QFI ADV Model Solutions
Fall 2013

1. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1f) Explain how deterministic shifts can be used to fit any given interest rate term structure and demonstrate an understanding of the CIR++ model.

(1g) Understand and explain the features of the G2++ model, including: The motivation for more than one factor, calibration approaches, the pricing of bonds and options, and the model's relationship to the two-factor Hull-White model.

Sources:

Interest Rate Models – Theory and Practice: With smile, Inflation and Credit, Brigo, D. and Mercurio, F., 2nd Edition, Chapters 4.1 – 4.2, pgs. 137 – 143, 147, 153, 159-160.

Commentary on Question:

This question tests the pros and cons of the G2++ model compared to the CIR2++ model. It tests some of the main features of the G2++ model and its link with the two-factor Hull-White model.

Solution:

(a) Compare and contrast the above models.

Commentary on Question:

The candidates performed relatively well on this section. In general, the candidates were able to recognize the main features of both models. Most candidates who lost marks did so because they listed characteristics rather than provide an explanation or they missed an important point (mostly flexibility of the model versus analytical tractability).

Similarities:

Both models are two-factor models, so they can better reproduce the imperfect correlation of continuously compounded spot rates of different maturities.

1. Continued

Differences:

The G2++ model is based on Gaussian distributions, which can lead to negative interest rates. The CIR++ model is based on CIR processes and, with the appropriate parameters, does not allow for negative spot rates.

The G2++ model is analytically tractable: it is possible to find analytical expressions for the price of bonds, options and forward rates. The CIR2++ model is only tractable when $\rho = 0$, which also makes the model less flexible. With the parameters presented here, the CIR2++ model cannot fit the humped shape of the volatility curve of the instantaneous forward rates.

(b) Calculate the G2++ parameters equivalent to the parameterization of the Hull-White Two-factor model above.

Commentary on Question:
The candidates performed well on this section. Most candidates used the correct formulas but some candidates did not use the right numbers in the calculations.

Using the formulas given on page 6 of the Formula Sheet, we have

$a = \bar{a} = 0.68$

$b = \bar{b} = 0.09$

$$\eta = \frac{\sigma_2}{\bar{a} - \bar{b}} = 0.012$$

$$\sigma = \sqrt{\sigma_1^2 + \frac{\sigma_2^2}{\left(\bar{a} - \bar{b}\right)^2} + 2\bar{\rho}\frac{\sigma_1\sigma_2}{\bar{b} - \bar{a}}} = 0.020189$$

$$\rho = \frac{\sigma_1\bar{\rho} - \eta}{\sigma} = -0.692319$$

(c) Propose a process to update the parameters of your G2++ model for this new development. You do not need to give any numerical results.

Commentary on Question:
The candidates performed relatively poorly on this section. Many candidates provided a description of the formula, but not the actual formula for which partial credit was given. However, some candidates did not realize that no additional data was required to update the parameters, while others did not explain that they needed to switch back and forth between the Hull-White two factor model and the G2++ model.

1. Continued

From the formula sheet, the covariance between two forward rates $f(t,T_1)$ and $f(t,T_2)$ in the G2++ model is

$$Cov(f(t,T_1),f(t,T_2)) = \sigma^2\exp(-a(T_1+T_2-2*t)+ \eta^2\exp(-b(T_1+T_2-2*t)+ \rho\sigma\eta[\exp(-a*T_1-b*T_2+(a+b)*t)+\exp(-a*T_2-b*T_1+(a+b)*t)]$$

We only want to change the correlation parameter of the HW model, but this will cause more than one G2++ parameters to change. So we first write the formula for the correlation in terms of the HW parameters. Then we can let $T_1 = 2$, $T_2 = 10$, and the whole formula equal to 0.000031758. There is only one unknown ($\bar{\rho}$), since the other HW parameters should remain unchanged. So we can solve for $\bar{\rho}$. Now to update the G2++ parameters, we use the new HW parameters along with the formulas used in b). We should get the same values for a, b and η, and updated values for σ and ρ.

2. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1a) Identify and differentiate the features of the classic short rate models including the Vasicek and the Cox-Ingersoll-Ross (CIR) models.

(1b) Understand and explain the terms Time Homogeneous Models, Affine Term Structure Models and Affine Coefficient models and explain their significance in the context of short rate interest models.

Sources:

Interest Rate Models – Theory and Practice: With smile, Inflation and Credit, Brigo, D. and Mercurio, F., 2nd Edition, Chapter 3.2

Commentary on Question:

The question tests candidates understanding of affine term structure models and the Vasicek model

Solution:

(a) Define the following three models and show how they are related.

1. Affine term structure model

2. Affine coefficients model

3. Time homogeneous model

Commentary on Question:

The candidates performed relatively well on this section. Candidates were successful in providing clear definitions of each short rate model and show a graphical representation of the relationship between them. However some candidates were not able to produce the spot rate functions for ATS and AFC. Full points were awarded to candidates who produced a clear definition of all three models along with a description of the appropriate spot rate model functions and demonstrated the relationship between them by using graphs.

1. Affine TS model are IR models where:
   The continuously compounded spot rate R(t,T) is an affine function in the short rate r(t)
   $R(t,T) = \alpha(t,T) + \beta(t,T)r(t)$
   This relationship is always satisfied when the zero-coupon bond price can be written as: $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$

2. Continued

2. Affine coefficient modes are IR models, which can be written as: $dr(t) = \big(\lambda(t)r(t) + \eta(t)\big)dt + \sqrt{\gamma(t)r(t) + \delta(t)}\, dW(t)$, where the greek letters are deterministic time functions

3. An IR model is time homogeneous when the greek function are constants

   Affine coefficients implies an affine term structure
   Affine term structure + time homogeneity implies affine coefficients

(b) Explain the advantages of working with an affine term structure model.

Commentary on Question:
The candidates performed relatively well on this section. Candidates were successful in producing a list of the advantages of an affine term structure model.

The general solution that the question was seeking is as follows:
Computationally and analytically tractable
Compound spot rate R(t,T) is an alinear function of the instantaneous rate r(t)
Bond and option prices can be easily derived
The entire interest rate term structure can be determined analytically.

However, other advantages were also accepted providing they were reasonable.
Additional responses included the following:

- Instantaneous forward rate can be determined easily
- Analytical formula for ZCB can be obtained by $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$
- ZCB price can be expressed as a closed form
- Volatility of the instantaneous forward rate can be calculated easily

(c) Solve for the $A(t, T)$ and $B(t, T)$ functions for the affine term structure model of the two above. (Hint: Only one of them is an affine term structure model.)

Commentary on Question:
The candidates performed relatively poorly on this section. Few candidates were able to successfully derive complete expressions via differential equations or recall the already-derived expressions of the Vasicek model as indicated in the formula sheet equation 3.25. Candidates could have solved for A(t,T) and B(t,T) by either solving the differential equations indicated below or directly by using the formula below which appears in the formula sheet. Since the question did not specify a particular approach, full points were awarded if the correct answer was arrived at using either method. Overall candidates were more successful at derivation of B(t,T).

2. Continued

Model 1 – Vasicek
Model 2 – Dothan
Vasicek has affine term structure and Dothan is not

$dr_t = -r_t\ dt + .01\ dW_t$
A(t,T) and B(t,T) can be solved from the differential equations:
$$\frac{\partial B(t,T)}{\partial t} + \lambda(t)B(t,T) - \frac{1}{2}\gamma(t)B(t,T)^2 + 1 = 0, B(T,T) = 0$$
$$\frac{\partial[\ln A(t,T)]}{\partial t} - \eta(t)B(t,T) + \frac{1}{2}\delta(t)B(t,T)^2 = 0, A(T,T) = 1$$
Lambda = -1, eta = 0, gamma=0, delta=.01^2

$$\frac{\partial B(t,T)}{\partial t} - B(t,T) + 1 = 0$$
$$\frac{\partial}{\partial t}[B(t,T)e^{-t}] = -e^{-t}$$

$$B(t,T)e^{-t} = -e^{-T} + e^{-t}$$
$$B(t,T) = 1 - e^{-(T-t)}$$

$$\frac{\partial[\ln A(t,T)]}{\partial t} + 0.00005\ B(t,T)^2 = 0, A(T,T) = 1$$

$$\frac{\partial[\ln A(t,T)]}{\partial t} + 0.00005\ B(t,T)^2 = 0, A(T,T) = 1$$
$$\ln A(T,T) - \ln A(t,T) = -0.00005\left(s - 2\,e^{-(T-s)} + 0.5\,e^{-2(T-s)}\right)_{s=t}^{s=T} + C$$

$$\ln A(t,T) = 0.00005\left(T - t - 2 + 2\,e^{-(T-t)} + 0.5 - 0.5\,e^{-2(T-t)}\right) + C$$
$$A(t,T) = K\,e^{0.00005\left(T-t-B(t,T)-0.5\,B(t,T)^2\right)}, A(T,T) = 1$$
$$A(t,T) = e^{0.00005\left(T-t-1.5+2\,e^{-(T-t)}-0.5\,e^{-2(T-t)}\right)}$$

(d) Derive an expression from the model (i) for the expected value of the instantaneous short rate 1 year from now as it is at 1% currently.

