 & 1.5 \\ 1 & 1 & 1.1 \\ 1 & 0.75 & 0.35 \end{pmatrix}$$

2. Continued

$$\begin{cases} 1 = \psi_1 + \psi_2 + \psi_3 \\ 1 = 1.2\,\psi_1 + \psi_2 + 0.75\,\psi_3 \\ 1 = 1.5\,\psi_1 + 1.1\,\psi_2 + 0.35\,\psi_3 \end{cases}$$

$$\begin{cases} \psi_1 = 0.5 \\ \psi_2 = 0.1 \\ \psi_3 = 0.4 \end{cases}$$

(c) Explain why a state price vector would not exist if the bond interest rate were above 20% per period in all scenarios.

Commentary on Question:
This part required Analysis on the part of the candidates as they needed to show an understanding of state price vectors and their underlying meaning. Besides the more detailed answer below, full credit was given if the candidate mentioned arbitrage was possible and also how to do it. Half credit was given if the candidate only stated either. No credit was given for simply stating the system of equations could not be solved.

A state price vector exists if there is a strictly positive solution to the linear equations with the new returns for bond X. All positive solutions to the linear equations would result in a (risk-neutral) expected payoff between 0.75 and 1.2 for stock Y. Since bond X has an expected return above 1.2, outside the range for stock Y, a strictly positive solution, and thus a state price vector, does not exist.

(d) Calculate the price of a one-period put option written on stock Y with strike price 1.05. Show your work.

Commentary on Question:
This part used the Comprehension cognitive level. Partial credit was given for having either the correct put option payoff or the correct formula.

Q(w)=(1+i)ψ(w),w∈Ω
Q(w)=ψ(w)
From (b): Q1 = 0.5, Q2 = 0.1, and Q3 = 0.4

Full credit was granted for correctly substituting Q1, Q2, and Q3 from part (b) even if Q1, Q2, and Q3 are incorrect.
Full credit was granted if the candidate recognized that the Q-measure is the same as the P-measure.

2. **Continued**

For a put option the cash flows are: x1 = 0, x2 = 0.05 and x3 = 0.3
Using the Q-measure the option price is Q1*x1 + Q2*x2 + Q3*x3 = 0 + 0.1*0.05 + 0.4*0.3 = 0.125

(e) Construct a replicating portfolio for the put option on stock Y using all three assets. Show your work.

Commentary on Question:
This part used the Analysis cognitive level. There was no penalty for using incorrect put payoffs (if consistent with d). Some credit was given for an incorrect answer where the candidate otherwise used the correct formulas with the incorrect payoffs from d, but had one calculation wrong (sign flipped).

$S(1,\Omega)\Theta = X$, where X1 = 0, X2 = 0.05, and X3 = 0.3

$$\begin{pmatrix} 1 & 1.2 & 1.5 \\ 1 & 1 & 1.1 \\ 1 & 0.75 & 0.35 \end{pmatrix}\begin{pmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{pmatrix} = \begin{pmatrix} X1 \\ X2 \\ X3 \end{pmatrix}$$

$$\begin{cases} \Theta_1 = -0.375 \\ \Theta_2 = 1.25 \\ \Theta_3 = -0.75 \end{cases}$$

This means that the option is replicated by borrowing 0.375 in bond X, investing 1.25 in stock Y and shorting 0.75 in commodity Z.

(f) Evaluate the student's statement.

Commentary on Question:
This part, which required Knowledge Utilization, had candidates apply the pricing approaches to business decision making. Some candidates incorrectly flipped the reasons between risk free rate and market rate. Also some failed to mention that it's not correct to use non arbitrage for the new product pricing.

As the existing models may not be no-arbitrage models, using those cash flows and then discounting at the risk-free rate may produce inappropriate results. Some products cannot be priced using the no-arbitrage approach because the current market may not have instruments available to replicate the cash flows for those products .

3.   **Learning Objectives:**
3.   The candidate will understand how and when to apply various stochastic techniques to situations which have uncertain financial outcomes.

Learning Outcomes:
(3g)   Explain the benefits and limitations of Value-at-Risk, Incremental Value-at-Risk, Component Value-at-Risk, and Expected Shortfall as tail risk measures.

Sources:
Stochastic Simulation and Applications in Finance, Huynh, Huu Tue, et. al.
- Chapter 14 Risk Management and VaR

Measures of Mark Risk, Dowd, Kevin, 2nd Edition
- Chapter 2 Measures of Financial Risk
- Chapter 11 Incremental and Component Risk

Commentary on Question:
This question tested whether the candidate had a basic understanding of VaR with an emphasis on calculation and a major limitation of VaR.

Very few candidates received full credit for this question. If the approach was correct, the candidate received partial credit even if the computations were not numerically accurate.

Solution:
(a)

(i)   Define the Value at Risk (VaR) measure without using formulas.

(ii)   Identify two parametric approaches to calculating the VaR.

Commentary on Question:
Cognitive Level: Retrieval
Very few candidates obtained full credit for part (a).
To obtain full credit for part (i), the candidate must mention that VaR is specific to a given confidence level as well as a given time frame.
For part (ii), only one of the distributions (Normal, Log-normal, Gamma) were accepted

(i)   VaR is an aggregate measure of risk that can be defined as the expected extreme loss emerging from the ownership of a portfolio or an asset during a specific period of time given a specific confidence level.

(ii)   Parametric approaches:
         Delta-Normal or Variance-Covariance method
         Monte-Carlo Simulation

3. **Continued**

(b)   Describe two examples of how financial firms or traders might exploit the fact
that VaR is not subadditive.

Commentary on Question:
Cognitive Level: Comprehension
*Very few candidates obtained full credit for part (b). Most candidates were able
to provide one example. In order to obtain full credit, the example must be well
explained.*

Traders may separate their accounts on organized exchanges to lower their margin
requirements, which would leave the exchange exposed to possible loss since the
margin requirements on separate accounts no longer cover the combined risk.

If regulators use non-subadditive risk measures to set capital requirements, then a
financial institution may be tempted to break itself up to reduce its capital
requirements because the sum of the capital requirements of the smaller entities
may be less than the capital require of the firm as a whole.

(c)   Calculate the 10-day 99.5% VaR for each of the following. Show your work.

   (i) $600 of asset A

   (ii) $400 of asset B

   (iii) Portfolio C

Commentary on Question:
Cognitive Level: Comprehension
*Most candidates received full credit for parts (i) and (ii), but did not receive full
credit for part (iii) due to incorrectly computing the portfolio standard deviation.
A recurring mistake for this part of the question was setting the portfolio variance
equal to the volatility of 1% given in the question. In this case, candidates
received appropriate partial credit.*

   (i)   VaR for $600 investment in asset A
          $\sigma_A^2 = (600 \times 1\%)^2 = 6^2 = 36$ which implies that $\sigma_A = 6$.
          Given that $z_\alpha = 2.576$ at the 99.5% confidence interval,
          $VaR_A = $ (square root of 10) $\times \sigma_A \times z_\alpha = 3.162 \times 6 \times 2.576 = 48.9$

          Or (in terms of a $1 portfolio times size of portfolio)

3. Continued

$\sigma = 1\%$
Given that $z_\alpha = 2.576$ at the 99.5% confidence interval,
$\text{VaR}_A = 600$ x (square root 10) x σ x $z_\alpha = 600$ x 3.162 x 0.01 x 2.576 = 48.9

(ii) VaR for \$400 investment in B
$\sigma_B^2 = (400 \text{ x } 1\%)^2 = 4^2 = 16$ which implies that $\sigma_B = 4$.
Given that $z_\alpha = 2.576$ at the 99.5% confidence interval,
$\text{VaR}_B = $ (square root of 10) x σ_B x $z_\alpha = 3.162$ x 4 x 2.576 = 32.6

Or (in terms of a \$1 portfolio times size of portfolio)

$\sigma = 1\%$
Given that $z_\alpha = 2.576$ at the 99.5% confidence interval,
$\text{VaR}_B = $ (square root of 10) x σ x $z_\alpha = 400$ x 3.162 x 0.01 x 2.576 = 32.6

