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**SOCIETY OF ACTUARIES**  
**Quantitative Finance and Investments Core**  
**Exam QFI CORE**  
**MORNING SESSION**

**Date:** Wednesday, October 30, 2013

**Time:** 8:30 a.m. – 11:45 a.m.

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**INSTRUCTIONS TO CANDIDATES**

**General Instructions**

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
  - a) The morning session consists of 11 questions numbered 1 through 11.
  - b) The afternoon session consists of 6 questions numbered 12 through 17.

The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

**Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFI Core.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.



**\*\*BEGINNING OF EXAMINATION\*\***  
**Morning Session**

- 1.** (4 points)
- (a) (1.5 points) Describe each of the following: actual volatility, implied volatility and realized volatility. Identify how they differ in terms of time period, number of associated time scales, and ability to be measured.
  - (b) (1.5 points) You wish to estimate 60-day future volatility using historical data. Describe ARCH and EWMA including their advantages and disadvantages.
  - (c) (1 point) Identify and specify a common model that combines the features of ARCH and EWMA.

2. (7 points) You are given the following:

- Stock price  $S_t$  is modeled as a geometric Brownian motion as follows:

$$dS_t = \left( \mu + \frac{\delta^2}{2} \right) S_t dt + \delta S_t dW_t$$

where

- $W_t$  is a Wiener process
  - $\mu$  and  $\delta$  are positive constants
- Discounted stock price process  $D_t$  is defined as,  $S_t e^{-rt}$ , where
    - $r$  is a positive constant with  $r < \mu$ .
  - $f(S_T)$  is the payoff of a derivative contract that matures at time  $T$ .

- (a) (1 point) State the Girsanov's Theorem.
- (b) (2 points) Show that  $D_t$  is not a martingale.
- (c) (2 points) Demonstrate how to use Girsanov's Theorem to convert  $D_t$  into a martingale.
- (d) (2 points) Explain how you can construct a replicating strategy for the derivative.

3. (7 points) Let  $W_t$  be a Brownian motion and  $I(T) = \int_0^T \sigma(S_t, t) dW_t$  be a well defined Ito-integral for the process  $\sigma(S_t, t)$ .

- (a) (1 point) State the conditions on the process  $\sigma(S_t, t)$  for which  $I(T)$  is well defined.
- (b) (1 point) Determine the mean and variance of  $I(T)$ .
- (c) (1 point) Determine whether or not  $\int_0^T W_{2t} dW_t$  is an Ito-integral.
- (d) (2 points) Suppose that  $F_t$  is a solution of the stochastic differential equation:

$$dF_t = \sigma F_t dW_t$$

Let  $V = V(F, T)$  be a twice continuously differentiable function and define a new process  $Y_t$  by

$$Y_t = e^{-rt} V(F_t, t)$$

Show that  $Y_t$  is a martingale if and only if  $V(F_t, t)$  satisfies the partial differential equation.

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2} F^2 \frac{\partial^2 V}{\partial F^2} - rV = 0$$

- (e) (2 points) Let  $T > 0$ , and suppose that  $V$  satisfies the equation in (d) for  $t < T$ , while for  $t = T$  we impose the boundary condition:

$$V(F, T) = g(F) \text{ where } g \text{ is some given function.}$$

Show that  $V(F_t, t) = e^{-r(T-t)} E[g(F_T) | I_t]$ .

4. (6 points) For a specific mandate, you need to produce monthly interest rate scenarios in a real world context. You have selected an AR(1) model:

$$r_t = \phi_0 + \phi_1 r_{t-1} + \varepsilon_t$$

with  $\varepsilon_t$  having a normal distribution with mean 0 and variance of  $\sigma^2$ .

To determine the parameters set of the model ( $\theta = \{\phi_0, \phi_1, \sigma^2\}$ ) you have access to monthly historical data of the interest rate  $(r_1, r_2, \dots, r_T)$ .