Commentary on Question:
The candidates performed well on this section. Most candidates were able to solve directly without integration by recalling the expression of the expected value for the Vasicek model. A common mistake was to improperly apply the formula by not plugging in the initial value for r_0 correctly.

2. Continued

$$dr_t = -r_t\,dt + .01\,dW_t$$
$$D(e^t r_t) = e^t\,.01\,dW_t$$
$$e^t r_t - e^0 r_0 = \int_0^t e^s\,.01\,dW_s$$
$$r_t = e^{-t} r_0 + .01\int_0^t e^{-(t-s)}\,dW_s$$
$$E[r_t] = E[e^{-t} r_0] + E[.01\int_0^t e^{-(t-s)}\,dW_s]$$

But since the dW term is a martingale, the expectation is 0.

$$E[r_t] = E[e^{-t} r_0] = e^{-t} r_0 = .01e^{-1} = 0.36\%$$

(e) Find the discount factor from the model (i) for a cash flow 3 years from now as the instantaneous short rate is 0.0025.

Commentary on Question:
The candidates performed relatively well on this section. Successful candidates were able to solve for the discount factor given their response to part (c). The common mistake candidates made was to use the short rate model r(t) to solve instead of P(t,T)

$$P(t,T) = A(t,T)\,e^{-B(t,T)r(t)}$$
$$B(0,3) = 1 - e^{-3} = 0.9502$$
$$A(0,3) = e^{0.00005\,(3-1.5+2\,e^{-3}-0.5\,e^{-6})} = 1.00008$$
$$P(0,3) = 1.00008\,e^{-.9502*.0025} = 99.77\%$$

3. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1q) Describe and explain various issues and approaches for fitting a volatility surface.

Sources:

Volatility Correlation – The Perfect Hedger and the Fox, Rebonato, R., 2nd Edition, Chapter 9

Commentary on Question:

Part (a) asks the candidate to discuss the merits and drawbacks associated with fitting the volatility surface to prices, transformed prices, implied volatilities and price densities. It is a straightforward retrieval from the text.

Part (b) is a special case of the mixture of 2 normal distributions described in the text.

Part (c) and part (d) asks the candidate to perform simple algebra around the items determined in part (b).

Solution:

(a) Discuss the merits and drawbacks associated with fitting the volatility surface to:

(i) Prices

(ii) Transformed Prices

(iii) Implied Volatilities

(iv) Price Densities

Commentary on Question:

Candidates overall performed relatively poorly on this part. The candidates who got partial credit generally were able to do so through from fitting prices and transformed prices.

(i) Fitting prices

Merit: simple and easy to implement

Drawback: Our input quantities are prices, we are going to mix and use on the same footing quantities of very different magnitudes (out-of the money and in-the-money options)

3. Continued

Drawback: The procedure used assumes that certain quantities are known with infinite precision. It does not address the question of whether a much more 'desirable' solution could be obtained if a few of the reference prices were modified even by a very small amount.

(ii) Transformed prices

Merit: fixes the fitting price problem by rescaling prices (e.g. log price)

(iii) Fitting implied volatilities

Merit: working in terms of implied volatility will remove the mix and use on the same footing quantities of very different magnitudes without polluting the market data by any smoothing

Drawback: The great sensitivity of the associated price density function to the details of the fitting to implied volatilities

(iv) Price Densities

Merit: the density-function-based approaches the most useful and robust.

Merit: If we obtain a smooth density function we can rest assured that the associated prices and implied volatilities will also be smooth

Merit: The safest route to obtain reliable input to the pricing model

(b) Determine the expression for the variance of the mixture $\left(\sigma_m^2\right)$ and the excess kurtosis of the mixture (κ).

Commentary on Question:
Candidates generally performed poorly on this section. Most candidates did not realize that this is a special case of the formulae in the text with the weights (w_i) equal to 0.5 and the mean (μ_i) = 0 for all i

$$\sigma_m{}^2$$
$$= 0.5 * \sigma_1{}^2 + 0.5 * \left(\sigma_1{}^2 + \sigma_2{}^2\right)$$

$$K$$
$$= 3\left\{\left[\left(0.5\sigma_1{}^4 + 0.5\sigma_2{}^4\right) / \left(0.25\left(\sigma_1{}^2 + \sigma_2{}^2\right)^2\right)\right] - 1\right\}$$

3. Continued

(c) Show that σ_1^2 satisfies the following quadratic equation.

$$A\sigma_1^4 + B\sigma_1^2 + C = 0$$

$$A = 1$$
$$B = -2\sigma_m^2$$
$$C = [1 - \kappa/3]\sigma_m^4$$

Commentary on Question:
Candidates generally performed poorly on this section. Only a few candidates realized part (c) can be derived from part (b).

Substituting $\sigma_2{}^2 = 2\sigma_m{}^2 - \sigma_1{}^2$ into the excess kurt osis equation

$$\kappa = 3\left\{\left[\left(0.5\sigma_1{}^4 + 0.5\left(2\sigma_m{}^2 - \sigma_1{}^2\right)^2\right)/\sigma_m{}^4\right] - 1\right\}$$

- $\sigma_1{}^4\left(0.5 + 0.5\right) + \sigma_1{}^2\left(-2 * 0.5\sigma_m{}^2 / 0.5\right) + \left(2 - \kappa/3 - 1\right)\sigma_m{}^4$
- $\sigma_1{}^4\left(1\right) + \sigma_1{}^2\left(-2\sigma_m{}^2\right) + \left(1 - \kappa/3\right)\sigma_m{}^4$

(d) Determine the condition such that feasible solutions exist for σ_1^2.

Commentary on Question:
Candidates generally performed poorly on this section. The key to this question was to recognize that in order for a solution to exist, $b^2 - 4ac$ had to be non-negative. Although this was not specifically listed in the book it was assumed the candidates would be able to make this connection.

The equation in part (c) is the same as:

$$A\left(\sigma_1{}^2\right)^2 + B\sigma_1{}^2 + C = 0$$

This is a quadratic equation of $\sigma_1{}^2$.

Solution of $\sigma_1{}^2$:

$$= \left(-B + /sqrt\left(B^2 - 4AC\right)\right)/2A$$

- A Solution for $\sigma_1{}^2$ exists if and only if $B^2 - 4AC >= 0$

By substitution we get $4\sigma_m{}^4 \quad -4\left(1 - \kappa/3\right)\sigma_m{}^4 >= 0$

and thus the condition is $4\kappa\sigma_m{}^4/3 >= 0$

4. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2h) Demonstrate an understanding of credit default swaps (CDS) and the bond-CDS basis, including the use of CDS in portfolio and trading contexts.

Sources:

V-C183010: Handbook JP Morgan – Credit Derivatives

Commentary on Question:

This question tested the candidate's understanding of a CDS: its utility, how it works and the cash-flows of a CDS.
Candidates performed relatively well on this question.

Solution:

(a) Describe five key considerations when establishing a negative basis trade.

Commentary on Question:

The candidates performed relatively well on this section. Most candidates were able to provide a satisfactory list, however the description or the quality of the description of items listed was lacking.

More points were given for describing and explaining the key considerations rather than just listing them. A brief description was sufficient. Below are listed 5 considerations that were most closely related to the questions. Other considerations were also accepted if the description was also included.

1. Cheapest to deliver or deliverability of bond
   Buyer of CDS has the option of delivering the cheapest bond upon default, thus the recovery rate of the deliverable bond and the CDS might be different.
2. Maturity mismatch
   CDS that have the exact same maturity than the bond to hedge
3. Bond price vs. coupon
   Similar CDS on 2 bonds having the same spread, thus the same perceived credit risk, will exhibit different Cash flows pattern and profits upon defaults.
4. CDS running spread vs. Bond coupon at default
   On default, bond coupons are lost while CDS accrued coupons are due
5. Funding cost
   Although two CDS may imply the same spread, the cash flow structure will affect the result in case of default.

4. Continued

(b) Explain why PECS is an appropriate measure to compare with CDS spreads.

Commentary on Question:
The candidates relatively well on this section. Most of the candidates were able to identify the two elements, but few also mentioned that the PECS was matching the market price and was sensitive to the cash flow structure.

It was important to highlight what characteristics of the PECS measure wasn't a characteristic of the other measures.

PECS is a function of bond assumed bond **recovery rate** and a function of the term structure of **default probabilities.**

It is the CDS spread which would **match the bond market price** respecting the recovery rate and term structure of default probabilities implied by the CDS market.

PECS is sensitive to the **cash flow structure of the bond**.

(c) Design the negative basis trade that minimizes credit risk.

Commentary on Question:
Candidates performed well on this section. Most of them identified the proper trade and gave the correct justifications.

It was required that the candidate identify the proper trade to execute but more importantly why it was the correct trade. Throughout the text, the benefit and basis of a negative trade are explained. There was only one possible trade that fit that requirement. Points were also given if you had explained why the other possible trades were not selected.