(iii) VaR for the portfolio
$\sigma_P^2 = \sigma_A^2 + \sigma_B^2 + 2 \rho \sigma_A \sigma_B = 36 + 16 + 2 \text{ x } 0.5 \text{ x } 6 \text{ x } 4 = 76$
which implies that $\sigma_P = 8.718$.
Given that $z_\alpha = 2.576$ at the 99.5% confidence interval,
$\text{VaR}_P = $ (square root of 10) x σ_P x $z_\alpha = 3.162$ x 8.718 x 2.576 = 71.0

Or
Take $\sigma_A = \sigma_B = 0.01$
$\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 \rho w_A w_B \sigma_A \sigma_B$
$= (0.6 \text{ x } 0.01)^2 + (0.4 \text{ x } 0.01)^2 + 2 \text{ x } 0.5 \text{ x } 0.6 \text{ x } 0.4 \text{ x } 0.01 \text{ x } 0.01$
$= 0.000076$
which implies that $\sigma_P = 0.008718$.
Given that $z_\alpha = 2.576$ at the 99.5% confidence interval,
$\text{VaR}_P = 1000$ x (square root of 10) x σ_P x z_α
$= 1000$ x 3.162 x 0.008718 x 2.576 = 71.0

(d)

(i) Assess the reliability and flexibility of the delVaR approach to estimating incremental VaR (IVaR).

(ii) Calculate the 1-day 99.5% VaR of portfolio C. Show your work.

(iii) Calculate the IVaR of portfolio C at the 99.5% confidence level for a purchase of \$10 in asset A using the delVaR approach. Show your work.

3. **Continued**

Commentary on Question:
Cognitive Level: Analysis
Few candidates received full credit for this part of the question.

Part (i) expects an evaluation of the reliability of the delVaR approach. A description of the delVaR approach did not receive credit.

Most candidates were able to score points for part (ii). Some candidates did not realize that they can use the formula for portfolio VaR which was provided.

Candidates did very poorly on part (iii), although this was an example that was clearly laid out in the source reading. Computing the new portfolio VaR using the portfolio VaR formula provided and taking the difference between this and the answer to part (ii) did not receive credit as the question requires candidates to use the delVaR approach.
Some candidates who approached this problem correctly did not compute the partial derivatives with respect to both assets, and did not use the correct IVaR formula
(Candidates who substituted values into one part of the formula pertaining to only one asset and ignored the second asset did not receive partial credit for the last step of the problem)

(i)   When the new position or trade is considered small relative to the size of the original portfolio, the delVaR approximation to IVaR is reliable. However, if the trade is large, then the first order approximation used here is poor.
Approximation also fails if there are a large number of small trades

(ii) $VaR_P = P\sigma (w_A^2 + w_B^2 + 2 \rho w_A w_B)^{1/2} z_\alpha$
$= 1000 \times 0.01 \times (0.6^2 + 0.4^2 + 2 \times 0.5 \times 0.6 \times 0.4)^{1/2} \times 2.576 = 22.4$

(iii)   *Relative Shares - Partial Derivatives*

$VaR_P = P\sigma (w_A^2 + w_B^2 + 2 \rho w_A w_B)^{1/2} z_\alpha$

$\partial VaR_P / \partial w_A = P\sigma (w_A + \rho w_B) (w_A^2 + w_B^2 + 2 \rho w_A w_B)^{-1/2} z_\alpha$
$= 1000 \times 0.01 \times (0.6 + 0.5 \times 0.4) \times (0.6^2 + 0.4^2 + 2 \times 0.5 \times 0.6 \times 0.4)^{-1/2} \times 2.576$

$= 23.64$

3. **Continued**

$$\partial \text{ VaR}_P / \partial \text{ w}_B = P\sigma (w_B + \rho w_A) (w_A^2 + w_B^2 + 2 \rho w_A w_B)^{-1/2} z_\alpha$$
$$= 1000 \text{ x } 0.01 \text{ x } (0.4 + 0.5 \text{ x } 0.6) \text{ x } (0.6^2 + 0.4^2 + 2 \text{ x } 0.5 \text{ x }$$
$$0.6 \text{ x } 0.4)^{1/2} \text{ x } 2.576$$
$$= 20.68$$

Change in Relative Shares - Add $10 of asset A
dwA = 610 / 1010 - 0.6 = 0.004
dwB = 400 / 1010 - 0.4 = - 0.004

Plugging all of the above calculations in, we find the incremental VaR
from adding $10 in asset A to the portfolio to be:
IVaR = 23.64 x 0.004 - 20.68 x 0.004 = 0.012.

4. **Learning Objectives:**
3. The candidate will understand how and when to apply various stochastic techniques to situations which have uncertain financial outcomes

Learning Outcomes:
(3b) Assess the appropriateness of a given stochastic simulation technique to quantify various market risk exposures.

(3c) Recommend the use of techniques to reduce the computational demand when applying stochastic methodology.

(3d) Assess the strengths and weaknesses of the calibration techniques for a given stochastic model.

Sources:
Stochastic Simulation and Applications in Finance, Huynh, Huu Tue, et. al.
- Chapter 15 VaR and Principal Component Analysis

Commentary on Question:
This question tested candidates' intuitive grasp of important concepts related to Principal Component Analysis. Parts (a)(i) and (b) were Analysis, part (a)(ii) was Comprehension and part (c) was Knowledge Utilization.

Solution:
(a) The actuarial student recommends that you retain all five principal components for analysis to avoid discarding valuable information on the underlying data.

    (i) Critique her recommendation.

    (ii) Propose a reasonable number of principal components to retain. Show your work.

Commentary on Question:
Few candidates got full credit on this part. Many candidates offered a good critique, but missed the point of the reduction in dimensionality. This reduction of dimensionality gives an improved analytical lens, but not necessarily a reduction in computational resources.

The calculation of the explained portion of variance of the data was done well. Candidates generally chose to keep 2 or 3 principal components and were given credit if they justified either answer.

4. Continued

        (i)   The purpose of PCA is to reduce dimensionality, and retaining all 5 principal components would not reduce dimensionality. While it is true that removing principal components removes information, most information can be retained even while eliminating several principal components.

        (ii)   Proportion of variance explained by the first X principal components is equal to the sum of their eigenvalues divided by the sum of all components' eigenvalues. Choosing 2 principal components would be reasonable. This should explain 99% of the variance of the model.

(b)   Describe one advantage and one disadvantage of using PCA instead of duration to measure interest rate risk.

Commentary on Question:
Several candidates said that a disadvantage of PCA was that it was more computationally intensive. This answer was not given credit.

*The model solution below contains more than one response to show examples of answers that were acceptable. However, credit was only given for **the first** advantage a candidate had written and **the first** disadvantage a candidate had written.*

Advantage
1. Principal components more precisely explain underlying movements in the yield curve.
2. Hedging principal components can immunize Mount Hardy Life against likely yield curve movements, whereas hedging duration protects against the less likely event of a sudden parallel shift.
3. Hedging based on principal components better recognizes that the yield curve is multidimensional.

Disadvantage
1. Having zero net exposure to the first X principal components does not guarantee that duration of surplus is zero, which may have adverse impacts vis-à-vis regulators and rating agencies.
2. PCA is more difficult to explain to the company's own management than duration matching.
3. PCA analysis is dependent on additional modeling assumptions not relevant to duration: how far back should data go, how many tenors should be used to derive principal components, how many components to retain?
4. PCA is more complicated.

4. **Continued**

(c)   Recommend an improvement on the student's suggestion. Justify your recommendation.

Commentary on Question:
A common answer was to suggest that principal components should be recalculated monthly or quarterly. This is a similar argument to the computational intensity argument from part (b) and was only given partial credit.

I would recommend removing older data at the same time that newer data is added, to keep the amount of data constant. Older data has less relevance for today's interest rate movements, so it should not be kept indefinitely.

5. **Learning Objectives:**
1.   The candidate will understand how a business enterprise funds its activities with considerations for its business model, and the cost and constraints of the sources of capital.

Learning Outcomes:
(1b)   Assess the various features and implications of various sources of capital funding and recommend the optimal approach for funding.

