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**SOCIETY OF ACTUARIES**  
**Exam FETE**  
**Financial Economic Theory and Engineering Exam (Finance/ERM/Investment)**

# Exam FETE

## AFTERNOON SESSION

**Date:** Thursday, May 2, 2013

**Time:** 1:30 p.m. – 4:45 p.m.

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### INSTRUCTIONS TO CANDIDATES

#### General Instructions

1. This afternoon session consists of 8 questions numbered 10 through 17 for a total of 60 points. The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam FETE.

#### Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.



**\*\*BEGINNING OF EXAMINATION\*\***  
**AFTERNOON SESSION**  
***Beginning with Question 10***

- 10.** (5 points) Company ABC currently has a debt ratio of 20% and can borrow at the risk free rate of 5%. The systematic risk of the company is 0.8 and the expected market return is 12%. The marginal tax rate is 50%. The company is considering accepting Project X, which has an unlevered beta of 0.5.

The new CFO of Company ABC decided to implement a new target capital structure and significantly increase the debt ratio from 20% to 50%.

- (a) (2 points) Calculate the hurdle rate the Company should use in evaluating Project X before and after the debt increase.
- (b) (2 points) Calculate the WACC of the Company using the new targeted capital structure.

Assume the Company's management is aligned with the shareholders.  
Assume the project manager is confident that Project X can earn 7% return.

- (c) (1 point) Explain if the Company should accept Project X.

- 11.** (8 points) You are an actuarial consultant working on a risk capital allocation project for SwapCo. SwapCo currently consists of one business unit, A, with \$50 million of expected profits and \$200 million stand-alone risk capital. The company is considering adding another business unit, B, with \$10 million of expected profits and \$100 million stand-alone risk capital. Mr. Alford has proposed to use Perold's model for his recommendation on whether or not to add business unit B.

Assume that the two business units A and B are independent and the risk capital is proportional to the standard deviation.

- (a) (2 points) Calculate the expected returns on a stand-alone basis, fully allocated risk capital, and marginal risk capital for business unit B.
- (b) (2 points) Recommend and justify whether or not adding business unit B based on a comparison of the expected profit after deducting the deadweight cost of risk capital, assuming that the deadweight cost of risk capital is 20%.

A year has passed and SwapCo added both business unit B and business unit C. You again are asked to look at the entire company consisted of these three business units (A, B, and C) from risk capital perspective. You plan to use Merton-Perold's approximate model of risk capital where risk capital  $\approx 0.4A_0\sigma\sqrt{T}$  ( $A_0$  is the gross asset,  $\sigma$  is the asset volatility,  $T$  is time) to assess the economic capital of the business units. The following information is gathered.

	Gross assets $A_0$	Volatility $\sigma$
Unit A	\$2000	37.5%
Unit B	\$2000	50.0%
Unit C	\$2000	62.5%

The correlation matrix among all three units is also determined.

	Unit A	Unit B	Unit C
Unit A	1.00	0.50	0.00
Unit B	0.50	1.00	0.00
Unit C	0.00	0.00	1.00

- (c) (2 points) Determine the marginal risk capital for business unit B using the above-mentioned Merton-Perold's approximate model with  $T = 1$ .

## 11. Continued

You feel that the risk capital using the above approximate model for business unit C is not adequate. You plan to use modeling as an alternative approach for re-assessment of the economic capital of this unit. One of your colleagues recommends an excellent reading titled “Economic capital modeling: practical considerations” which might answer some of your concerns about adopting the modeling approach.

- (d) *(1 point)* Compare VaR to Tail VaR based on the above-mentioned paper.
- (e) *(1 point)* Explain why a real-world technique would be preferred over a risk-neutral technique in an economic capital calculation.

- 12.** (8 points) During October 2012, your company has decided to apply a mixed stable strategy to bond portfolio trading in 2013 and will hire 10 traders to begin work on January 1, 2013. Effective January 1, 2013, many existing bonds will begin to receive tax-exempt treatment in certain cases. It is known that U.S.-domiciled companies may take full tax benefits but that foreign companies will not be able to.

You and two colleagues have been charged with hiring staff to implement the strategy. During your first planning meeting one of your colleagues suggests that asset prices are best explained by the intrinsic value hypothesis while the other insists that asset prices have to be based on the rational expectations of investors.

- (a) (2 points) Describe and differentiate these two positions specifically with respect to tax-exempt bonds.
- (b) (1 point) Differentiate between markets that fully aggregate information and those that only average it.

You know that other firms which trade bonds in the market – all of which your company may trade with – support one or the other of the two hypotheses being argued.