Buy a CDS and a bond on SafeCo with 4% coupon.

Because the basis is negative for this bond, the 3 others bonds have a positive basis.

Basis = CDS spread – bond spread = 200bps – 250bps = -50 bps

Bond and CDS position offset each other, and then investor has no credit risk.

(d) Calculate the total net profit on default if it happens in exactly one year, just before coupon payment.

4. Continued

Commentary on Question:
The candidates performed well on this section.

We expected the candidates to show their understanding of how a CDS works and what are the cash flows during the life of a CDS. Hence, you could have an answer either using the formula in the textbook, or illustrate the cash-flows produced by that transaction to get credit.

Also, some candidates commented that the recovery rate was missing. However it was expected that you could obtain it, either by an understanding the recovery rate was irrelevant or by simplifying the formula to prove that it was irrelevant as long as the same notional amount is used of the bond and the CDS.

The CDS upfront in this case is negative. It is neither an error nor an accident. It was important to get that item right to show that you really understood the concept of a CDS. That was a common error among candidates.

First, using the formula in the textbook:
= CDS notional * (100 – recovery – CDS upfront - CDS coupon paid – CDS funding cost paid) + Bond notional * (Recovery + Bond coupons received – Bond Price – Bond funding cost paid)

Per the question, there is no funding cost.
= 1000 (100 – recovery – (-.0165*100) – 4*.01*100 – 0) + 1000 * (recovery + 1*(.04/2)*100 – 96.1 – 0)

=97,650 – 1000*recovery + (-94100 + 1000*recovery)

=97650 – 1000*recovery -94100 + 1000*recovery

Hence, if same notional for bond and CDS then recovery rate is not a factor.

=3,550

4. Continued

Second, by listing cash-flows:

Time	Cash in	Cash out	Net Cash
0	CDS upfront	Cost of bond	
3 months		CDS running	
6 months	Bond coupon	CDS running	
9 months		CDS running	
1 year	Notional paid by CDS	CDS running	
Total			
Time	Cash in	Cash out	Net Cash
0	1,650	96,100	-94,450
3 months		1,000	-1,000
6 months	2,000	1,000	1,000
9 months		1,000	-1,000
1 year	100,000	1,000	99,000
Total	103,650	100,100	3,550

(e) Calculate the timing and amount of each cash flow realized during the first seven months of the selected transaction if there is no default.

Commentary on Question:
The candidates performed relatively well on this section. A common mistake was to not understand a semi-annual bond means that half the coupon is paid twice a year and that a quarterly frequency on the CDS running spread means ¼ is paid each quarter. It was important to get the correct cash-flows at the right time. Once again, the CDS upfront is negative and many candidates were confused by this. There was a high correlation between the performance on this section and section (d).

T = 0 (company pays for the structure)
= -CDS upfront – price of bond
= +1.65% 100,000 - 96,100 = (94,450)

T= 3 months
= -CDS running = -1% x100,000 = -1,000

T = 6 months
= bond coupon – CDS running
Bond coupon = (4%/2) x 100,000 = 2,000
CDS running = 1,000
= 2,000 – 1,000 = 1,000

5. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2a) Demonstrate an understanding of events and causes of the recent global credit crisis.

Sources:

QFIA-102-13: Credit Risk Measurement IN and Out of the Financial Crisis, Saunders, A., Allen, L., 3rd Edition, Chapters 1 and 2

Commentary on Question:

This question tests the candidates' understanding of various securitization vehicles, the role of subprime mortgage market in the 2007-2008 financial crisis and the consequences it has.

Solution:

(a) Explain the differences between an SPV and an SIV.

Commentary on Question:

The candidates did relatively well on this question. The question asks to explain the differences between the two vehicles. Some candidates only defined the vehicles without contrasting them and hence did not receive points.

- SPV sells ABSs directly to investors to raise cash; SIV sells bonds or commercial paper to investors.
- Under SPV, investors have direct rights to the cash flows of the underlying loans. Under SIV, investors do not have direct rights to the cash flows of the underlying loans. Instead, they are entitled to the payments specified on the SIV's debt instruments.
- SPV only pays out what it receives from the underlying loans. SIV is responsible for payments on its ABCP obligations whether or not the underlying pool of assets generates sufficient cash flow to cover those costs.
- SPV earns only a fee from the creation and servicing of ABSs. SIV earns a spread as the loan assets is expected to generate higher returns than its cost of funds from ABCP.
- SPV terminates when it's underlying ABSs mature. SIV's lifespan is not tied to any security.

(b) Identify the risks which are present in Bears Bank's loan financing and underwriting.

5. Continued

Commentary on Question:
The candidates did relatively well on this section. Candidates were expected to pick up the following information from the data given in the question and utilize it to answer the question:

- *A significant portion of the portfolio is substandard loans.*
- *Substandard loans have a very low down payment.*
- *The national LEPI index is increasing rapidly, consistent with the appearance of a bubble and similar to the 2007-2008 financial crisis initiated by the breakdown of the subprime mortgage market.*

The best answers took the information a step further by explaining the consequences in the event of a financial crisis, drawing from the readings on the subprime mortgage problem in the 2007-2008 financial crisis. Those that listed a generic set of risks received little credit.

- A large portion of the portfolio is substandard loans, indicating borrowers have low credit rating and/or high default probability.
- Low down payment rate (i.e. high debt-to-equity ratio) on top of loans being substandard further indicate a high likelihood of default.
- Increase in demand index is rapid and likely unsustainable. If the bubble bursts, many loans will default.
- If such event happens (many loans default), SPV won't have enough cashflow to back its ABS/CLO obligations. This will lead to higher liquidity risk.
- Higher defaults will also result in credit rating downgrades, which leads to higher credit risk.
- In the event of credit rating downgrades, investors will lose confidence and turn to government issued securities.
- This makes Bears Bank harder to issue private loans, thus exacerbating the liquidity problem and furthering the crisis.
- The index is a nationwide index, indicating that there is no geography diversification because the fall in demand is nationwide.

(c) Assess the benefits and risks that are created by this securitization.

Commentary on Question:
The candidates performed well on this section. Candidates did better in answering the benefits of securitization than the risks. Since the question asks for both, candidates who answered only the benefits received a maximum of half the points.

5. Continued

Benefits:

- Instead of having loans as assets on bank's BS, banks receive cash proceeds from SPV from the sale of ABS/CLO. This moves the liquidity risk off of bank's BS
- Credit risks can be moved off of bank's BS as well, as investors of ABS/CLO now own the cashflow of the mortgages. This frees up capital for the bank.
- Interest rate risks are moved off of bank's BS. Should rates on long-term loans drop below rates credited to short-term deposits, banks can use the proceeds from SIV to issue new loans at a higher rate.
- With fewer loans on the book, banks can now issue more loans (and start the cycle of securitization all over again), generating more earnings.
- Banks can reach out to more investors via the varying tranches through securitization than it otherwise would.
- Banks can take advantage of the tax benefits of securitization.

Risks:

- There is less regulation for off BS items (lack of standard measuring and reporting structure).
- The separation of ownership (those who underwrite the loans and those who bear the risk of the loans) leads to underwriting risk.
- There is moral hazard to do less due diligence, leading to less transparency around the structure of ABS/CLO.
- It is hard to assign a credit rating to the ABS/CLO.
- As securitization frees up bank's capacity to finance more loans, banks start the cycle of securitization all over again, thus magnifying the risks of securitization.

6. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.

Sources:

Introduction to Credit Risk Modeling, Bluhm, Christian, 2nd Edition, Chapters 1 and 2

Commentary on Question:

This question tests candidates understanding of basic concepts of credit analysis (probability of default, loss given default and exposure at default) and the application of these concepts in a numerical problem-solving framework.

Solution:

(a) Describe in words the key assumptions of the Moody's KMV model by focusing on "default-only" mode and ignoring the "mark-to-model" approach.

Commentary on Question:

The candidates performed relatively well on this section. Candidates were generally able to describe at least two assumptions with "loss is assumed to occur when the firm's asset value is below a critical threshold" being the most common one. Few candidates, however, identified the assumption of "loss variable is Bernoulli mixture model".

1. KMV assumes the loss variable is Bernoulli mixture model.

2. Loss is assumed to occur when the firm's asset value is below a critical threshold.

3. The log-return of the firm's asset value is assumed to be normally distributed and is driven by a composite factor and a firm-specific factor.

4. Asset correlations between counterparties are exclusively captured by the correlations between the respective composite factors.

(b) Determine the amount of cash, if any, withdrawn by Sax Glass during the first year.

6. Continued

Commentary on Question:
The candidates performed relatively poorly on this section. Most candidates were able to compute the loss given default (LGD) required in the question, but unable to correctly compute exposure at default (EAD). Partial credit was given if the candidates successfully completed partial steps required to arrive at the final solution.

