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**SOCIETY OF ACTUARIES**  
**Quantitative Finance and Investments**

# **Exam QFI ADV**

## **MORNING SESSION**

**Date:** Thursday, October 31, 2013  
**Time:** 8:30 a.m. – 11:45 a.m.

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### **INSTRUCTIONS TO CANDIDATES**

#### **General Instructions**

1. This examination has a total of 100 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 40 points).
  - a) The morning session consists of 9 questions numbered 1 through 9.
  - b) The afternoon session consists of 6 questions numbered 10 through 15.The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

#### **Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas. When you are asked to recommend, provide proper justification supporting your recommendation.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam QFI ADV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.





**\*\*BEGINNING OF EXAMINATION\*\***  
**Morning Session**

- 1.** (5 points) You are an investment actuary at Soldier Life and are proposing to use a two-factor interest rate model to replace the one-factor model used currently to price long-term financial guarantees. You are considering the following models:

G2++

$$r(t) = x(t) + y(t) + \varphi(t)$$

$$dx(t) = -ax(t)dt + \sigma dW_1(t)$$

$$dy(t) = -by(t)dt + \eta dW_2(t)$$

$$dW_1(t)dW_2(t) = \rho dt$$

Shifted Longstaff & Schwartz CIR2++

$$r(t) = x(t) + y(t) + \varphi(t)$$

$$dx(t) = k_x(\theta_x - x(t))dt + \sigma_x \sqrt{x(t)}dW_1(t)$$

$$dy(t) = k_y(\theta_y - y(t))dt + \sigma_y \sqrt{y(t)}dW_2(t)$$

$$dW_1(t)dW_2(t) = 0$$

- (a) (2 points) Compare and contrast the above models.

You are given the following parameters from a Hull-White Two-factor model:

$\bar{a}$	0.680000
$\bar{b}$	0.090000
$\sigma_1$	0.014702
$\sigma_2$	0.007108
$\bar{\rho}$	-0.134500

- (b) (1 point) Calculate the G2++ parameters equivalent to the parameterization of the Hull-White Two-factor model above.

You believe that the correlation parameter of the Hull-White two-factor model has changed. In particular, you are given that the instantaneous covariance between the forward rates  $f(0, 2)$  and  $f(0, 10)$  is 0.000031758.

- (c) (2 points) Propose a process to update the parameters of your G2++ model for this new development. You do not need to give any numerical results.

**2.** (8 points) You are reviewing affine term structure models.

(a) (2 points) Define the following three models and show how they are related.

1. Affine term structure model
2. Affine coefficients model
3. Time homogeneous model

(b) (1 point) Explain the advantages of working with an affine term structure model.

You are given the following two short rate models:

(i)  $dr_t = -r_t dt + 0.01dW_t$

(ii)  $\frac{dr_t}{r_t} = -0.01dt + 0.4dW_t$

(c) (2 points) Solve for the  $A(t, T)$  and  $B(t, T)$  functions for the affine term structure model of the two above. (Hint: Only one of them is an affine term structure model.)

(d) (2 points) Derive an expression from the model (i) for the expected value of the instantaneous short rate 1 year from now as it is at 1% currently.

(e) (1 point) Find the discount factor from the model (i) for a cash flow 3 years from now as the instantaneous short rate is 0.0025.

3. (7 points) You are exploring various approaches of fitting an exogenous volatility surface. One approach would be applying the mixture of two normal price densities  $(\varphi_1(x), \varphi_2(x))$  to the data.

$\varphi_1(x)$  has a mean of zero and variance of  $\sigma_1^2$

$\varphi_2(x)$  has a mean of zero and variance of  $\sigma_2^2$

Assume they are equally weighted.