- (c)
- (i) (1 point) Identify the hypothesis that you should seek in the bond traders to be hired.
- (ii) (1 point) Outline a bond trading strategy that will be advantageous given the competing trading firms.

Each trader hired will either use costly information (Analysts) or will not (Random Selectors). On November 1, your team hires the 10 traders. The competitive advantage at November 1 is known to be 4.00, but for each of the next two months (until the new hires actually begin working) the competitive advantage of Analysts in the market is expected to either increase 0.75 or decrease 0.75 relative to the previous month's value. You also know the following:

- Upward or downward moves in competitive advantage are equally likely.
- The base fee for trading is 2%.
- The normal rate of return is 4%.
- The cost of analysis is the competitive advantage of Analysts in the market times the base fee for trading.
- Analysts command a salary of 100,000 per year.
- Random Selectors command a salary of 25,000 per year.

## 12. Continued

Both of your colleagues believe that you should hire no more than 5 Analysts but you believe that at least 6 Analysts is appropriate.

- (d) (3 points) Recommend and justify mathematically the expected number of each type of trader.

**13.** (9 points) Takakkaw General Insurance (TGI) wants to raise more capital to fund its current rapid business expansion. TGI's management is considering issuing convertible bonds as one way to raise capital.

- (a) Prepare a briefing to the management identifying and describing the following:
- (i) (1 point) Benefits to TGI of a convertible bond issue relative to equity and conventional debt issues.
  - (ii) (1 point) Drawbacks of raising capital using convertible bonds.

Assume the following:

- Maturity of the convertible bond is 20 years.
- Yield of conventional debt is 15%.
- Convertible bond coupon rate is 8% semiannually.
- Conversion price is \$26 per share.
- Convertible bond will sell at par value of \$1000.
- Value of a straight debt bond with the same maturity, coupon payments, face value and sold at par is \$580.

You also know:

- Current stock price is \$22.
- Standard deviation of the firm's equity rate of return is  $\sigma = 0.3$ .
- Risk-free rate is 14% for a 20-year Treasury bond.
- Expected return on market portfolio is 20%.
- Equity beta is 1.8.
- Firm does not pay any dividends.
- $d_2 = 1.29$

- (b) (3 points) Calculate the convertible bond's before-tax cost of capital  $k_{CV}$ .

### 13. Continued

Later, after the convertible bond has been issued, but before any conversions are exercised, the following is known:

- The firm had 80,000 shares outstanding.
- Total debt before conversion was \$1,500,000 of which \$500,000 is senior debt and the remainder is convertible bond with face amount \$1,000,000.
- Per \$1000 par value, the conversion ratio is 20:1.
- Marginal tax rate is 30%.
- Critical firm value is \$3,000,000.

Also assume a +1% change in stock price due to conversion causes the systematic risk to *decrease* by 0.25%.

- (c) (2 points) Calculate the maximum value of the convertible bond below which investors should exercise the conversion at the given critical firm value.
- (d) (1 point) Demonstrate mathematically that the beta after the conversion is 1.89.
- (e) (1 point) Calculate the WACC after conversion, assuming the stock price increases to \$30 per share at conversion.

- 14.** (8 points) As the actuary of ABC Life Insurance Company, you decide to introduce 2 new 5 year Equity-Linked GIC products with the S&P 500 Total Return as the underlying index.

The products differ by their payout to the contract holder:

- Product A: Return of premium plus 110% of the percentage increase of the index over the 5 year period.
- Product B: Return of premium plus 100% of the percentage increase of the index over the 5 year period.
- Product B includes a feature under which an increase of 50% in the index return is locked in.

ABC Life is expected to sell 1 million in premium of Product A and Product B. Market conditions are as follows:

Current index	1,000
Stock-price process is geometric Brownian motion. No dividend is assumed.	
Expected annual index return	15%
Implied volatility	18%
Continuous risk-free rate	4%
Term of available futures	1 year
Contract size of available future contracts	\$100,000

The following two investment strategies are being considered for Product A:

- Strategy 1: Does not hedge and set aside a fund that accumulates with risk-free rate
- Strategy 2: Buy fixed income bonds and replicate the embedded option through delta hedging with one year index futures

## 14. Continued

- (a) (2 points) Calculate the number of futures contracts required to hedge your company's position at issue for Product A using Strategy 2.
- (b) (2 points) Calculate the fund amount in Strategy 1 so that there will be a 99% probability of being able to pay the guarantee out of the accumulated value of this fund for Product A.
- (c) (2 points) Recommend and justify the combination of assets instruments (bonds, nonstandard and standard options) needed to statically hedge Product B from issue. Define the exercise prices of the options.
- (d) (2 points) Compare and contrast Strategy 2 with the static option hedging strategy in part (c) with respect to:
  - (i) Costs
  - (ii) Risks