Let X = cash withdrawn by Sax Glass during the first year
Expected Loss = EAD * LGD * PD = 156,000
Collateral in the event of default = 100,000 * 20 = $2,000,000
PD = 1%
EAD = X
+ 80% * 60%* (20,000,000 – X)
+ 40% * 70% * (30,000,000 – 20,000,000)
= 52% * X + 12,400,000

LGD = [EAD – Collateral]/EAD = [EAD – 2,000,000]/EAD
156,000 = [52% * X + 12,400,000 – 2,000,000] * 1%
X = 10,000,000

(c)

(i) Explain, in general, why the independence assumption for variables underlying the expected loss analysis is not a good assumption.

(ii) Provide, in particular, an example from the Sax Glass case to support each of your explanations in (i) above.

Commentary on Question:
The candidates performed relatively poorly on this section. Some were able to describe correlations between loss given default and probability of default as they relate to Sax Glass and Grant Auto. However, many candidates did not adequately explain why the three variables underlying the credit analysis (EAD, LGD and PD) are correlated. Partial credit was given as long as the candidate's answer included references related to some of the three variables.

6. Continued

In general	In particular
PD and LGD: Defaults and recoveries to some extent are influenced by the same underlying systemic risk drivers so that they cannot be independent.	When Sax Glass defaults due to bad economy, it's likely that the share price of Grant Auto is lower than $20 due to bad economy, too. This will reduce Blues Bank's recovery when Sax Glass defaults
PD and EAD: In times of financial stress, firms tend to draw on their open credit lines, this increases EADs in times when default rates are going high systematically, so even EAD cannot safely be considered as independent from default risk.	Sax Glass tends to max out its credit line in a bad economy when its default rate is likely to be high.
EAD and LGD: since PD is positively correlated to EAD and LGD, respectively, it's not reasonable to assume EAD and LGD is independent. Instead, it's likely that EAD and LGD tends to move in the same direction (the higher the EAD, the higher the LGD).	EAD will be large when Sax Glass max out its credit line if it is about to default due to bad economy. Meanwhile, LGD also tends to be large as the expected stock price of Grant Auto is lower in a bad economy.

7. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2g) Demonstrate and understanding of and be able to apply the concept of Duration Times Spread (DTS).

(2k) Understand and apply various approaches for managing credit risk in a portfolio setting.

Sources:

Ben Dor Chapters 1-4

Quantitative Credit Portfolio Management, Ben-Dor, et. al., Chapters 1 - 4

Commentary on Question:

The intent of the question was to test the candidates understanding of Duration Times Spread (DTS) by requiring both application of formulas from the text, as well as making recommendation as to the appropriateness of using the measure under different situations. Maximum points were given to those that not only recognized when DTS should or should not be used, but also explained why it demonstrated a strength or weakness of DTS.

Solution:

(a)

(i) Calculate the Percentage of Portfolio limit that ABC Corporate Bond can obtain and still comply with the DTS contribution limit.

(ii) Calculate the current contribution to DTS for XYZ CDS.

(iii) Find the maximum spread that Country of Zeus could reach before breaching the DTS contribution limit.

Commentary on Question:

The candidates performed well on this section. The majority of candidates were able to show the formulas behind their calculations which was important as some students did not use the correct figures but still received partial credit.

Contribution to DTS = Percent of Portfolio * Spread * Duration
Solve for % Portfolio→ = Max Contribution / Spread / Duration
Solve for Spread → = Max Contribution / % Port / Duration

7. Continued

(i) ABC = 8 / 40 / 8 = 2.5 bps
(ii) XYZ = 100 * 1% * 5 = 5 bps
(iii) Zeus = 8 / 0.5% / 3 = 533 bps

(b)

(i) Compare the nature of absolute spread volatility with relative spread volatility and recommend which metric should be used.

(ii) Recreate the graph above and plot a best estimate of where the three relative spread volatility data points would lie.

Commentary on Question:
The candidates performed relatively well on this section. Almost all candidates recognized that relative spread is the more appropriate measure and most were able to give some reason as to why. However, many candidates struggled to recreate the graph and recognize that the point of the question was to illustrate that relative spread remains stable over time (plotting x = y).

- Relative spread is a more appropriate measurement
- Relative spread is more stable than absolute volatility
- Volatility and spread tend to move together
- When recreating the graph, the point is to plot the 3 relative spread points on or very near to the x = y line. This shows the stability of relative spread.

(c) Evaluate each of Jimbo's statements by either defending why DTS is still appropriate or why it exemplifies a weakness of DTS.

Commentary on Question:
Candidates performed relatively poorly on this section. Many were able to identify that DTS is appropriate for Credit Default Swaps. However, candidates tended to perform poorly on part I and III. For all 3 statements, few offered an adequate explanation of their assessment.

I. DTS may not be appropriate because one weakness of DTS is that it can allow large exposures to low spread assets.
II. DTS is appropriate as spread volatility of CDS contracts are found to be linearly proportional to the level of spreads
III. DTS is appropriate. Distressed debt and other High Yield securities can still utilize DTS because it is fairly stable across the maturity spectrum.

8. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

Learning Outcomes:

(2k) Understand and apply various approaches for managing credit risk in a portfolio setting.

Sources:

QFIA-101-13: Managing Credit Risk: The Great Challenge for Global Financial Markets, Caouette, John B., et. al., 2nd Edition, 2008, Chapter 20

Commentary on Question:

This question tests the candidates' understanding of credit risk and the risk-return trade-off.

Solution:

(a) Outline the following alternative portfolio approaches as discussed in Managing Credit Risk by Caouette and their key assumptions.

(i) Altman's Optimization

(ii) MKMV Corporation's Monte Carlo Simulation

(iii) RAROC 2020's Monte Carlo Simulation

(iv) CreditRisk+'s Analytical approximation

(v) CreditMetric's Monte Carlo Simulation

(vi) McKinsey & Co's Monte Carlo Simulation

Commentary on Question:

The candidates performed poorly on this section. Many candidates listed the models and some details of each approach but were unable to accurately outline what each approach calculates and its underlying assumptions.

- Altman's Optimization: Calculates optimum portfolio weights assuming historic correlations will prevail in the future
- MKMV Corporation's Monte Carlo Simulation: Calculated expected loss, unexpected loss and portfolio distributions assuming asset value correlations approximate credit quality correlations
- RAROC 2020's Monte Carlo Simulation: Calculates risk-adjusted return on capital, daily price volatility and limit usage assuming a normal distribution of prices

8. Continued

- CreditRisk+'s Analytical approximation: Calculates expected loss, risk contribution and 99th percentile loss assuming the volatility of default probabilities incorporates the effect of default correlations
- CreditMetrics' Monte Carlo Simulation: Calculated portfolio value, standard deviation of portfolio, 1% value and marginal risk, assuming econometric estimates of parameters will continue to prevail in the future
- McKinsey & Co's Monte Carlo simulation: Calculates portfolio value distribution assuming econometric estimates of parameters will continue to prevail in the future

(b) Calculate the portfolio ratio η for a portfolio consisting of:

(i) Bond 1 only

(ii) Bond 2 only

(iii) The equal weighted portfolio

(iv) A portfolio containing 75% Bond 1 and 25% Bond 2

Commentary on Question:
The candidates performed well on this section. Most candidates got full credit on parts (i) and (ii) but did not do as well on parts (iii) and (iv) because they did not correctly calculate sigma.

(i)

Expected Annual Return = Yield to Maturity – Expected Annual Loss = 2.0% - 0.5% = 1.5%
Portfolio Ratio = Expected Annual Return / Sigma = 1.5% / 1.0% = 1.5

(ii)

Expected Annual Return = 4.0% - 1.0% = 3.0%
Portfolio Ratio = 3.0% / 3.0% = 1.0

(iii)

Expected Annual Return = weight1*EAR1 + weight2*EAR2 = 0.5*1.5% + 0.5*3.0% = 2.25%
Sigma = sqrt(weight1^2*sigma1^2 + weight2^2*sigma2^2 + 2*weight1*weight2*sigma1*sigma2*correlation) = sqrt(0.5^2*1%^2 + 0.5^2*3%^2 + 2*0.5*0.5*3%*1%*10%) = 1.63%
Portfolio Ratio = 2.25% / 1.63% = 1.38

8. Continued

(iv)

Expected Annual Return = 0.75*1.5% + 0.25*3.0% = 1.88%
Sigma = sqrt(0.75^2*1%^2 + 0.25^2*3%^2 + 2*0.75*0.25*3%*1%*10%) = 1.11%
Portfolio Ratio = 1.88% / 1.11% = 1.69

(c) Discuss how a diversified portfolio can improve your risk-return trade-off, as it relates to this example.

Commentary on Question:
The candidates performed relatively well on this section. Almost all candidates mentioned diversification and most candidates identified the 75/25 portfolio as the best option. However, most candidates missed the key relationship between correlation and diversification, thus were unable to relate to the results in (b) and get full credit.

The 75/25 portfolio has a better portfolio ratio than either 100% bond 1 or 100% bond 2. This is because they are not perfectly correlated – the lower the correlation, the greater the diversification benefits. Bond 1 has a better portfolio ratio than Bond 2, so want to use a higher proportion bond 1 than bond 2.