- (a) (1.5 points) Determine the mean and variance of the unconditional distribution of  $r_t$ .
- (b) (1.5 points) Develop the log-likelihood function,  $\ln f(r_2, \dots, r_T | r_1, \theta)$ , that you will need to maximize to obtain the MLE parameters from all the data provided.
- (c) (2.5 points) Determine the system of algebraic equations that needs to be solved to get the MLE parameters for the AR(1) model.
- (d) (0.5 points) Calculate the half-life measure of the speed of mean-reversion if the estimated parameters are  $\theta = \{0.015, 0.7, 0.0025\}$ .

5. (4 points) Suppose that the quarterly log earnings  $w_t$  of airline company Eastket follows the following model:

$$w_t = (1 - B)(1 - B^4)y_t = (1 - 0.58B)(1 - 0.17B^4)a_t$$

$a_t$  follows a normal distribution with mean 0 and variance of  $\sigma_t^2$ .

$$(\sigma_t)^2 = 8.08 \times 10^{-5} + 0.235 (a_{t-1})^2 + 0.723 (\sigma_{t-1})^2$$

The last 5 log earnings, residuals and volatilities are summarized in the following table:

time	46	47	48	49	50
$y_t$	1.06	1.40	0.89	1.14	1.19
$a_t$	0.0223	-0.0316	0.0011	-0.0106	0.0115
$\sigma_t$	0.03509	0.03418	0.0332	0.02953	0.02677

- (a) (0.5 points) Identify the periodicity and the parameters of the time series.
- (b) (1.5 points) Calculate the 1-step ahead prediction of the log earnings, assuming that the forecast origin is 50.
- (c) (2 points) Determine the Autocorrelation Function (ACF) of  $w_t$ .

6. (5 points) Consider the Vasicek spot rate model,  $dr_t = \alpha(\mu - r_t)dt + \sigma dW_t$ , where the parameters  $\alpha$ ,  $\mu$ , and  $\sigma$  are known positive constants.  $W_t$  is a Wiener process.

(a) (4.5 points) Show for  $t < s$  that

- $E[r_s | r_t] = \mu + (r_t - \mu)e^{-\alpha(s-t)}$

- $Var[r_s | r_t] = \frac{\sigma^2}{2\alpha}(1 - e^{-2\alpha(s-t)})$

(b) (0.5 points) Describe what these two equations imply for the conditional mean and variance of spot rates as  $s \rightarrow \infty$ .



7. (6 points)

- (a) (1 point) Describe the reasons for and against yield curve fitting using a one-factor interest rate model.

You are doing yield curve fitting work with a Ho & Lee spot interest rate model.

The process for the risk-neutral spot rate is  $dr = \eta(t)dt + cdX$ ,

where

- the standard deviation of the spot rate process,  $c$ , is constant, and
- the drift rate  $\eta$  is time dependent.

The solution of the bond pricing equation for a zero-coupon bond is

$$Z(r, t; T) = e^{A(t; T) - r(T-t)}$$

where  $A(t; T) = -\int_t^T \eta(s)(T-s)ds + \frac{1}{6}c^2(T-t)^3$

- (b) (3 points) Show that the functional form for  $\eta(t)$  must be

$$\eta^*(t) = c^2(t - t^*) - \frac{\partial^2}{\partial t^2} \log Z_M(t^*; t)$$

in order for zero coupon bonds for all maturities to have the correct value.

where

- $t^*$ , to be fitted, is today's date, and
- $Z_M(t^*; t)$  are the discount factors in the market.

- (c) (2 points) Express  $A(t; T)$  in terms of  $Z_M(t^*; T)$ ,  $Z_M(t^*; t)$ ,  $t$ ,  $T$  and  $c$ .

**8.** (4 points)

(a) (2 points) Describe how the following derivatives can be used to manage the duration of a bond portfolio and comment on their advantages and disadvantages.

- (i) Interest Rate Futures
- (ii) Interest Rate Swaps
- (iii) Interest Rate Options

You are given the following information:

A portfolio consists of bonds A, B and C:		
Bond	Duration	Market Value
A	5	1 million
B	7	2 million
C	10	1.5 million

The target dollar duration of the portfolio is 24.3 million.