Sources:
Corporate Finance, Berk, Jonathan and Demarzo, Peter, 2rd Edition, Chapter 15

Corporate Finance, Berk, Jonathan and Demarzo, Peter, 2rd Edition, Chapter 18

Commentary on Question:
This question tested candidates' ability to calculate, re-lever, and use a firm's weighted average cost of capital to value an investment project. It also tested candidates' knowledge of interest tax shields, free cash flows, and free cash flows to equity.

Solution:
(a)

        (i)   Calculate Linville's weighted average cost of capital before issuing the new debt. Show your work.

        (ii)   Calculate Linville's weighted average cost of capital after issuing the new debt and repurchasing $25 million of equity. Show your work.

        Commentary on Question:
        Part (a) tested candidates' comprehension. Candidate performance on this part was mixed. Successful candidates re-levered the return on equity before calculating the new WACC.

        (i)
                $E = 100, D = 60, rD = 4\%, rE = 12\%, t = 25\%$

                $WACC = E / (E + D) * rE + D / (E + D) * rD * (1 - t)$
                $WACC = 100 / (100 + 60) * 12\% + 60 / (100 + 60) * 4\% * (1 - 25\%) = 8.63\%$

        (ii)
                $E = 100, D = 60, rD = 4\%, rE = 12\%, t = 25\%$

                $rU = E / (E + D) * rE + D / (E + D) * rD$
                $rU = 100 / (100 + 60) *12\% + 60 / (100 + 60) * 4\% = 9.0\%$

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5. **Continued**

E' = E + PV(Interest Tax Shield) - Share Repurchase
E' = 100 + (25% x 20) - 25 = 80

rD' = 60 / 80 * 4% + 20 / 80 * 6% = 4.5%

rE' = rU + D' / E' * (rU - rD')
rE' = 9.0% + 80 / 80 * (9.0% - 4.5%) = 13.5%

WACC' = E' / (E' + D') * rE' + D' / (E' + D') * rD' * (1 - t)
WACC' = 80 / (80 + 80) * 13.5% + 80 / (80 + 80) * 4.5% * (1 - 25%) = 8.44%

<u>Alternate solution</u>

WACC' = rU - d' * t * rD'
WACC' = 9.0% - 80/160 * .25 * 4.5% = 8.44%

(b)   Calculate the following, showing your work:

   (i)   The unlevered value of the project

   (ii)   The levered value of the project

   (iii) The value of the interest tax shield generated by the project

Commentary on Question:
Part (b) tested candidates' comprehension. Candidates generally performed well on this question. Successful candidates used the appropriate discount rate for each part.

(i)
   *rU = 80/160 * 13.5% + 80/160 * 4.5% = 9.0% (from question (a)(ii))*
   FCF = EBIT * (1 - t) + Depreciation
   FCF = 2 * (1 - .25) + 4 = 5.5 per year for 2 years
   Vu = 5.5 / 1.09 + 5.5 / 1.09 ^ 2 = 9.68

(ii)
   WACC = 8.44% (from question (a)(ii))
   Vl = 5.5 / 1.0844 + 5.5 / 1.0844 ^ 2 = 9.75

(iii)
   Vl = Vu + PV(tax shield)
   PV(tax shield) = Vl - Vu = 9.75 - 9.68 = 0.07

5. Continued

Alternate solution

	Year 0	Year 1	Year 2
Vl (rL = 8.44%)	9.75	5.07	0
Debt Level (50% of Vl)	4.88	2.54	0
Interest (4.5% of Dt-1)		0.22	0.11

PV(tax shield) = 0.25 * (0.22 / 1.09 + 0.11 / 1.09 ^ 2) = 0.07 *(unlevered discount rate is used because of constant D/E ratio)*

(c)   Calculate the project's free cash flows to equity (FCFE). Show your work.

Commentary on Question:
Part (c) tested candidates' analysis ability. Candidate performance on this question was mixed. Successful candidates had a strong grasp of the components of an income statement and statement of cash flows

FCFE = FCF - Interest (1 - t) + Net Borrowing

	Year 0	Year 1	Year 2
FCF		5.50	5.50
CapEx	-8.00		
Interest * (1 - .25)		-0.14	-0.07
Net Borrowing	4.00	-2.00	-2.00
FCFE	**-4.00**	**3.37**	**3.43**

Alternate solution

FCFE = Net Income + Depreciation - CapEx + Net Borrowing

	Year 0	Year 1	Year 2
Net Income		1.36	1.43
CapEx	-8.00		
Net Borrowing	4.00	-2.00	-2.00
Depreciation		4.00	4.00
FCFE	**-4.00**	**3.36***	**3.43**

*Difference due to rounding

6. Learning Objectives:

3. The candidate will understand how and when to apply various stochastic techniques to situations which have uncertain financial outcomes.

4. The candidate will understand how to critique the appropriateness of advanced risk assessment methods for a given situation.

Learning Outcomes:

(3d) Assess the strengths and weaknesses of the calibration techniques for a given stochastic model.

(4c) Describe the limitations and modeling dependences in risk variables.

(4d) Apply techniques to estimate tail correlation for long dated liabilities.

Sources:

Measures of Market Risk, Dowd, Kevin, 2nd Edition
- Chapter 5 Forecasting Volatilities Covariances and Correlation Including Appendix – Modeling Dependence: Correlations and Copula

A Practical Concept of Tail correlation, 2008 Enterprise Risk Management Monograph

Commentary on Question:
Commentary listed underneath question component.

Solution:

(a) Contrast implied and historical volatilities.

Commentary on Question:
Cognitive Level: Comprehension
Most candidates did well on this question. At least two points were required to get full credit.

- Historical volatility estimates use historic returns to calculate volatility, while implied volatility estimates use available market prices to calculate volatility.
- Implied volatility incorporates information that is not present in the historical data. Historical volatility does not.
- Implied volatility provides an estimate of volatility on which experienced market operators have confidence to bet real money.
- Historic volatilities are easier to get, because implied volatilities can only be obtained for assets on which options have been written.
- Implied volatilities are contingent on the accuracy of the model being used. This is a problem because models have well-known limitations, including "Holes in Black-Scholes."

6. **Continued**

(b)   Compare and contrast using equal-weighted and exponentially weighted moving average models for estimating the volatility of equity returns.

Commentary on Question:
Cognitive Level: Comprehension
Most candidates did ok on this question. At least two points were required to get full credit and some candidates only gave one point.

Equal-weighted moving average	Exponentially-weighted moving average
Assumes that true volatility is constant	Assumes that true volatility changes over time in a stable way
More distant events have the same effect as more recent once. This produces ghost effect - an event influences next n volatility estimates (where n is the number of estimations used) and then volatility suddenly returns to normal.	Assigns greater weight to more recent events. Less prominent ghost effects.
Predicts that volatility will remain at its current level	Predicts that volatility will remain at its current level
Sample size is fixed	Sample can be extended
Doesn't allow volatility clustering	Allows for volatility clustering
Accounts for new observations relatively slowly	Accounts for new observations quickly

(c)   Describe three shortcomings with using correlations as a measure of dependency.

Commentary on Question:
Cognitive Level: Comprehension
Few candidates got full credit for this question. Most candidates got half credit. Many answers were either incomplete or not precise enough. Three points were required for full credit.

- For correlation to exist, it is necessary that x and y are jointly stationary and have finite covariances.
- Both series have to come from elliptical distributions.
- Estimators of correlation are often very volatile.
- Correlations can appear to be fairly stable in normal market conditions, and then jump to very high or very low values in stressful situations, when they are needed the most.
- Zero correlation does not necessarily imply that risks are independent.
- Correlation is not invariant to transformations of the underlying variables.

6. Continued

(d)   Calculate the aggregate diversification factor. Show your work.

Commentary on Question:
Cognitive Level: Comprehension
Most candidates got full credit on this question.