9. Learning Objectives:

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
   - Real Estate
   - Private Equity
   - Commodities
   - Hedge Funds
   - Managed Futures
   - Distressed Securities
   - Farmland and Timber

Learning Outcomes:

(6a) Demonstrate an understanding of the types of investments available in each market, and their most important differences for an investor.

(6b) Demonstrate an understanding of the benchmarks available to evaluate the performance of alternative investment managers and the limitations of the benchmarks.

(6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.

Sources:

QFIA-111-13: Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Chapter 8

QFIA-114-13: CAIA Level II: Advanced Core Topics in Alternative Investment, 2nd Ed., Chapter 21

Commentary on Question:

This question tests the candidates on the knowledge of the various types of real estate investments describing their investment characteristics that help to address inflation, and risk diversification, and to understand the limitations of using the just the benchmark to make investment decisions.

Solution:

(a) Compare and contrast the abilities of farmland investments, direct investment in real estate, and indirect investment in real estate in addressing the two key concerns of the CIO.

9. Continued

Commentary on Question:
The candidates performed relatively well on this section. They were able to identify that these asset classes can be used to hedge against inflation, and provide diversification benefits. However, candidates had less success comparing the different assets abilities to hedge inflation/provide diversification.

To get full points the candidates were expected to describe the ability of each investment to hedge inflation risk, and diversify risk. The candidate was expected to state that certain assets are better at hedging inflation than others and some assets have higher correlation with equities than others.

Direct Investment in Real Estates:
Ability to hedge inflation:
- Some ability to hedge inflation risk
- Apartments tend to have negative correlation with inflation
- Office, retail, industrial sectors tend to have an inflation component

Ability to diversify risk:
- Real estate returns, on average, have lower volatility than public equities
- Typically less affected by short-term economic conditions
- Not highly correlated with performance of other assets
- Geographical diversification can reduce exposure to catastrophic risks
- Values of real estate investments in different locations can have low correlations

Indirect Investment in Real Estates:
Ability to hedge inflation:
- Research has found some long-run but no short-run inflation-hedging ability
- Lower transaction cost than direct investments does not eat into fund returns

Ability to diversify risk:
- Higher correlation with equities/bond assets than direct investments, but still low
- Higher volatility due to higher use of leverage in REIT

Farmland Investments
Ability to hedge inflation:
- Significant hedge against inflation risk
- Higher interest rate associated with lower farmland return
- Land prices are procyclical

Ability to diversify risk:
- Return of holding farmland negative correlation with S&P500
- Real estate market lags behind publicly traded securities

9. Continued

(b) Describe how smoothing might undermine the validity of the NCREIF and the associated Sharpe ratio metric.

Commentary on Question:
The candidates performed relatively well on this section. Candidates were able to identify that smoothing resulted in volatility and correlations with other assets being understated causing the smoothed NCREIF index to overstate the benefits of real estate. Candidates generally did not identify the source of the smoothing, which is lower volatility stemming from infrequent property appraisals.

To get full points the candidates are expected to state the source of the smoothing as well as state the limitations of the NCREIF index which uses the Sharpe ratio. The candidate is expected to identify how it overstates the benefit of direct real estate investment.

- The NCREIF Index is based on property appraisal values rather than market values.
- Property appraisals are conducted infrequently, so appraisal-based property values can exhibit inertia.
- Appraisal values tend to be less volatile than market values, an effect known as smoothing.
- As a result of smoothing, volatility and correlations with other assets tend to be understated.
- Therefore, using smoothed NCREIF Index may overstate the benefits of real estate in a portfolio.

(c) Describe the key factors that may prevent this portfolio from achieving investment objectives for the insurance product.

Commentary on Question:
Candidates did relatively poorly on this section. Many candidates were able to state liquidity concerns associated with backing a product that is entirely liquid in 5 years with a relatively illiquid asset such as real estate. However, most candidates did not mention the concerns related to returns for real estate being just barely above inflation, and the possibility that future returns could deviate from NCREIF.

To get full credit, candidates are expected to state that the expected return of the portfolio with 20% NCREIF is barely exceeding inflation targets, the liquidity profile of the direct real estate conflicts with the liquidity profile of the liabilities, and there is a risk that returns could be different from the NCREIF as it is not investable.

9. Continued

Meeting inflation target:
- Expected return is slightly higher than the target of 5%
- However, high transaction cost may jeopardize the return reaching 5%
- High maintenance and operation costs

Liquidity concerns:
- May have difficulty in liquidating the asset at the end of 5 years

Other concerns:
- The NCREIF Index is also not investable
- As a result, actual returns achieved from direct investments may not track closely with the Index

(d) Recommend and justify one of these two alternatives to the CIO.

Commentary on Question:
Candidates did relatively poorly on this question. Most candidates were able to provide only one or two reasons to justify their recommendation, but were not able to provide the additional benefits to fully justify their recommendation.

To get full points the candidates must justify in detail either indirect real estate investments or farmland. The candidates did not receive extra points for justifying both – credit was based on the superior answer.

<u>If indirect investment in real estate is suggested:</u>
- The portfolio has similar expected return to direct investments
- Can invest into commercial REITs to increase correlation with inflation
- Does not require specific farmland / agricultural knowledge
- More liquid than farmland investments
- Foreign farmland investments may be subject to risk of expropriation
- Lower transaction cost decreases the chance of fund return not reaching inflation target
- Higher volatility than direct investments, but still relatively low, and is still good at diversifying risk of the portfolio
- Higher liquidity than direct investments

<u>If farmland investment is suggested:</u>
- Although the portfolio performance exhibits lower expected return, farmland investments are expected to track closer to inflation than the other two asset classes
- Current global economic and demographic growth will likely cause demand of agricultural products to outgrow supply, leading to future appreciation of existing farmland assets; therefore, the expected return is more certain

9. Continued

- Relatively free from wholesale disruptions in market structure, organizational form, and political economy
- A relatively more mature and stable asset class; expected to have lower volatility and stable properties against macroeconomic factors
- Need to find someone who understands and can analyze crop yields
- Expected return higher than target inflation
- Low transaction cost does not eat into expected return
- Low correlation with other assets makes it a good diversifier

10. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
   - One and two-factor short rate models
   - LIBOR market models

   The candidate will understand approaches to volatility modeling.

Learning Outcomes:

(1b) Understand and explain the terms Time Homogeneous Models, Affine Term Structure Models and Affine Coefficient models and explain their significance in the context of short rate interest models.

(1g) Understand and explain the features of the G2++ model, including: The motivation for more than one factor, calibration approaches, the pricing of bonds and options, and the model's relationship to the two-factor Hull-White model.

Sources:

Interest Rate Models – Theory and Practice: With Smile, Inflation and Credit (2nd Edition), Brigo, D. & Mecurio, F., Chapters 3.2, 4.1 and 4.2

Commentary on Question:

The question attempts to test certain understanding of certain one- and two-factor short rate standard yield curve models. Also, it tests candidates' ability to distinguish the properties between some one- and two-factor models, and to apply them in pricing a given financial instrument.

Solution:

(a) Interpret each of the parameters in the model given above.

Commentary on Question:

The candidates performed extremely well on this section. They had successfully interpreted the meaning of the parameters of the Vasciak model. However, a handful of candidates mixed up the speed (k) of mean reversion and the long term mean level (θ) of the interest rate.

k: speed (rate) of mean reversion
θ: long term mean level (average) of interest rate
σ: volatility of the interest rate
r(0): interest rate at time 0

(b) Explain why the Vasciek model is an affine term-structure model.

Commentary on Question:

The candidates performed relatively well on this section. They had successfully wrote down the expression of P(t,T) only, but most of them did not write out the expressions of A(t, T) and B(t, T) at all.

10. Continued

The Vasicek model is an affine term-structure model because the zero-coupon bond can be written in the following form:

$P(t, T) = A(t, T) = EXP(-B(t, T)r(t))$, where

$A(t, T) = EXP[(q - s^2 / 2k^2) [B(t, T) – T + t] – (s^2 / 4k) B(t, T)^2]$
$B(t, T) = (1 / k) [1 – EXP(-k(T – t))]$

(c) Assess the appropriateness of each model.

Commentary on Question:
The candidates performed well on this section. Most of them were able to state that the Vasicek and CIR models are inappropriate because of assumed perfect correlation of forward rates on the yield curve, and that CIR disallows negative rate. Most candidates also were able to recall the basic property of the G2++ model which is that it allows negative interest rates. However, most candidates did not recall the more advanced properties of the G2++ model which would allow you to connect them to the attributes of European Swaptions and conclude that it is appropriate to use the G2++ model to price European swaptions.

1. The Vasicek model is <u>not appropriate</u> because, as one-factored model, it assumes perfect correlations among rates of different maturities.

2. The CIR model is <u>not appropriate</u> because of the following reasons:
   - As one-factored model, it assumes perfect correlations among rates of different maturities.
   - It does not allow negative interest rates.