The cheapest to deliver bond underlying the interest rate futures contract has the following properties:

- The price is 100,000
- The duration is 6, and
- The conversion factor is 1.1.

(b) (2 points) Calculate the number of futures contracts the portfolio manager should buy or sell to achieve the target duration.

9. (7 points) Bull & Bear Life Insurance Company introduces an Equity Indexed Annuity (EIA) with 10-year maturity.

The annual crediting rate is equal to:  $\text{Min}(\text{Max}(3\%, \text{ACRC}), \text{Max}(0.5\%, \text{IR}))$ , where:

- IR is 1-year total return on S&P 500 index from the previous anniversary; and
- ACRC (Annual Crediting Rate Cap) is determined in the beginning of each anniversary.

Investment Strategy = 1-year zero-coupon bond

Interest Rate = 2% flat across all the term structure (continuous compounding)

S&P 500 Dividend rate = 3% (continuous compounding)

S&P 500 1-year implied volatility = 20%

Target profit margin and expenses combined are 0.9% per year.

No early withdrawal is allowed.

You may approximate option pricing using Greeks.

- (1 point) Calculate the static hedging budget for the first year to achieve the target profit margin and expenses.
- (1 point) Recommend a static hedging strategy for this crediting rate policy.
- (3 points) Determine the first year cap of the crediting rate (ACRC) given the profit margin and expenses.

The company is considering investing in the 10-year Treasury bond to pick up additional yield and adding a feature that allows early withdrawals with no penalties or adjustments.

- (2 points) Assess the disintermediation risk and recommend a risk mitigation strategy.

- 10.** (6 points) JNH offers a diversified portfolio of products from participating whole life, universal life and variable universal life to fixed and variable annuities.

A new CIO, Mr. Yao, has recently heard about portfolio segmentation and is debating whether he should adopt this approach for JNH.

- (a) (1 point) Describe the portfolio segmentation approach.
- (b) (1.5 points) Recommend and justify your recommendation to Mr. Yao to adopt the portfolio segmentation approach.

The asset allocation of the participating insurance segment is as follows:

Asset Classes	Current Allocation	Permissible Range
Cash and Short Equivalents	2%	1%-5%
Public Bond	30%	26%-34%
Private Placement	30%	26%-34%
Mortgage	20%	16%-24%
Equities	18%	14%-22%

The participating insurance segment has an overall implicit annual return guarantee of 5%. The portfolio yield is at 5.6% currently.

Mr. Yao would like to perform a tactical asset allocation shift to express his views on the capital markets. He expects credit spread to widen. Therefore, he plans to temporarily re-allocate assets from private placement to public bond as much as permitted.

A 1% allocation to public bond from private placement is expected to result in an 8 bps reduction in the overall portfolio yield. There is also a 20 bps transaction cost for such a tactical adjustment.

- (c) (1.5 points) Recommend a tactical asset allocation for the participating insurance segment that incorporates Mr. Yao's plan and the segment's objectives.
- (d) (1 point) Evaluate what additional information should be considered before implementing a tactical asset allocation for JNH.

Mr. Yao wants to use fixed income Exchange Traded Funds (ETFs) to perform the re-allocation, but worries about the ETF prices.

- (e) (1 point) Describe three factors that affect the price at which an ETF trades.

**11.** (4 points) Your firm, which operates strictly in the bond universe, has received a mandate to manage \$100 M of assets from a pension plan. Your managing partner has given you the task to develop the benchmark that will be used to track the performance of this mandate.

(a) (1 point) Describe the considerations that come in play when selecting a benchmark.

It has been decided that the performance of the mandate/portfolio will be benchmarked on index ABC.

(b) (2 points) List and describe the various strategies that can be used when creating a fixed income portfolio based on an index.

The head of your firm has stated that, based on the exceptional abilities of its staff, the portfolio should be managed actively.

(c) (1 point) Identify the advantages and disadvantages of active management of fixed-income portfolios.

**\*\*END OF EXAMINATION\*\***  
**Morning Session**

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