Aggregate Capital = C = D1 * c1 + D2 * c2

C = 0.8 * 4 + 0.45 * 6

Aggregate Diversification factor = Aggregate Capital / Sum of Marginal Capital Requirements

Aggregate Diversification factor = C / (c1 + c2) = 5.9 / (4 + 6) = 59%

(e)   Explain why each of the following formulas is not appropriate for aggregating equity and interest rate risks:

   (i) $C(u, v) = u + v$

   (ii) $C(u, v) = uv$

   (iii) $C(u, v) = \min(u, v)$

Commentary on Question:
Cognitive Level: Analysis
Very few candidates got full credit for this question. The question tests whether the candidate knows what a copula is, realizes that the first choice is not a copula at all, and knows whether the other two copulas are appropriate for modeling dependency between interest and equity.

   (i) $C(u,v) = u + v$ By definition, a copula may not be below zero nor above one. This function is not a copula, because it's not always less than or equal to one. For example C (0.75, 0.75) = 1.5

   (ii) $C(u,v) = uv$ This is the copula for independent variables. It cannot be used to model dependency between interest rate and equity which are not independent.

   (iii) $C(u,v) = \min(u,v)$ This copula is used when variables are perfectly positively correlated, which is not the case for interest rate and equity returns.

7. **Learning Objectives:**
1. The candidate will understand how a business enterprise funds its activities with considerations for its business model, and the cost and constraints of the sources of capital.

2. The candidate will understand how an enterprise's structure and policies allow its management to prioritize and select among projects or business activities that are competing for scarce capital resources.

Learning Outcomes:
(1d) Assess whether the risky return from a new project or ongoing business is sufficient to employ investor capital

(1e) Evaluate the return on employed capital using NPV, IRR and Payback period.

(1g) Describe the methods of allocating risk capital.

(2c) Recommend an optimal capital structure and how to implement it for a given business or strategy.

Sources:
Corporate Finance, Berk, Jonathan and Demarzo, Peter, 2nd Edition, Chapter 6

Corporate Finance, Berk, Jonathan and Demarzo, Peter, 2nd Edition, Chapter 14

F-101-13: Capital Allocation in Financial Firms

Commentary on Question:
Overall, candidates did well except for part (e).

For part (a), a retrieval question, successful candidates were able to define the payback rule and its disadvantages.

For part (b), an analysis question, successful candidates were able to calculate the effect that issuing new equity and new debt have on a firm's equity cost of capital.

For part (c), a comprehension question, successful candidates correctly defined Modigliani-Miller Proposition I and applied it to the given market conditions and cash flows.

For part (d), an analysis question, successful candidates used the correct discount rate to evaluate the acquisition. Most were also able to compare the value of the acquisition with its purchase price in order to give the right recommendation.

7. Continued

For part (e), an analysis question, most candidates performed poorly. Successful candidates recalled the relationships between risk capital, volatility, and an optimal cash cushion from the source and were able to identify and use critical parameters from the formulas.

Solution:

(a)

(i) Define the payback rule.

(ii) List two disadvantages of the payback rule compared to the NPV rule.

(iii) Describe two general situations when the payback rule is useful despite its disadvantages.

Commentary on Question:
Other solutions from the source were given credit in (ii).

(i) The payback rule says only accept a project if its cash flows pay back its initial investment within a specified period.

(ii) It ignores cost of capital and time value of money;
It ignores cash flows after the payback period;

(iii) Use for small investment decisions
Provides budgeting information regarding the length of time in which capital will be provided.

(b) Calculate the equity cost of capital in each of the following scenarios. Show your work.

(i) Juney issues $8.82 million of new equity and uses the proceeds to purchase the distributor.

(ii) Juney issues $8.82 million of debt and uses the proceeds to purchase the distributor.

E = 1.5 million shares * $42 per share = $63 million
D = 0.1 * E = $6.3 million
rD = 2% because the firm can borrow at the risk-free rate (given as 2%).
rU = (E/E+D) x rE + (D/E+D) x rD = (63/69.3)x12.56% + (6.3/69.3)x2%
= 11.6%

7. **Continued**

w/ an equity issue, Enew = 63 + 8.82 = 71.82

(i) New rE = rU + D/E x (rU - rD) = 11.6% + (6.3/71.82) x (11.6% - 2%) = 12.442%

(ii) w/ a debt issue, Dnew = 6.3 + 8.82 = 15.12
New rE = rE + D/E x (rU - rD) = 11.6% + (15.12/63) x (11.6% - 2%) = 13.904%

(c) Explain, using Modigliani-Miller Proposition I, why Juney is indifferent to the two financing options in part (b).

MM Proposition I states that in a perfect capital market, the total value of a firm is equal to the market value of the total cash flows generated by its assets and is not affected by its choice of capital structure.

Juney is indifferent to the financing options since the project's cash flows are unaffected by the financing and therefore firm's market value is unaffected.

(d)

(i) Calculate the value of the acquisition. Show your work.

(ii) Recommend whether Juney should purchase the distributor.

(i) Calculate the Value of the Acquisition
WACC = 11.6%, same as rU from part (b)
PV = (Annual Cash Flow / WACC) since the cash flow is perpetual

= $1 million / 0.116 = $8,620,690
NPV = PV - initial outlay = $8,620,690 - $8,820,000 = -$199,310

(ii) Juney should decline the acquisition because the NPV of the project is negative.

(e) Calculate the optimal cash cushion required for Juney to minimize their deadweight cost of risk capital. Show your work.

R = S/(sqrt(2π)*(1+r))

$50 million = S/(sqrt(2$\pi$)*1.02)

Solve for S = 127,837,988

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7.   **Continued**

d = 0.2, m = 0.3

C* = Z(m/(d+m)) * S / (1+r)

= Z(0.6) * 127,837,988 / 1.02

= $31,746,434

8. **Learning Objectives:**

4. The candidate will understand how to critique the appropriateness of advanced risk assessment methods for a given situation.

Learning Outcomes:

(4b) Compare and contrast the methods that quantify the cost of capital within a risk valuation framework.

Sources:

F-107-13: A Market Cost of Capital Approach to Market Value Margins

Commentary on Question:

The question tests whether the candidate understands the advantages of the MCoC approach over the Percentile approach to calculate market value margins for non-hedgeable risks. Candidates should be able to demonstrate why the MCoC approach is superior and perform the calculations.

Solution:

(a) Define market-consistent value of liabilities (MVL).

Commentary on Question:

Cognitive Level: Retrieval

Candidates generally did well on this part of the question, but few candidates got full credit. Candidates fell in two camps, those that mentioned the PV of liabilities and MVM, and those that mentioned arm's length transaction and cost of managing risk.

The MVL is the cost of managing the risks underlying the business on an ongoing basis. It also represents the market consistent value at which the liabilities could be transferred to a willing, rational, diversified counterparty in an arms' length transaction under normal business conditions.

The calculation includes the expected present value of future liability cash flows and an explicit, additional cost of risk (the MVM) for non-hedgeable risks

(b) Describe the advantages of the market cost of capital (MCoC) approach over the percentile approach for calculating MVMs for non-hedgeable risks in terms of:

(i) Implementation

(ii) Reflection of risk

(iii) Transparency

(iv) Solvency in run-off mode

8. **Continued**

Commentary on Question:
Cognitive Level: Comprehension
Candidates generally knew answers to the first 3 parts of the question. They had a harder time describing the Solvency in Run-off in part (iv). In general, when a candidate lists an advantage for the MCoC, it should not also be an advantage of the Percentile Approach.

(i) **Implementation**: MCoC is easy to implement even for small insurers. Percentile approach is complicated because it requires stochastic modeling.

(ii) **Reflection of Risk**: MCoC provides appropriate reflection of risk by considering the tails of the distribution of the risks. Percentile does not consider the distribution at all.

(iii) **Transparency**: MCoC is transparent, verifiable, and comprehensible. There is only one unknown parameter. Percentile approach is much more difficult and results in wide variations from insurer to insurer.

(iv) **Solvency in Run-Off**: MCoC is more prepared for a crisis by using the SCR as opposed to a 75% CI used by the percentile approach which is unlikely to cover future liabilities if faced with a stressed situation.

(c) Calculate the MVM for the non-hedgeable risks of the term block using the MCoC approach. Show your work.