3. The Gaussian G2++ <u>is appropriate</u> because of the following reasons:
   - It allows negative interest rates for the given situation.
   - Analytical tractability eases the task of pricing of the exotic European swaption.
   - The Gaussian distribution allows the derivation or a number of non-plain vanilla instruments such that it can combine with the analytical expression for non-zero bonds, leading to efficient and fairly fast numerical procedures for pricing any possible payoffs.
   - Being a two-factored model, a non-perfect correlation among rates of different maturities is introduced (it means this results in a more precise calibration to correlation-based products like European swaptions).

11. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

Learning Outcomes:

(3c) Understand the levels of liquidity available with various asset types, and the impact on a company's overall liquidity risk.

(3d) Understand liability termination provisions such as book-value surrender and the impact on a company's overall liquidity risk.

(3f) Apply liquidity scenario analysis with various time horizons.

(3g) Create liquidity risk management plans and procedures, including addressing appropriate product design, investment guidelines, and reporting given a desired liquidity risk level.

Sources:

QFIA-105-13: Report of the Life Liquidity Work Group of the American Academy of actuaries to the Life Liquidity Risk Working Group of the NAIC

QFIA-106-13: Liquidity Risk: Measurement and Management – A Practitioner's Guide to Global Best Practices, Matz, Leonard and Neu, Peter, 2006, Chapter 3

Commentary on Question:

This question tests the candidate's ability of understanding the impact of surrender charges, the calculation and analysis of cash flow cushion, and of the inclusion of various asset classes on the liquidity risk of the company.

Solution:

(a) Discuss implications of permitting the policyholder to surrender with little or no penalty.

Commentary on Question:

Candidates performed relatively well on this section. Almost everyone identified the increase liquidity risk, which was a key element. However, many candidates did not get the other comments below.

- Liquidity risk increases as surrender charges grade down.
- Market value adjustment based on a fixed formula may result in value greater than the immediate sale of assets.
- Book value "put" provisions and others that permit surrender without penalties are effectively free withdrawals.
- Knowledgeable institutional contract holders with free puts could create dramatic drains on a company's liquidity in a stress scenario.

11. Continued

(b) Discuss considerations when setting up back-stop liquidity lines.

Commentary on Question:
Candidates performed relatively well on this section. Some candidates did not provide answers relevant to the question asked for example candidates frequently answered "soft credit events" and "bond covenants" as considerations.

- Be able to draw millions of dollars on short notice, with a guaranteed price.
- Typically best to have these outside of the "family".
- Key is for liquidity lines to be durable.
- If the counterparty can get out of lending when the company needs it the most, the counterparty's promises may not be all that valuable.

(c) Calculate the cash flow cushion at the end of the first year under the base scenario and the stress scenario.

Commentary on Question:
Candidates performed well on that question. Most candidates were able to calculate the base scenario correctly. However many candidates missed a few elements of the stress scenario calculation and hence got partial credit. A common mistake made was defining the cash flow cushion as a dollar value when it is a ratio.

The Cash Flow Cushion is defined as the ratio of the forecast cash inflows for the period divided by the forecast cash outflows for the period.

i.e.: Cash Flow Cushion = Forecast cash inflows for the period / Forecast cash outflows for the period

For the base scenario:

Forecast cash inflows = income payable on assets = 20 million
Forecast cash outflows = death benefits and expenses = 10 million
Cash flow cushion = 20/10 = 2.0

Stress scenario:

Forecast cash inflows:

Proceeds from sale of CMBS = .80 * .25 * 500 = 100 million
Proceeds from sale of corporate bonds = .75 * .25 * 500 = 93.75 million
Total income received, as above in base scenario = 20 million (this is the total income received since it is assumed that the coupon will be received prior to sale.

11. Continued

Borrowing line available from parent = 5 million
Total = 100 + 93.75 + 20 + 5 = 218.75 million

Forecast cash outflows:

Death benefits and expenses = 10 million
Surrenders = 0.50 * 500 * (1 – 0.18) = 205 million
Total = 215 million

Cash flow cushion = 218.75 / 215 = 1.017

(d) Determine the additional funding, if needed, under both scenarios to achieve the target ratio.

Commentary on Question:
Candidates performed extremely well on this section. This question relied on the answer from the part (c). It was graded assuming the part (c) answer was correct. Thus full credit could be obtained even if part (c) was incorrect. Oddly, some candidates got the definition of the cash-flow cushion correct here and wrong in part (c).

- No additional funding needed under the base scenario.
- Additional funding needed for the stress scenario.

Target ratio = (forecast cash inflow + additional funding)/forecast cash outflow
1.1 = (218.75 + X)/ 215
Additional funding = 1.1x215 – 218.75 = 17.75

(e) Recommend and justify changes to the asset portfolio to reduce liquidity risk given the investment policy.

Commentary on Question:
Candidates performed extremely well on this section. This was a qualitative question that asked for a qualitative answer. However, a few candidates tried to quantify the different changes in allocation based on the existing portfolio using the cash-flow cushion from previous sections. Most candidates provided a correct recommendation however full credit was only given if the recommendation came with a justification.

- Reduce allocation to commercial mortgages, as this asset can be difficult to sell.
- Add an allocation to cash to increase the liquidity of the portfolio.
- Add an allocation to U.S. Treasuries to increase the liquidity of the portfolio.
- Reduce percentage allocation to CMBS due to reduce bid-ask spread.
- Shift the quality of the corporate bonds higher (to A) to reduce bid-ask spread.

12. Learning Objectives:

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

(4c) Understand and apply various techniques for analyzing multivariate time series.

(4d) Understand the concept of cross correlation in multivariate time series.

Sources:

Analysis of Financial Time Series, Tsay, Ruey S., 3rd Edition, 2010

- Chapter 8 Multivariate Time Series Analysis and its Application (excluding Appendices)

Commentary on Question:

This question tests cross correlation and the multivariate portmanteau test in multivariate time series.

Solution:

(a) Determine the values of correlation coefficients $\rho_{22}(0)$, $\rho_{52}(0)$ and $\rho_{55}(0)$.

Commentary on Question:

Candidates performed well on this section. Those that did miss points generally missed getting $\rho_{52}(0) = \rho_{25}(0) = 0.3$ *and assumed the answer was 1. Candidates had to get all correlation coefficients correct to receive full credit.*

$\rho_{52}(0) = \rho_{25}(0) = 0.3$, by symmetry.

Since $\rho_{ij}(0) = 1$, for $i = j$, $\rho_{22}(0) = \rho_{55}(0) = 1$.

(b) Explain whether $\rho_{52}(3)$ can be obtained using the same approach as for (a).

Commentary on Question:

Candidates performed relatively well on this section. Many candidates correctly mentioned that $\rho_{52}(3)$ *cannot be obtained using the same approach under part (a) with a correct reason. Candidates had to provide a reason to get full credit.*

No, $\rho_{52}(3)$ cannot be obtained using the same approach as for (a) since the lag-l cross correlation matrix is NOT symmetric.

(c) Describe the situation in which $\{r_{5t}\}$ and $\{r_{10t}\}$ are said to be uncoupled.

12. Continued

Commentary on Question:
Candidates performed relatively poorly on this section. They generally only mentioned one of $\rho_{510}(l)=0$ *and* $\rho_{105}(l)=0$ *but not both which was required. In addition, a significant number of candidates did not mention that the conditions are for all lag l >0. As a result, they did not receive full credit.*

$\rho_{510}(l)=0$ and $\rho_{105}(l)=0$ for all $l>0$, we say time series $\{r_{5t}\}$ and $\{r_{10t}\}$ are uncoupled.

(d) Determine whether we can conclude that $\{r_{5t}\}$ and $\{r_{10t}\}$ have no linear relationship.

Commentary on Question:
Candidates performed poorly on this section. Many answered there was no linear relationship when there could be.

No, two uncoupled series may still have linear relationship. Please note that when l=0, $\rho_{510}(l)$ or $\rho_{105}(l)$ can still not equal to zero.

(e) Outline the steps required to apply the multivariate Portmanteau test.

Commentary on Question:
Candidates performed relatively well on this section. In general, candidates answered correctly Step 1 and Step 2 correctly. Some candidates mixed up the meaning of p value and critical value of the test statistics.

Step 1. Set up the null hypothesis: No auto- and cross correlations at lag 1.
Step 2. Calculate the test statistic $Q_k(m)$ and identify the parameters.
Step 3. Determine whether p value < significant level or alternatively the test statistic > critical value of the test statistic using the asymptotic distribution of $Q_k(m)$

(f) Identify the asymptotic distribution of the test statistic $Q_k(m)$, under the null hypothesis.

Commentary on Question:
Candidates performed relatively well on this section. In general, candidates correctly identified the distribution as chi-square. However, they either did not write down the correct degrees of freedom or an incorrect one. As a result, they did not receive the full credit.

12. Continued

It's chi-square distribution with degrees of freedom $k^2 m$ where k=2 and m=1.

(g) Conclude whether you can reject the null hypothesis of the test and justify your answer.

Commentary on Question:
Candidates performed relatively well on this section. One of the most common mistakes made were to not take into account of the sample size and calculate 0.02 x 500=10 correctly. Another common mistake was to mix up the meaning of p value and critical value of the test statistics.