- (a) (2 points) Discuss the merits and drawbacks associated with fitting the volatility surface to:
- (i) Prices
  - (ii) Transformed Prices
  - (iii) Implied Volatilities
  - (iv) Price Densities
- (b) (1 point) Determine the expression for the variance of the mixture ( $\sigma_m^2$ ) and the excess kurtosis of the mixture ( $\kappa$ ).
- (c) (2 points) Show that  $\sigma_1^2$  satisfies the following quadratic equation.

$$A\sigma_1^4 + B\sigma_1^2 + C = 0$$

$$A = 1$$

$$B = -2\sigma_m^2$$

$$C = [1 - \kappa / 3]\sigma_m^4$$

- (d) (2 points) Determine the condition such that feasible solutions exist for  $\sigma_1^2$ .

4. (8 points) The following bonds are currently available at 12/31/2012 (PECS = Par Equivalent CDS spreads):

	Price	Coupon	Maturity Date	Next Coupon	Frequency	Par Value	PECS (bps)
SafeCo Bond1	96,100	4%	12/31/2014	6/30/2013	Semi-annual	100,000	250
SafeCo Bond2	94,510	3%	12/31/2014	6/30/2013	Semi-annual	100,000	150
RiskyCo Bond1	92,700	4%	12/31/2014	6/30/2013	Semi-annual	100,000	450
RiskyCo Bond2	88,550	4%	12/31/2015	6/30/2013	Semi-annual	100,000	450

And the following Credit Default Swaps (CDS) are also available:

	Equivalent Full Running Spread (bps)	Upfront Spread (bps)	Running Spread (bps)	Frequency	Maturity	Next Coupon
SafeCo	200	-165	400	Quarterly	12/31/2014	03/31/2013
RiskyCo	480	-115	400	Quarterly	12/31/2014	03/31/2013

Your company wants to enter into a negative basis trade. Given the bonds and CDS available:

- (a) (2 points) Describe five key considerations when establishing a negative basis trade.

For the CDS market, bonds are quoted using PECS as a measure for spread.

- (b) (1 point) Explain why PECS is an appropriate measure to compare with CDS spreads.
- (c) (2 points) Design the negative basis trade that minimizes credit risk.

Assume that all recovery rates are equal but unknown and that there is no funding cost.

- (d) (2 points) Calculate the total net profit on default if it happens in exactly one year, just before coupon payment.
- (e) (1 point) Calculate the timing and amount of each cash flow realized during the first seven months of the selected transaction if there is no default.

5. (6 points) Bears Bank is a large national bank. Its Equipment Financing department offers loan financing to commercial and residential construction companies to purchase construction equipment. Loans are underwritten in three classes: super-standard, standard, and substandard. Larry, the department head has brought you in as a risk management consultant to draft a report on the use of Special Purpose Vehicles (SPV).

Larry wants to add a section in the report on Structured Investment Vehicles (SIV).

- (a) (2 points) Explain the differences between an SPV and an SIV.

Larry provides you with the following information related to new business the Equipment Financing department underwrote last year:

Super-standard loans as a % of total new business	10%
Standard loans as a % of total new business	40%
Substandard loans as a % of total new business	50%
Down payment rate for super-standard & standard loans	20%
Down payment rate for substandard loans	5%

Larry also provides you with the following proprietary information: “Larry’s Equipment Pricing Index” (LEPI). He tells you that LEPI measures the demand for construction equipment nationwide.

Period	LEPI
Q4/2012	1200
Q1/2013	1400
Q2/2013	1800
Q3/2013	2600

- (b) (2 points) Identify the risks which are present in Bears Bank’s loan financing and underwriting.

Bears Bank is considering a securitization of these loans using an SPV.

- (c) (2 points) Assess the benefits and risks that are created by this securitization.



**6.** (6 points) Sax Glass is a private company that manufactures auto glass exclusively for Grant Auto, a publicly traded automobile company. To further expand the business, Sax Glass applied for a credit at Blues Bank. You are the Chief Credit Officer of Blues Bank and your bank uses Moody's KMV model for its credit risk modeling.

(a) (1 point) Describe in words the key assumptions of the Moody's KMV model by focusing on "default-only" mode and ignoring the "mark-to-model" approach.