- 15.** (7 points) At time 0, the prices of three zero-coupon bonds, which mature at time  $t$ ,  $P(0, t)$  are as follows:

$$P(0, 1) = 0.9608$$

$$P(0, 2) = 0.9241$$

$$P(0, 3) = 0.8896$$

Assume the following:

- The one year interest rate  $r(t, j)$  follows a recombining binomial equilibrium model where time  $t = 0, 1, 2$  and state  $j = 0, 1, 2$ .
- $r(t, j)$  is continuously compounding.
- The Risk Neutral Probability is 0.4.

$t / j$	0	1	2
0	0.04	0.045	0.05
1		0.035	0.04
2			0.03

- (a) (2 points) Verify that each value of  $P(0, t)$  given above is correct.
- (b) (1 point) Determine the annual coupon rate for a 3 year bond issued at par at time zero.

Assume annual payments for all interest rate swaps.

- (c) (2 points) Demonstrate that the swap rate on a 2-year interest rate swap which starts in 1 year is 3.923%.

You are given the following information about a European swaption:

- Notional of \$10,000,000
  - Exercisable after 1 year into a 2-year interest rate swap paying 4% fixed
  - The volatility for the forward swap is 15% per annum
- (d) (2 points) Calculate the value of the swaption.

- 16.** (7 points) You are an Actuary at Company ABC, and you are investigating the launch of a new product. You want to launch the product as a part of a two year program. The following table shows the demand matrix (in policies, at a cost of \$75 each) for the program, with the probabilities in parenthesis:

Present (Dec 31, 2013)	Year 1 (Dec 31, 2014)	Year 2 (Dec 31, 2015)
0	600 (1/3)	2000 (1/9)
	150 (2/3)	500 (4/9)
		125 (4/9)

You can hire up to three new staff to support this program, at a cost of \$10K each (for hiring and training fees). Each new staff could handle up to 500 policies a year. Alternatively, the current staff can handle up to 500 policies a year through overtime, at a cost of \$60/policy. The normal hourly rate is \$40/policy. Assume 3% annual discounting for all calculations.

- (a) (1 point) Calculate the Net Present Value of the program if you decide to hire three new staff right now.
- (b) (1 point) Calculate the Net Present Value of the program if you decide to fund it using overtime.

Assume you will not use overtime and will satisfy the full market demand.

- (c) (4 points) Show that a higher Net Present Value can be obtained using Real Options Analysis.
- (d) (1 point) Explain why Real Options Analysis gives a higher value.

**17.** (8 points)

- (a) (1 point) Compare and contrast the Hull-White and Ho-Lee interest rate models.

A Ho-Lee model has been calibrated to an at the money caplet. The caplet expires in 5 years with a tenor of 1 year and volatility of 1.3%.

The following was used in the calibration:

- (i) Zero coupon bond volatility formula

$$\sigma_p = \sigma \cdot (s - T) \cdot \sqrt{T}$$

- (ii) Interest rate term structure

Time (year)	Forward Rate (%)	Zero Coupon bond's Price
0	3.00	1
1	3.00	0.9705
2	3.25	0.9418
3	3.25	0.9117
4	3.50	0.8825
5	3.50	0.8521
6	3.75	0.8228
7	3.75	0.7926
8	4.00	0.7634
9	4.00	0.7335

Assume that for the Hull-White model  $\alpha = 0.05$ .

- (b) (1 point) Solve for the Hull-White volatility such that the caplet price is equal under both models and explain why the volatility parameters between models are different.

## 17. Continued

You are considering using one of the above calibrated models to price a caplet with a tenor of 1 year and a very long time to expiration

- (c) (2 points) Calculate the volatility of the caplet by taking the limit as the expiration approaches infinity. Recommend and justify which model is most suitable based on the volatility derived from both models.

Three simulations of the short rate to year 5 have been produced and the resulting values are 2.5%, 3.5%, and 4.5%.

- (d) (3 points) Calculate the spot rates for a zero coupon bond that matures at year 6 for each model and each of the three simulations.
- (e) (1 point) Interpret the results from part (d) in regards to the sensitivity between short rate movements and spot zero coupon bond movements and explain the difference in model results.

**\*\*END OF EXAMINATION\*\***

**Afternoon Session**

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