The test statistic equals 0.02x500=10>9.488. Thus, we reject the null hypothesis and conclude that there are cross-correlations between the two time series at lag 1.

13. Learning Objectives:

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

(4h) Describe and assess performance measurement methodologies for assets, liability and hedge portfolios.

(4i) Describe and assess techniques that can be used to select or build a benchmark for a given asset, portfolio.

(4j) Recommend a benchmark for a given asset, portfolio.

(4k) Calculate and interpret performance attribution metrics for a given asset, portfolio.

Sources:

QFIA-107-13: Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012, Chapter 69 and 70

Commentary on Question:

This question tests candidates on performance attribution, calculating attribution metrics such as outperformance, and recommending a performance attribution model based on a given set of criteria.

Full credit is given to candidates who demonstrate accurate understanding of the concepts and correct implementation of the formulas.

Solution:

(a) Briefly describe Performance Attribution.

Commentary on Question:

The candidates performed well on this section. Most candidates were able to describe performance attribution definition. However, a few candidates did not describe the purpose and usage.

A mathematical model framework that attempts to split total outperformance of a portfolio versus a benchmark into the contributions of individual decisions and actions.

An important function of portfolio management that brings clarity to the sources of portfolio risk and performance.

Quantifies the contributions of individual decision makers and identifies structural issues.

13. Continued

(b) Calculate the outperformance (in bps) for each of the following:

(i) The Indian Rupee portion of the portfolio versus the Indian Rupee portion of the index.

(ii) The China Renminbi portion of the portfolio versus the China Renminbi portion of the index.

Commentary on Question:
The candidates performed poorly on this section. Candidates generally did not calculate outperformance contribution of a portion versus index correctly; a few were able to figure out the outperformance contribution, but missed the weight.

Outperformance contribution of the Indian Rupee portfolio = +22 bps. From table above; intersection between Local Management (bps) and Indian Rupee

Indian Rupee portfolio weight = 0.40 or 40%. From table above; intersection between Portfolio Weight (%) and Indian Rupee

Outperformance (in bps) of the Indian Rupee's portion of the portfolio version the Indian Rupee portion of the index = (Outperformance contribution of the Indian Rupee) / (Indian Rupee portfolio weight) = +22 bps / 0.40 = 55 bps

Outperformance contribution of the China Renminbi portfolio = +18 bps. From table above; intersection between Local Management (bps) and China Renminbi

China Renminbi portfolio weight = 0.30 or 30%. From table above; intersection between Portfolio Weight (%) and China Renminbi

Outperformance (in bps) of the China Renminbi portion of the portfolio version the China Renminbi portion of the index = (Outperformance contribution of the China Renminbi) / (China Renminbi portfolio weight) = +18 bps / 0.30 = 60 bps

(c) Conclude whether your client's investments in the Indian Rupee outperformed their investments in the China Renminbi.

Commentary on Question:
The candidates performed well on this section. Most candidates were able to correctly draw the conclusion based on the results from prior part. Credit was given in this part for the conclusions regardless of the accuracy of the prior part.

Since the Indian Rupee Outperformance of 55 bps is less than the China Renminbi Outperformance of 60 bps, it can be concluded that investments in the Indian Rupee did NOT outperform investment in the China Renminbi

13. Continued

(d) Calculate each of the following for the Corporate Sector:

(i) The outperformance due to asset allocation.

(ii) The outperformance due to security selection.

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were able to calculate the results correctly show their work. Partial credit was given for intermediate steps shown if the final answer was incorrect.

Outperformance due to Asset Allocation
= (Market Weight (%) Average Portfolio Corporate – Market weight (%) Average Benchmark Corporate) x (Return Ex Common Factors Benchmark Corporate – Return Ex Common Factors Benchmark Total)
= (29%-16%) x (185-124)
= 7.93%

Outperformance due to Security Selection
= (Market Weight (%) Average Portfolio Corporate) x (Return Ex Common Factors Portfolio Corporate – Return Ex Common Factors Benchmark Corporate)
= (29%) x (114-185)
= -20.59%

(e) Assess the portfolio manager's decisions with respect to asset allocation and security selection for the Corporate sector.

Commentary on Question:
The candidates performed well on this section. Most candidates were able to correctly assess the portfolio manager's decisions.

The portfolio manager had great skill in choosing the asset allocation shown by the 7.93% outperformance.

On the other hand, the portfolio manager did a poor job of selecting securities shown by the -20.59% outperformance.

(f) Recommend an appropriate performance attribution distribution model for your client, and support your recommendation.

13. Continued

Commentary on Question:
The candidates performed relatively well on this section: a few candidates were able to recommend the fully analytical model and fully justify their recommendations. However some were able to recommend the correct model but did not provide an adequate explanation and hence got partial credit. It was also possible to get partial credit for recommending a different model as long as a consistent explanation was provided.

Recommendation: Fully Analytical Model

Justification:
It takes full advantage of the return splits by isolating returns from each factor individually. A Fully Analytical model allows portfolio managers to use measures other than market value as allocation weight. Managers have the ability to allocate their capital based on risk exposures rather than market values.

Client wants to include Mortgage Backed Securities. Mortgage securities have additional exposure to prepayment risk and mortgage spreads. Managers want to manage these additional risk factors separately.

Client wants to include Inflation-Linked Bonds. Inflation-Linked Bonds have additional exposure to their reference inflation indices. Managers want to manage these additional risk factors separately.

This model gives managers the ability to break down outperformance due to specific factor exposures which becomes very valuable to explain the impact on their portfolio decisions.

14. Learning Objectives:

4. The candidate will understand important quantitative techniques for analyzing financial time series Performance Measurement and Performance Attribution.

Learning Outcomes:

(4f) Understand the concept of a factor model in the context of financial time series.

(4g) Apply various techniques for analyzing factor models including Principal Component Analysis and Statistical Factor Analysis.

(4m) Understand and apply various techniques of adjusting autocorrelated returns for certain asset classes.

Sources:

Analysis of Financial Time Series, Tsay, Ruey S., 3rd Edition, 2010

- Chapter 9 Principle Component Analysis and the Factor Models

Commentary on Question:

The question was testing the ability of the candidate to apply some of the concepts regarding Principle Component Analysis and Statistical Factor Analysis numerically.

Solution:

(a) Interpret the first and second principal components in principal component analysis, and identify any constraints.

Commentary on Question:

The candidates performed relatively well on this section. One common error was to not identify the constraints which prevented the candidates from getting full credit. Another common mistake made was to identify the first principal component as market and the second principal component as economic and this was not what the question was trying to test.

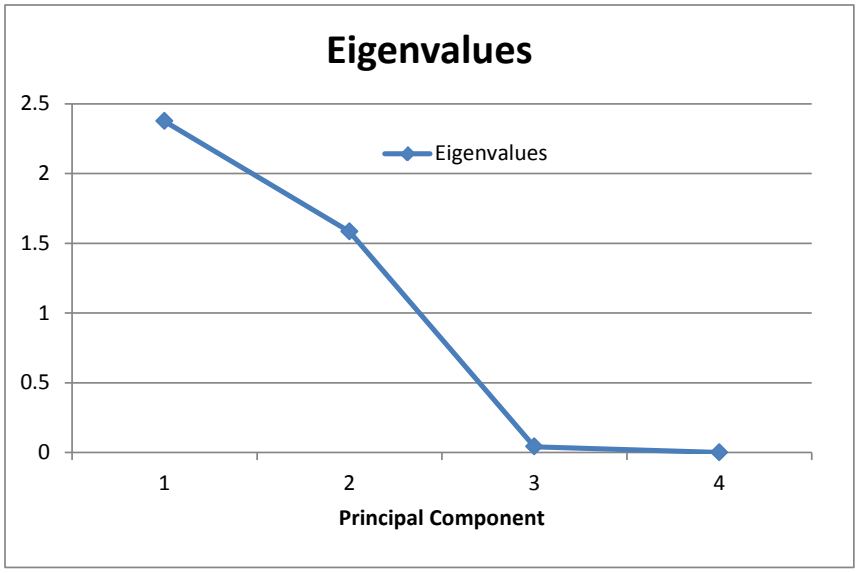
- The first principal component of the stock returns is the linear combination $y_1 = w_1 * r_t$ where w_1 is the vector of weightings for each stock in the portfolio that maximizes the var(y_1) subject to the constraint that $w_1' * w_1 = 1$. (The magnitude of the weighting vector is 1)
- The second principal component is the linear combination $y_2 = w_2 * r_t$ where w_2 is the vector of weightings for each stock in the portfolio that maximizes the var(y_2) subject to the constraints that $w_2' * w_2 = 1$(the magnitude of the weight vector w_2 must be equal to 1) and the additional constraint that the cov(y_1, y_2) = 0.

(b) Recommend how many principal components should be used in your statistical factor analysis and justify your recommendation.

14. Continued

Commentary on Question:
The candidates performed well on this section. Most people were able to correctly identify 2 principal component which was necessary for full credit. Partial credit was available if a different answer was provided with a justification but this was a rare occurrence.