Commentary on Question:
Cognitive Level: Analysis
Candidates did very well on this part of the question. A common mistake we saw here was the mis-application of the discount rate in calculating the MVM.

Calculating SCR as a % of PV Future Claims
SCR(0)/PV(0) = (5,860/47,312) = 12.4%
SCR(1): \$31,978*0.124= \$3,965
SCR(2): \$16,214*0.124= \$2,011

Calculate the Capital Charge at each time period
Cost of capital rate = 3.5%

CapitalCharge(0) = \$5,860*3.5% = \$205.10
CapitalCharge(1) = \$3,965*3.5% = \$138.78
CapitalCharge(2) = \$2,011*3.5% = \$70.39

Calculating MVM
Swap rate = 4.5%

8. Continued

Year 0 = $205.10 - *no discount needed*
Year 1 = ($138.78/(1.045)^1) = $132.80
Year 2 = ($70.39/(1.045)^2) = $64.46

Summing all discounted Capital Charges gives:
Total MVM = ($205.10 + $132.80 + $64.46) = **$402.36**

(d)   Looking Glass is considering offering single premium whole life insurance, investing the proceeds in long-term corporate bonds.

    (i)   List two examples of financial non-hedgeable risks with this product.

    (ii)   List two examples of non-financial non-hedgeable risks with this product.

Commentary on Question:
Cognitive Level: Analysis
Most candidates did fairly well on this portion of the question. Some candidates wrote that "long-term bonds" were not hedgeable. Credit was not given for this answer unless long-term was clarified to be 40+ years.

*It should be noted that the question asked for **two** examples. If more were given, credit was not given for those examples. More examples are given in the model solution to give candidates an idea of the answers that were acceptable.*

Financial: 60 year USD, Yen, or EUR cash flow or interest rate option; 15 year emerging market CF; Corporate bond default risk.

Non-Financial: Mortality risk, policyholder lapse risk, expense [or inflation] risk, operational risk.

(e)   Explain to the CEO why you agree or disagree with her point.

Commentary on Question:
Cognitive Level: Comprehension
This question was addressing the concept of circularity in the MVM calculation using the MCoC approach. The important concepts to note in this part were the circularity in the calculation as well as the relative insignificance of the MVM in relation to the SCR.

The CEO makes a good point that the MVM is calculated directly from the SCR, and the SCR is calculated from the MVL which should already include the MVM risk charge.

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8. **Continued**

However, it is clear in the calculations that the MVM risk charge is very small in comparison with the present value of the liabilities, that it can be deemed negligible in the SCR calculation. Therefore, when taking a percentage of the SCR to calculate the MVM, it is not raising an issue of circularity.

9. **Learning Objectives:**
3. The candidate will understand how and when to apply various stochastic techniques to situations which have uncertain financial outcomes.

Learning Outcomes:
(3b) Assess the appropriateness of a given stochastic simulation technique to quantify various market risk exposures.

(3c) Recommend the use of techniques to reduce the computational demand when applying stochastic methodology.

Sources:
Measures of Market Risk, Dowd, Kevin, 2nd Edition
• Chapter 4 Non-Parametric Approaches INCLUDING Appendices 1 - 4

Commentary on Question:
This question tested candidates' understanding of non-parametric historical simulation.

Solution:
(a) List four advantages for Turtleback of using a non-parametric approach.

Commentary on Question:
Candidates generally did well on part (a), a retrieval part.

The advantages are:
Relatively simple (i.e. can be implemented in excel)
Data is readily available from public sources or internal systems
The method is often times easy to explain to management/lay person
Can be adapted to more complex positions (i.e. derivatives, hedging etc.)

(b)

(i) Describe each of the three approaches.

(ii) Outline general advantages and disadvantages of each approach.

Commentary on Question:
Candidates performed poorly on this comprehension part of the question. Many of them confused the HS methods with more general stochastic methods. Other possible answers received credit for subpart (ii).

(i) Basic HS: Put data on histogram and read VaR from the histogram.

Bootstrapped HS: Resample from existing data (from Basic HS) with replacement. Each sample will then have its own VaR and the best estimate is the average of the resulting VaR results.

9.   Continued

HS using Non-Parametric Density: Treat the data as if being sampled from an unknown empirical distribution function.

(ii)   Basic HS Advantages/Disadvantages:
Adv.: Intuitive/easy to implement
Dis.: You need large data samples

Bootstrapped HS:
Adv: Intuitive and easy to apply because it does not require difficult programming
Dis: You need very large sample sizes for this approach to work

HS Using Nons-Parametric Density:
Adv. Allows for non-discrete CI's
Dis: Requires complex mathematical procedures and programming.

(c)   Recommend the most appropriate non-parametric HS approach for Turtleback. Justify your recommendation.

Commentary on Question:
This analysis part of the question was generally well done by candidates. There was one optimal recommendation but some credit was awarded for other recommendations if they were well supported.

Turtleback should adopt the Bootstrapped HS method.
The reasons are:
Does not want to purchase new software
Data is weighted towards recent history
Does not want any model that is too complex

10. **Learning Objectives:**

2. The candidate will understand how an enterprise's structure and policies allow its management to prioritize and select among projects or business activities that are competing for scarce capital resources.

Learning Outcomes:

(2b) Describe the factors impacting short-term capital needs.

(2c) Recommend an optimal capital structure and how to implement it for a given business or strategy.

Sources:

Corporate Finance, Berk, Jonathan and Demarzo, Peter, 2nd Edition Chapter 2

Corporate Finance, Berk, Jonathan and Demarzo, Peter, 2nd Edition Chapter 26

Commentary on Question:

This question tested candidates on how to retrieve information from a firm's financial statements, the difference between the book value and market value of equity, and how to use the DuPont Identity. This question was answered very well by most candidates. Some candidates did not recognize that part (c)(iii) could be done by applying the work from (c)(i) and (c)(ii) to get the answer.

Parts (a) and (d) were retrieval questions. Part (b) was a comprehension question. Part (c) was an analysis question.

Solution:

(a) Calculate the following for Frenz as of 12/31/2012. Show your work.

    (i)   Market-to-book ratio

    (ii)   Book debt-equity ratio

    (iii)   Market debt-equity ratio

    (iv)   Enterprise value

    M/B Ratio = Market value of Equity/Book Value of Equity = 188.2%
    Book D/E ratio = Total Debt/Book Value of Equity = 27.8%
    Market D/E ratio = Total Debt/Market value of equity = 14.8%
    Enterprise value = MV of Equity + Debt – Cash = 795

10. Continued

(b)

       (i)   Contrast book value and market value of equity.

       (ii)   Explain how enterprise value can be used to evaluate the business.

       (i)   Book value of equity is the value of equity on the balance sheet (assets-liabilities). Market value of equity is share price multiplied by number of shares.
Book value of equity excludes off-balance sheet items. Market value of equity includes off-balance sheet items

       (ii)   Enterprise value is the purchase price of the firm. It is the cost to acquire the business unencumbered by debt.

(c)   Referring to Frenz's Financial Statements as of 12/31/2012:

       (i)   Calculate Frenz's ROE. Show your work.

       (ii)   Calculate the net profit margin, asset turnover, and equity multiplier. Show your work.

       (iii)   Calculate the increase in sales needed to increase ROE to 35%, assuming Frenz is unable to increase its net profit margin or change its total assets. Show your work.

ROE = Net Income/Book Value of equity = 33%

Net Profit Margin = Net Income/Sales = 12.1%
Asset Turnover = Sales/Total Assets = 149.7%
Equity Multiplier = Total Assets/Book Value of Equity = 180.3%

ROE = Net Profit Margin x Asset Turnover x Equity Multiplier = 35%
ROE = 12.1% x Y x 180.3% = 35%
Y = 160.4%
(Sales + z)/Total Assets = 160.4%
Sales + z = 1250
Sales have to increase by 7.2%

10. Continued

(d)

        (i)   Calculate the accounts receivable days, inventory days, and accounts payable days as of 12/31/2012. Show your work.

        (ii)   Calculate the cash conversion cycle (CCC) as of 12/31/2012. Show your work.