After routine analysis, you approved the application with the terms of:

- Total credit line granted to Sax Glass is \$30 million.
- Sax Glass can draw at most \$20 million as cash and use the remaining \$10 million credit line for contingent liabilities.

One year after the credit was granted, your analyst showed you that the expected loss from Sax Glass is now \$156,000 based on the following assumptions:

- Sax Glass default probability is 1%.
- Sax Glass tends to draw on the free part of the credit line in 80% of the cases and on average uses 60% of the available cash.
- Sax Glass tends to use the credit line supporting contingent liabilities in 40% of the cases and the cash equivalent exposure factor is 70%.
- Sax Glass holds 100,000 shares of Grant Auto and this holding is the only collateral available for Blues Bank in the event of Sax Glass default.
- The market price of Grant Auto is \$20 per share and the share transaction cost is assumed to be \$0.
- All random variables underlying the expected loss analysis are assumed to be independent.

(b) (3 points) Determine the amount of cash, if any, withdrawn by Sax Glass during the first year.

(c) (2 points)

- (i) Explain, in general, why the independence assumption for variables underlying the expected loss analysis is not a good assumption.
- (ii) Provide, in particular, an example from the Sax Glass case to support each of your explanations in (i) above.

7. (6 points) Blackhawk Life Insurance's portfolio manager, Billy Bob, is contemplating utilizing Duration Times Spread (DTS) as a constraint for portfolio management. The following assets are currently held in Blackhawk's portfolio:

Asset	Rating	Absolute Spread Volatility	Spread Over Treasury	Percent of Portfolio	Duration
ABC Corporate Bond	AAA	0.07	40	1.50%	8
XYZ Credit Default Swap	AA	0.15	100	1.00%	5
Country of Zeus Debt	B	0.2	350	0.50%	3

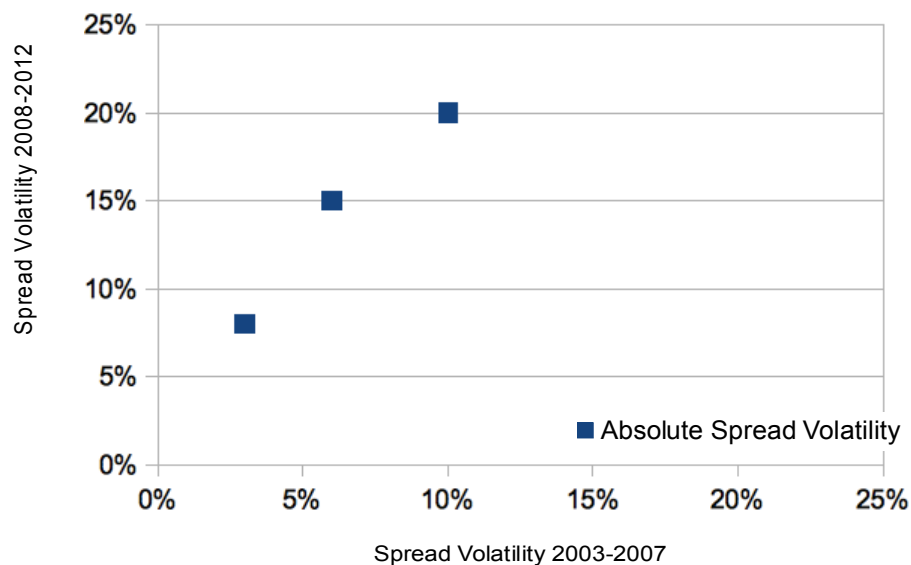
Billy Bob concludes that the DTS maximum contribution limit for each asset should be set at 8. Assume that market value of the assets remains constant.

- (a) (2 points)
- (i) Calculate the Percentage of Portfolio limit that ABC Corporate Bond can obtain and still comply with the DTS contribution limit.
  - (ii) Calculate the current contribution to DTS for XYZ CDS.
  - (iii) Find the maximum spread that Country of Zeus could reach before breaching the DTS contribution limit.