- $\lambda_i/(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)$ where λ_i is the eigenvalue for the i^{th} component
- First component Proportion: $\lambda_1/(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) = 2.374/(2.374 + 1.584 + 0.041 + 0.001) = 59.35\%$
- Second component Proportion: $\lambda_1/(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) = 1.584/(2.374 + 1.584 + 0.041 + 0.001) = 39.6\%$
- Third component Proportion: $\lambda_1/(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) = 0.041/(2.374 + 1.584 + 0.041 + 0.001) = 1.025\%$
- Fourth component Proportion: $\lambda_1/(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4) = 0.000/(2.374 + 1.584 + 0.041 + 0.001) = 0.025\%$
- Based on the proportion of variability two components explains 99% of the total variability.



- The Scree plot shows a sharp elbow with the third component confirming that two components is the best answer.

(c) Estimate the communality and specific variance of the returns on stock VVV using the principal components recommended in (b).

14. Continued

Commentary on Question:
The candidates performed poorly on this section. There were only a couple of candidates who got nearly perfect marks on this section. Many candidates did demonstrate in their answers that they are familiar with the communality concept but were not able apply their knowledge and hence many points were missed.

- $\beta = (\sqrt{\lambda_1}*e_1 \mid \sqrt{\lambda_2}*e_2)$ is the formula for the factor loading matrix with two components.
- Communality = $\beta_{11}^2 + \beta_{12}^2$
- $\beta_{11} = \sqrt{\lambda_1}*e_{11} = \sqrt{2.374} * 0.610 = 0.93988$
- $\beta_{12} = \sqrt{\lambda_2}*e_{21} = \sqrt{1.584} * 0.271 = 0.34107$
- Communality = $\beta_{11}^2 + \beta_{12}^2$ = 0.93988^2 + 0.34107^2 = 0.8834 + 0.1163= .9997
- Specific Variance = $\sigma_1^2 = \sigma_{11,r} - \beta_{11}^2 - \beta_{12}^2$ where $\sigma_{11,r}$ is the variance for stock VVV. So the trick to calculating the Specific Variance is to calculate the variance of VVV.
- $\sigma_{11,r} = \beta_{11}^2 + \beta_{12}^2 + \beta_{13}^2 + \beta_{14}^2 =$
  $= 0.93988^2 + .34107^2 + 0.041*(-0.165)^2 + 0.001*(-0.726)^2 = 1.0013$
- Specific Variance = 1.0013 – 0.9997 = 0.0016

(d) Explain why the communality and specific variance is not affected by the rotation.

Commentary on Question:
The candidates performed relatively poorly on this section. Many candidates realized that the orthogonality of the rotation played a role but failed to clearly articulate <u>how</u> this affected the communality and specific variance.

- The rotation is achieved by an *mxm* orthogonal matrix *P* such that the rotated factor loading matrix β^* is related to β by the following:
  $\beta^{*'} = P'\beta'$, $PP' = I$. The communality is equal to $\beta\beta' = \beta I \beta' = \beta PP' \beta' = \beta^*\beta^{*'}$ which is the communality for the rotated factors.
- The specific variance is also unchanged because 1- $\beta^*\beta^{*'}$ is equal to the specific variance of the rotated factors and 1- $\beta^*\beta^{*'} = 1 - \beta\beta'$ which is the specific variance of the unrotated factors.

(e) Determine the value of the missing rotated factor loading *a* for component 2.

Commentary on Question:
The candidates performed poorly on this section. Those that struggled with sections (c) and (d) also struggled with section (e). Some candidates realized that there was a missing unrotated factor loading that could be calculated using the fact that the communality is equal for both rotated and unrotated factors.

14. Continued

- Use the fact that the communality is equal for both rotated and unrotated factors.
- $\beta_{21} = \sqrt{\lambda_2} * e_{12} = \sqrt{2.374} * -0.121 = -0.186$
- $\beta_{22} = \sqrt{\lambda_2} * e_{22} = \sqrt{1.584} * 0.774 = 0.974$
- Communality of WWW is -0.186^2 + 0.974^2 = 0.984
- Communality of WWW for the rotated factors is a^2 + 0.210^2 = .984
- a = (.984 - .210^2)^(1/2) = .969

15. Learning Objectives:

5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

Learning Outcomes:

(5a) Explain how behavioral characteristics of individuals or firms affect the investment or capital management process.

(5c) Identify and apply the concepts of behavioral finance with respect to individual investors, institutional investors, portfolio managers, fiduciaries and corporate managers.

Sources:

QFIA-108-13: Behavioral Finance and Investment Committee Decision Making, by A. Wood, CFA Institute Conference Proceedings, December 2006

Commentary on Question:

This question explores the topic of behavioral finance in the context of an investment committee. For full credit, candidates needed to relate the information in the reading to the situation described in the question.

Solution:

(a) Describe two tests that will illustrate how the decisions of the investment committee are affected by these biases.

Commentary on Question:

The candidates performed relatively poorly on this section. Only minimal credit was given for defining the two given biases. Most candidates simply provided examples from the reading but did not describe test sufficiently or how it would test the bias.

Some possible examples of the test could include the following:

1. Survey to predict the return of a stock in the next period given ten random numbers. With no other information check to see if they pick the mean. The key would be to have each member or small groups do this independently. You may also ask for a justification of the results given. (Misconceptions of randomness)
2. The syllabus example for misconception of randomness was seeing results of an 80/20 choice and predictions made for it. (Optimal behavior would be to always pick the same choice.)

15. Continued

3. For anchoring, any test that plausibly would *indicate the Committee falling victim to "past performance indicates future performance*" is valid. One such example would to have a test where the committee was broken into two groups each given a different view of the expectation. Then see how much they use that given view to come up with a future prediction.

(b) Identify and explain in detail four additional biases that are likely being introduced by the meeting structure and behaviors of the Chairman.

Commentary on Question:
The candidates performed relatively well on this section. Common mistakes included candidates having the wrong name of the bias being explained. Also many candidates described the same thing many times. Better solutions related the biases back to the situation described. A simple list of biases without explanation got minimal credit.

The biases below represent a good sample of the information needed for full credit. Four good descriptions would receive full credit:

Bounded Rationality – the Board is likely reviewing too many numbers and too many reports to come up with reasonable conclusions. Only a limited number of pieces of data can be assimilated at once and fewer are really required to achieve reasonable accuracy

Hindsight Bias – the volume of reports focusing on historicals likely indicates that the Board is inclined to see past events as predictive of future events.

Survivorship Bias – the long ten-year historicals in the reports indicate that the Board is not accounting for survivorship. Long periods of data may not indicate sufficient observations for poor performance of a strategy.

Representativeness – the Chairman has focused on a single bad version of the strategy and may assign excessive weight to it in terms of evaluating the true performance of it or similar strategies being discussed.

Confirmation or Committee bias – Board members are unlikely to challenge the Chairman as he likely states his own opinions. Committees are often unable to challenge the opinions of those in either leadership or perceived expert positions.

(c) Recommend modifications to the Investment Advisor Committee structure that could improve the Committee's performance in the three areas suggested by the CRO.

15. Continued

Commentary on Question:
The candidates performed relatively well on this section. Most candidates were able to give a description of the characteristics desired which got partial credit. However this does not answer the question which asked for a recommendation. Most candidates failed to clearly make recommendations for the selection of the chairman or how to make the agenda better. Candidates often did not link the change suggested back to something observed that was not working. Many candidates did accurately make a recommendation for the composition of the board.

The answer should have three distinct sections addressing each of the items the CFO has suggested. Each section should have a justification for the change as to why it should help with the getting the committee working better. The three sections below are simply the ideal characteristics for the three topics:

1. Ideal Chairman:
   Less autocratic, current chairman has been described as tyrannical, surrounded by hand selected people who don't challenge him.
   The "Effective Chair" - focus, diplomacy, and communication.
   Empower others to prevent "railroading"
   Assign Tasks to specific members of the committee

2. Composition of the Committee:
   Allow Board members terms to expire and appoint a smaller Board both with a less autocratic Chairman as well as more confident Board members who are appointed independently from the Chairman. This will allow more effective discussions to proceed while limiting behavioral biases.
   In addition to the size of the Committee, it is likely that it is not diverse enough in terms of experience and expertise. Including a variety of financially-inclined but non-investment professionals on the Board would likely improve its ability to avoid crowd biases
   Independently minded individuals. Getting people who don't necessarily think like each other should allow for better discussions.

3. Meeting Agenda:
   Attempt to avoid committee bias by requiring opinions to be shared openly within the meetings. Any methods which achieve independence of thought (blind voting, reports prepared by each member before the meeting, polling responses before the meeting, etc) are all acceptable responses.
   Pre-reads – move the emphasis of the meeting from the reports to actual decision making.

15. Continued

Task oriented versus process planning agendas. (Want process planning, like setting investment policy rather than task oriented like picking stocks.)
Pressure to conform – trying things to make sure that this behavior is minimized. (See chairman)
Brainstorming outside of the meeting, writing down ideas to bring to the meeting.