Account Receivable days = Account Receivable / Average Daily Sales = 60/(1166/365) = 18.8

Inventory Days = Inventory/Average Daily Cost of Goods Sold = 96/(495/365) = 70.8

Accounts Payable Days = Accounts Payable/Average Daily Cost of Goods Sold = 24/(495/365) = 17.7

Cash Conversion Cycle = Accounts Receivable days + Inventory days – Accounts Payable Days = 71.9

11. **Learning Objectives:**
1.   The candidate will understand how a business enterprise funds its activities with considerations for its business model, and the cost and constraints of the sources of capital.

Learning Outcomes:
(1b)   Evaluate the return on employed capital using NPV, IRR and Payback period.

(1c)   Apply real options analysis to recommend and evaluate firm decisions on capital utilization.

Sources:
Corporate Finance, Berk, Jonathan and Demarzo, Peter, 2nd Edition, Chapter 22

Commentary on Question:
The focus of this question was to test candidate's understanding on the real option analysis as it applies to decision making on capital utilization. Candidates are further required to analyze the impact on option values to certain parameter changes and recommend simpler alternatives to option analysis. The question is placed in the context of the case study. Successful candidates were expected to demonstrate their knowledge and thorough understanding of the concept. Parts (a), (c), and (d) were Comprehension. Part (b) was Analysis.

Solution:
(a)   Calculate the value of upgrading its fleet today.

> **Commentary on Question**:
> *Candidates did well on this section. Most candidates were able to calculate the NPV correctly. About 25% of the candidates didn't consider the initial investment and didn't receive full points.*
>
> NPV of Upgrading Blue Jay's Fleet in Favorable State:
> Additional Revenue / (CoC minus Growth Rate) - Cost
> = 18M / (12%-2%) - 100M = 80M
>
> NPV of Upgrading Blue Jay's Fleet in Unfavorable State:
> Additional Revenue / (CoC minus Growth Rate) - Cost
> = 0M / (12%-2%) - 100M = -100M
>
> Probability Weighted Avg NPV using Pr(Favorable) = 60%,
> Pr(Unfavorable) = 40% (given)
> NPV = 60% x Favorable NPV + 40% x Unfavorable NPV
> = 60% x 80M + 40% x -100M = 8M

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11. Continued

(b)

    (i)   Complete the three missing elements in the table above. Show your work, where applicable.

    (ii)   Calculate the value of the option to delay one year. Show your work.

    (iii)   Recommend whether Blue Jay Air should upgrade immediately or wait one year to upgrade. Justify your recommendation.

Commentary on Question:
This section asks candidates to recognize and perform a European call option valuation using the Black-Schole formula given.

Candidate did moderately well on (b)(i). A common mistake was the misuse of cost of capital and risk-free rate on calculation of Sx and PV(K). No points were taken off if discount rate was interpreted as force of interest.

Candidates did very well on (b)(ii). No penalty on using Sx and PV(K) from b(i) even if the values were incorrect. Only a few candidates failed to do the correct normal table substitutions.

Candidates did well on (b)(iii). Candidates that did not provide any quantitative support for a recommendation and gave only qualitative commentary received little credit. Full credit was given if incorrect calculations in (b)(ii) and part (a) resulted in the opposite recommendation as long as the justification was correct. Some candidates didn't link back to part (a) but used loss of dividend as an explanation and therefore didn't receive full credit.

    (i)   Sx=S - PV(CF in year 1) = 108M - 10.8 / 1.12 = 98.36M

           PV(K) = 100M /1.02 = 98.04

           Dividend = [Expected] FCF (future cash flows) Lost from Delay

    (ii)   C = Sx x N(d1) - PV(K) x N(d2)
           d1 = 0.11, given
           d2 = d1 - 0.18 = -0.07, given
           C = 98.36*N(d1) - 98.04*N(d2) = 98.36 x 0.5438 - 98.04 x 0.4721 = 7.2M

    (iii)   Recommendation: Upgrade immediately.
           The value of the option to wait one year and only perform the upgrade if it is profitable is $7.2M from subpart (ii). The NPV of performing the upgrade immediately is $8M from part (a).

11. Continued

The NPV of the upgrade ($8M) exceeds the value of the option to wait ($7.2M).

(c)   Describe the effect on the value of the option to delay from each of the following:

   (i)   Competition from a new high-speed rail corridor increases the volatility of returns.

   (ii)   The delay of the upgrade is two years instead of one year.

   Commentary on Question:
   In general, candidates did well on this section but only a few received full credit on (c)(ii). Many candidates didn't realize the conflicting impact of longer time vs. incremental loss of dividends on option values.

   (i)   Higher volatility of returns increases the value of the option to delay.

   (ii)   Longer T would increase the value of the option to delay. However, delaying the upgrade will increase the expected lost revenue. The more lost revenue, the lower the value of the option to delay.

(d)   Blue Jay Air's CFO is concerned about the difficulty of implementing option analysis on an ongoing basis and has asked you for a simpler approach.

   (i)   Describe two commonly used rules of thumb to account for an option to delay.

   (ii)   Recommend which rule of thumb is more applicable to Blue Jay Air. Justify your recommendation.

   Commentary on Question:
   Few candidates received full credit on this section. Some were able to describe the rules and received partial credit.

   (i)   The Profitability Index Rule
          The profitability index = NPV/Initial Investment
          The profitability rule directs you to invest whenever the profitability index exceeds some predetermined number. When the investment cannot be delayed, the optimal rule is to invest whenever the profitability index is greater than 0; when there is option to delay, a common rule of thumb is to invest only when the index exceed a higher threshold, such as one.
          It's a way of accounting for the option to wait when there is cash flow uncertainty.

11.   Continued

The Hurdle Rate Rule
Hurdle Rate rule = cost of capital * callable annuity rate/risk free rate
The hurdle rate rule raises the discount but then applies the regular NPV
rule: Invest whenever the NPV calculated using this hurdle rate is positive.
It's a way of accounting for the option to wait when there is interest rate
uncertainty.

(ii)   Profitability index rule is more applicable as there is cash flow uncertainty
in this case.

12. **Learning Objectives:**

5. The candidate will understand how to identify and recommend appropriate risk assessment and monitoring techniques for financial risk management.

Learning Outcomes:

(5b) Describe the types of models and the sources of model risk.

(5c) Assess the methods and process for quantifying and managing model risk within a financial institution.

Sources:

Measures of Market Risk, Dowd, Kevin, 2nd Edition, Chapter 16

Commentary on Question:

This question is testing candidates' basic understanding of models and model risk. Successful candidates demonstrated knowledge of model definition, sources of model risk, and guidelines of how to manage model risk as practitioners in conjunction with the case study.

Part (a) tested candidates' comprehension ability. Part (b) tested candidates' analysis ability. Part (c) tested candidates' knowledge utilization ability.

Solution:

(a)

(i) Describe each of the three model types.

(ii) Identify an example of each model type and its purpose within either Frenz or Blue Jay Air.

Commentary on Question:

Candidates were expected to describe all 3 different models for question (i) and provides model examples and model purposes from the case study for either Frenz or Blue Jay Air. Candidates generally did well on this part.

(i)

1. Fundamental Models - formal systems tying outputs to inputs based on assumptions about dynamic processes, interrelationships between variables, and so on.
2. Descriptive Models – can be regarded as short cuts to fundamental model.
3. Statistical Models - capture the relationship between variables using some sort of statistical best fit with the emphasis usually on the correlation between them rather than any attempt at a causal explanation as such.

12. Continued

    (ii)
1. Fundamental Models – Blue Jay Air uses Black-Scholes model for hedging work
2. Descriptive Models – Frenz uses supply-demand economic model to evaluate business strategy or risk profile
3. Statistical Models – Blue Jay Air uses statistical regression model for customer base study or loyalty program

(b) For Blue Ocean's proposed renewable energy business:

    (i) Identify four types of model risk present in its reserving model.

    (ii) Describe how each type of model risk identified in part (i) relates to its reserving model.

Commentary on Question:
Candidates are expected to answer types of model risk and its justification from Blue Ocean's reserving model. Candidates generally did well.