You are given the following:

	Absolute Spread Volatility		Relative Spread Volatility	
	2003 -2007	2008 – 2012	2003 – 2007	2008 – 2012
ABC Corporate Bond	0.03	0.07	0.05	
XYZ Credit Default Swap	0.06	0.15	0.06	
Country of Zeus Debt	0.1	0.2	0.15	

## 7. Continued



(b) (2 points)

- (i) Compare the nature of absolute spread volatility with relative spread volatility and recommend which metric should be used.
- (ii) Recreate the graph above and plot a best estimate of where the three relative spread volatility data points would lie.

Blackhawk's CIO, Jimbo, believes that DTS should be used in conjunction with a ratings-based portfolio constraint system. Jimbo mentions the following reasons:

- I. ABC Corporate Bond has historically had very low spreads.
- II. Blackhawk is considering utilizing more Credit Default Swaps for strategic purposes.
- III. The Country of Zeus Debt is in distress.

(c) (2 points) Evaluate each of Jimbo's statements by either defending why DTS is still appropriate or why it exemplifies a weakness of DTS.

8. (6 points) You are given the following information for two bonds:

Bond	Yield to Maturity	Expected Annual Loss	Sigma
1	2.0%	0.5%	1.0%
2	4.0%	1.0%	3.0%

The correlation between the two bonds is 10%.

- (a) (2 points) Outline the following alternative portfolio approaches as discussed in *Managing Credit Risk* by Caouette and their key assumptions.
- (i) Altman's Optimization
  - (ii) MKMV Corporation's Monte Carlo Simulation
  - (iii) RAROC 2020's Monte Carlo Simulation
  - (iv) CreditRisk+'s Analytical approximation
  - (v) CreditMetric's Monte Carlo Simulation
  - (vi) McKinsey & Co's Monte Carlo Simulation

You have decided to use Altman's Optimization Approach.

- (b) (3 points) Calculate the portfolio ratio  $\eta$  for a portfolio consisting of:
- (i) Bond 1 only
  - (ii) Bond 2 only
  - (iii) The equal weighted portfolio
  - (iv) A portfolio containing 75% Bond 1 and 25% Bond 2
- (c) (1 point) Discuss how a diversified portfolio can improve your risk-return trade-off, as it relates to this example.

9. (8 points) You are an investment actuary at Cubs Life. Cubs Life currently invests 100% of its assets into a Balanced Fund with a blend of 50% in equity and 50% bond investments. The company currently offers an insurance contract that provides an inflation-linked cash value that is entirely liquid in 5 years. The Chief Investment Officer is concerned about the increasing risk of inflation and the volatility of the current investment portfolio. She has asked you for advice on the following alternative investments: farmland, direct, or indirect real estate investments.

(a) (3 points) Compare and contrast the abilities of farmland investments, direct investment in real estate, and indirect investment in real estate in addressing the two key concerns of the CIO.

You are given the following:

- Expected annual inflation over the next 10 years is 5%
- Direct investments in real estate are benchmarked by the NCREIF Index
- Indirect investments in real estate are benchmarked by the NAREIT Index

Portfolio	Sharpe Ratio	Expected Annual Return
80% Balanced Fund, 20% NCREIF Index	0.29	5.3%
80% Balanced Fund, 20% NAREIT Index	0.25	7.0%
80% Balanced Fund, 20% Farm Investment	0.26	5.1%

Based on the information above, the CIO asks for your opinions on a strategy of investing 20% of the portfolio into direct investments in real estate.

- (b) (1 point) Describe how smoothing might undermine the validity of the NCREIF and the associated Sharpe ratio metric.
- (c) (2 points) Describe the key factors that may prevent this portfolio from achieving investment objectives for the insurance product.

The CIO responds by indicating an interest in considering indirect real estate and/or farmland but has limited experience with these investment classes.

(d) (2 points) Recommend and justify one of these two alternatives to the CIO.

**\*\*END OF EXAMINATION\*\***  
**Morning Session**

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