A common candidate answer: Leveraging travel insurance reserve calculation engine for Blue Ocean's reserve models is an incorrect use of models. But this should be referred to as other model risk types such as mis-specified relationships or missing risk factors.

Other answers from the source received credit for (i).

    (i)
1. Missing risk factors
2. Misspecified relationships
3. Implementation risk
4. Incorrect calibration

    (ii)
1. Missing risk factors - captured current risk factors are Weather, Mechanics, Default rate, Energy conversion ratio, and Counterparty risk. Due to it being an emerging industry, there might be other risk factors.
2. Misspecified relationships - since Jay borrowed the reserving method from Travel insurance product, renewable energy business reserving method might misspecify relationships between variables
3. Implementation risk - Since Jay developed the renewable energy reserving model by himself without full specification of the reserving method and no detail peer review was performed, there is potential for implementation risk.

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12. Continued

       4. Incorrect calibration - Due to it being an emerging market and lack of available data, all the parameters are roughly estimated.

(c)   Recommend four improvements that would reduce the model risk of Blue Ocean's proposed renewable energy reserving model. Justify your recommendations.

Commentary on Question:
Candidates were expected to answer general modeling improvements with respect to Reserving models.

No credit was granted for just listing out an item without a description of the item. No credit was granted for the recommendation of post-implementation review, risk based capital assessment, or stress-testing since these 3 items were already presented as risk monitoring under the risk management framework.

Peer-review and its documentation were granted as one of the recommendations. Candidates generally did not do as well on this part as the prior two.

1. Be aware of model risk - The reserving model not only was built from the Travel Insurance reserving model but also it is still a model. In order to use this model for production level, the reserving methodology should be reviewed and developed for the renewable energy product and the model user still needs to be aware this is a model.

2. Identify, evaluate and check key assumptions - since renewable energy is an emerging industry, practitioners should perform the following: (1) identify other key risk factors in addition of existing 5 risk factors, (2)the roughly estimated key assumptions need to be re-evaluated as more data becomes available, and (3) validate the assumptions based on model output.

3. Don't ignore small problems – The reserving model has not been peer reviewed. When any peer reviews the model, peer should not ignore any small problems.

4. Plot results and use non-parametric statistics – A graphical view would make it easier to understand the relationship between liability amount and # of homes insured or between liability amount and other variables.

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13. **Learning Objectives:**

5. The candidate will understand how to identify and recommend appropriate risk assessment and monitoring techniques for financial risk management.

Learning Outcomes:

(5a) Evaluate the methods and processes for measuring and monitoring market risk positions.

Sources:

Measures of Market Risk, Dowd, Kevin, 2nd Edition

- Chapter 10 Option Risk Measures
- Chapter 12 Mapping Positions to Risk factors

Commentary on Question:

Overall candidates did poorly on this question, which relied on the case study. Part (a) was Comprehension, parts (b)-(d) were Analysis, & part (e) was Knowledge Utilization.

Part (a): Candidates who understood risk mapping did well here even without having to reference the case study.

Part (b): The primary problem was a lack of understanding of how to do the risk mapping process. The main pitfalls were candidates either 1) attempting to use effective duration to estimate the impact or 2) shifting the entire yield curve instead of the individual spot rates.

Part (c) and (d): Candidates who studied the Case Study tended to do well as it contained many obvious answers. Candidates that scored poorly incorrectly linked hedging asset losses to liability movements such as policyholder behavior.

Part (e): Candidates who did well on (c) or (d) should have been able to do well on part (e) since it required minimal additional explanation of how problems already identified in parts (c) and (d) could be addressed. Candidates who did poorly here were unable to make this connection.

Solution:

(a)

      (i)   Describe the risk mapping process.

      (ii)   Describe the benefits of risk mapping for Darwin Life.

Commentary on Question:

Part (i), for full credit, candidates had to make it clear the mapping process was specifically about mapping securities (assets such as stocks) to risk factors or indices (like Large Cap / S&P500). Half credit was given if there was a generic description of mapping of an asset to a reference instrument.

13. Continued

Part (ii), to get full credit, the candidates needed to mention 2-3 benefits. More than that number are listed below to show possible acceptable answers.

    (i)   Risk Mapping is the process of describing a portfolio with a lesser number of benchmark securities.
             Steps:
1. Create a benchmark portfolio by selecting core reference instruments
2. Map the actual portfolio to the benchmark portfolio's reference instruments
3. Estimate the risk measures using the benchmark portfolio's reference instruments instead of the actual portfolio's securities

    (ii)   Generic Benefits of Risk Mapping
- Reduces dimensionality
- Reduces computing time
- Requires less data
- Removes problem of highly correlated risk factors

           Specific Benefits to Darwin Life
- Group Annuity Block is unhedged
- Darwin has a wide variety of holdings and risk mapping makes it easier to model

(b)   You map the portfolio to just the 5-year and 10-year spot rates.

    (i)   Calculate the reference rate for the portfolio. Show your work.

    (ii)   Calculate the sensitivity of the portfolio to an up 1% parallel shift in the spot rate curve. Show your work.

    (iii)   Calculate the reference point cash flows at 5 years and 10 years that have the same interest rate sensitivity as the portfolio cash flows. Show your work.

Commentary on Question:
Part (i) was basic linear interpolation

Part (ii): Candidates needed to realize the sensitivity was to an upward shift of the SPOT rate not the yield curve. No credit was given to any candidates for estimating sensitivity using duration or key-rate durations (KRD). Force of interest (not compound) should have been applied.

13. Continued

Part (iii): Candidates had to demonstrate how to calculate the cash flows. Most successful candidates answered the majority of part (iii) in part (ii) by calculating the PV100's.

(i)   r(6) = (10 - 6) / (10 - 5) x r(5) + (6 - 5) / (10 - 5) x r(10)
        = .8 x 3% + .2 x 7% = 3.8%

(ii)   First, calculate PV of initial portfolio:

PV using r(6) = $1000 EXP(-3.8% x 6) = $796.12 million

Second, calculate the new reference rates for separate PV100 (1% shifts) in the spot rates:

r_L(6) = (10 - 6) / (10 - 5) x [r(5) + 1%] + (6 - 5) / (10 - 5) x r(10)
        = .8 x 4% + .2 x 7% = 4.6%

r_H(6) = (10 - 6) / (10 - 5) x r(5) + (6 - 5) / (10 - 5) x [r(10) + 1%]
        = .8 x 3% + .2 x 8% = 4.0%

Third, calculate the PV based on the new reference rates after separate 1% shifts in the spot rates:

PV using r_L(6) = $1000 EXP(-4.6% x 6) = $758.81 million
PV using r_H(6) = $1000 EXP(-4.0% x 6) = $786.63 million

Finally, calculate the differences to get the sensitivities:

PV100 of r(5) = 758.81 – 796.12 = - $37.31 million
PV100 of r(10) = 786.63 – 796.12= -9.49 million

(iii) CF(5) = PV100 of r(5) / [EXP((-r(5) + 1%) x 5) - EXP(-r(5) x 5)]
        = -37.31 / [EXP(-4% x 5) – EXP(-3% x 5) = $888.82 million

CF(10) = PV100 of r(10) / [EXP((-r(10) + 1%) x 5) - EXP(-r(10) x 5)]
        = -9.49 / [EXP(-8% x 10) – EXP(-7% x 10) = $200.82 million

(c)   Describe three reasons for Darwin's large losses due to hedging breakage in the fixed income portfolio.

13. Continued

Commentary on Question:
Acceptable answers are below (of which only three were needed). When giving credit, only the first 3 answers were considered (unless 2 of the answers were similar). For full credit, at least 2 of the answers had to relate specifically to the case study. No credit was given for any mention of the liabilities or policyholder behavior as hedging breakage only considers the assets.

Fixed Income Hedging Breakage Reasons:
- The portfolio was mapped fully to just 2 points on the yield curve when there is interest-rate exposure to other parts of the curve such as the 30% of securities with effective duration under 3 years
- There is optionality in the fixed income portfolio from the 70% callable bonds (which was not considered during mapping / hedging)
- The portfolio has (unhedged) credit / default risk from 40% of the public corporate being below investment grade securities
- The portfolio was not rebalanced
- Hedging was done to PV100 which may not be suitable for this portfolio (depending on how often it is rebalanced and the volatility of the portfolio returns)
- The method of hedging was not described and the securities used to offset the interest rate risk may not have been suitable
- Non-parallel shifts in the yield curve

(d) Describe three reasons for Darwin's large losses due to hedging breakage in the equity portfolio.

Commentary on Question:
Acceptable answers are below (of which only three were needed). When giving credit, only the first 3 answers were considered (unless 2 of the answers were similar). For full credit, at least 2 of the answers had to relate specifically to the case study. No credit was given for any mention of the liabilities or policyholder behavior as hedging breakage only considers the assets.

Equity Portfolio Hedging Breakage Reasons:
- (The portfolio was mapped entirely to 100% US Large Cap when) there are 26% bond or bond-equity combination funds
- (The portfolio was mapped entirely to 100% US Large Cap when) there are 11% international equities and specialty funds
- (The portfolio was mapped entirely to 100% US Large Cap when) there are 26% Small and Mid-cap equities
- There are bond funds mapped to US Large Cap
- The portfolio was not rebalanced

13. Continued

- The method of hedging was not described and the securities used to offset the equity risk may not have been suitable

(e)   Recommend four improvements to the hedging program described above. Justify your recommendations.

Commentary on Question:
The purpose of this part was to get candidates to suggest solutions to problems listed in parts (c) and (d).

Acceptable answers are below, of which 4 were needed. Points were given for each correct answer regardless of whether it applied to the case study. Only the first 4 answers were considered (unless 2 answers were similar).

Credit was not given for mentioning any of the following:
- *Adding an ALM / risk report (unless included with a statement about rebalancing the hedge more frequently)*
- *Better monitoring of liability*
- *Adding a more efficient hedge like delta-gamma or rho (since it is not stated which type of hedge Darwin uses based on the results of the risk mapping process)*
- *Specific instruments to hedge like equity options or interest-rate swaps (since it is not stated which type of hedge Darwin uses based on the results of the risk mapping process)*

Possible Improvements:
- Rebalance more frequently
- Map the fixed income to other points on the yield curve such as under 3 years and over 15 years
- Hedge or monitor Credit risk
- Map the international equities to a foreign equity index instead of US Large Cap
- Map the small/mid cap securities to a Small / Mid cap index instead of US Large Cap
- Consider the optionality of the callable bonds when mapping risks
- Consider the optionality of the callable bonds when hedging risks
- Map bond funds to bond index instead of US Large Cap
- Consider impact of policyholder behavior

14.

Learning Objectives:

3. The candidate will understand how and when to apply various stochastic techniques to situations which have uncertain financial outcomes.

4. The candidate will understand how to critique the appropriateness of advanced risk assessment methods for a given situation.

Learning Outcomes:

(3d) Assess the strengths and weaknesses of the calibration techniques for a given stochastic model.

(3e) Interpret the results of a given application of stochastic modeling and the impact of the chosen calibration process used.

(4e) Explain how to quantify risk when there is limited data.

Sources:

F-106-13: Investment Guarantees, Hardy, Chapters 3 and 4

Commentary on Question:

This question tested candidates' understanding of Maximum Likelihood Estimation and how to calculate it for a lognormal distribution. Candidates were expected to be able use judgment to select an appropriate model given its application and the desire to balance fit and complexity. Finally, candidates needed to demonstrate a grasp of what is involved in bootstrapping and recognize its appropriateness when data is limited. The cognitive level increased throughout the question with part (a) starting at Retrieval, moving to Comprehension in part (b), increasing to Analysis in parts (c) and (d), and finishing with Knowledge Utilization in part (e).

Solution:

(a) Explain how parameters are calibrated using Maximum Likelihood Estimation (MLE).

Commentary on Question:

Most candidates understood that parameters were chosen to maximize the likelihood function. Many did not mention anything about using the joint probability function of the data. Candidates could also get credit by using formulas in addition to their explanations.

Parameters are chosen in order to maximize the likelihood function given sample data. The likelihood function is the joint probability function of the given data as a function of its parameters.

14. Continued

(b)   Calculate the necessary parameters to fit the given stock returns to a lognormal model using MLE. Show your work.

Commentary on Question:
Several candidates derived the mean and variance formulas from first principles. Credit was given for this approach as well. Additionally candidates were given credit for solving for either the variance or the standard deviation.
A common error was candidates failing to divide by n when computing the variance.

$$\mu = \frac{\Sigma(Y_i)}{n} = \frac{4.65}{600} = 0.76\%$$

$$\sigma^2 = \frac{\Sigma(Y_i^2)}{n} - \mu^2 = \frac{2.17}{600} - (0.76\%)^2 = 0.36\%$$

$$\sigma = 5.96\%$$

(c)

(i)   Recommend a more appropriate calibration method for the model based on the characteristics of the product and the given data. Justify your recommendation.

(ii)   Calculate the model parameters for the lognormal model using the method you chose in part (i). Show your work.

Commentary on Question:
Most candidates realized the model should be calibrated to the tail percentiles. Part (ii) proved more difficult, perhaps because of a lack of time. Several candidates were able to get partial credit by setting up the equations to solve and demonstrating an understanding of how one would fit the tail of the distribution.

(i)   Because the product in question deals with embedded guarantees on the minimum credited return, it is more appropriate to calibrate this model to the left tail (1st and 2.5^{th} percentile) than it is to calibrate to the overall mean and standard deviation.

(ii)   Given the left tail of the returns, and a standard normal table, have to solve:

$$P\left(Y < \frac{-0.12 - \mu}{\sigma}\right) = 0.01$$

$$P\left(Y < \frac{-0.097 - \mu}{\sigma}\right) = 0.025$$

14. Continued

Using the standard normal table, we get

$$\frac{-0.12 - \mu}{\sigma} = -2.33$$

$$\frac{-0.0.97 - \mu}{\sigma} = -1.96$$

Solving for the variables, we see

$$\mu = \frac{0.12 \times 1.96 - 0.097 \times 2.33}{2.33 - 1.96} = 2.48\%$$

$$\sigma = \frac{-0.12 - 0.0248}{-2.33} = 6.22\%$$

(d) Describe how the following quantitative measures for evaluating parameter calibration might inform your choice between the lognormal and RSLN-2 models above:

   (i) Likelihood Function

   (ii) Schwarz-Bayes Criterion

Commentary on Question:
To get full credit on this question, candidates needed to do two things: 1) explain what the measures were and 2) demonstrate which model should be chosen under the specified measure.
Because the question asked how the quantitative measures might inform the choice of model, a strict likelihood ratio test was not required. Some candidates were able to construct a likelihood ratio test. These candidates were given credit for their calculations and the choices derived from their calculations.

   (i) Likelihood function: High value gives a good indication of the overall fit of the data to the parameters. Likelihood function suggests that the RSLN-2 has the best fit.

   (ii) SBC: The fit of a model to the underlying data is weighted against the number of parameters in that model and sample size. Hence, more complex models need a significant improvement in fit to be worthwhile. RSLN-2, while it gives the best fit, has six parameters vs. two for the lognormal model. The additional complexity will outweigh the improvement in fit and RSLN-2 will be discarded and lognormal chosen.

14. Continued

(e)   Your manager suggests that you recalibrate your existing model to the cumulative 5-year return at the end of each historical month, where available.

   (i)   Identify the error in your manager's proposal.

   (ii)   Recommend an improvement upon your manager's suggestion. Justify your recommendation.

Commentary on Question:
Candidates generally struggled with this question. Many did not understand the point that there were overlapping data points which leads to the dependence within the data.
For the second part, alternative acceptable solutions included: calibrating to monthly returns and getting more data.

   (i)   59 out of 60 months are overlapping for each data point using the manager's suggested methodology. There is significant correlation between successive observations, and many calibration methods assume independence.
   (ii)   Can generate extra data (on 5-year returns) using the "Bootstrap method", which will generate a sufficient number of independent 5-year samples that can be used for calibration