
SOCIETY OF ACTUARIES
Exam FETE
Financial Economic Theory and Engineering Exam (Finance/ERM/Investment)

Exam FETE

MORNING SESSION

Date: Thursday, May 2, 2013

Time: 8:30 a.m. – 11:45 a.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 120 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 60 points).
 - a) The morning session consists of 9 questions numbered 1 through 9.
 - b) The afternoon session consists of 8 questions numbered 10 through 17.

The points for each question are indicated at the beginning of the question. The templates are provided to be used in answering question 9.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet and each template included in your exam booklet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets and the provided templates into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam FETE.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

****BEGINNING OF EXAMINATION****
MORNING SESSION

Question 9 must be answered using the templates provided in your exam booklet.
Please include these sheets in your Answer-Essay Envelope upon completion of the exam.

- 1.** (7 points) You are an actuarial student at Browne Life and you are considering innovative modeling and hedging strategies to deal with mortality risk in Browne Life's insurance products.

You decide to use a one-factor stochastic model to project future mortality for a given age x , rather than traditional static mortality tables.

Let $\mu_x(t)$ be the force of mortality for any age x over the time period t . You assume:

- $\mu_x(t)$ is always non-negative.
 - The volatility of $\mu_x(t)$ increases as the force of mortality increases.
 - You ignore catastrophe mortality jumps that might be caused by epidemics, wars, or natural catastrophes.
- (a) (2 points) Design a continuous stochastic process to model the force of mortality $\mu_x(t)$ and explain all parameters in the process.
- (b) (2 points) Describe how each of the following instruments can be used to hedge catastrophe mortality:
- (i) (1 point) Catastrophe Bonds
 - (ii) (1 point) Catastrophe Options
- (c) (1 point) Identify any credit risk considerations associated with the above instruments.

Your hedge team suggests two other alternatives to hedge catastrophe mortality:

- Policy Conditions
 - Catastrophe Equity Puts
- (d) (2 points) Determine if these alternatives are effective in your situation. Justify your answer.

2. (6 points) You supervise a capital manager at a financial services firm. You have been asked to provide a report to the firm's board regarding debt and equity capital structures, including commentary on the manager's compensation.

(a) (2 points) Identify and explain the four major sources of conflict between stockholders and bondholders.

At the current date, the market believes the following facts about the firm:

- There are two separate views of the firm's current value, $V(0)$, either 50 or 100.
- Between now and exactly 1 year from now, the value of the firm will not change.
- At exactly one year from now, the firm's value, $V(1)$, will become publicly known.
- The maximum amount of debt the firm can carry without going into bankruptcy is 30 if $V(0) = 50$.
- The firm will issue a debt of amount D today.
- The manager's compensation, M , at time 1 is
$$M = 0.1 * V(0) + 0.2 * V(1) \quad \text{if} \quad V(1) > D \quad \text{or}$$
$$M = 0.1 * V(0) + 0.2 * (V(1) - 10) \quad \text{if} \quad V(1) \leq D$$

Based on discussions with the manager:

- (i) The manager believes that the firm has a value of 50 now.
 - (ii) The manager has a choice of issuing a debt of either 20 or 60 today, and the manager is unaware of any debt restrictions.
 - (iii) The manager plans to retire in exactly 1 year (at time 1) and apparently needs a large amount of money to buy a nice house then.
- (b) (2 points) Determine the amount of debt that you expect the manager to issue today. Show all your work.
- (c) (2 points) Discuss three ways for the board to adjust its agreement with the manager so that he has the incentive to issue the more appropriate amount of debt while the manager's compensation remains linked to the value of the firm.

- 3.** (5 points) Your company has been given an opportunity to purchase an American put option for \$10 with a strike price of \$105 at time 1 and \$110 at time 2. The current stock price at time 0 is \$100. The risk free rate is 10% per time period (Use $e^r = 1.1$). The stock pays no dividends.

You have decided to use a risk neutral re-combining binomial tree using the Cox, Ross, and Rubenstein conditions. The parameters of the model are as follows:

Probability of an upshock: 0.60
Return growth in an upshock: 25%

- (a) (1 point) Calculate the growth factor implied by this model.
- (b) (2 points) Calculate value of the put under this model.
- (c) (1 point) Estimate the value of Delta and Gamma of the American put option from this model.
- (d) (1 point) Recommend and justify whether the company should buy the option or not.

4. (7 points) You are using the following lognormal model for an equity index:

$$Y_t = \mu + \sigma \varepsilon_t,$$

where

- μ and σ are constants.
- ε_t follows the standard normal distribution; ε_t and ε_s are uncorrelated for $t \neq s$.
- $Y_t = \ln(S_{t+1} / S_t)$ is the log return random variable.
- S_t is the stock price at time t .

You are given a sample of n historical log returns, y_1, y_2, \dots, y_n , with mean \bar{y} and sample standard deviation s_y .

- (a) (2 points) Show that the log-likelihood function $l(\mu, \sigma)$ is given by:

$$l(\mu, \sigma) = -\frac{n}{2} \ln(2\pi) - n \ln(\sigma) - \frac{1}{2} \sum_{t=1}^n \left(\frac{y_t - \mu}{\sigma} \right)^2.$$

- (b) (2 points) Derive the maximum likelihood estimates of parameters μ and σ in terms of \bar{y} , n , and $y_t, t = 1, 2, \dots, n$, using the result in (a).
- (c) (1 point) Show that the maximum likelihood estimate of μ is unbiased.
- (d) (1 point) Describe, other than the property of asymptotically unbiasedness, two other desirable properties of maximum likelihood estimators.
- (e) (1 point) Describe two limitations of maximum likelihood estimation.

5. (5 points) Your company, ABC Insurance Co., has a block of variable deferred annuity business (VA) with a Guaranteed Minimum Maturity Benefit (GMMB) feature and all of its funds invested in an emerging market indexed fund (EMIF).

The company has decided to implement a hedging program to mitigate the downside risk of this business. Your manager has recommended using the Black-Scholes model to value its GMMB.

You are given the following:

Current Account Value of this VA block	500 million
GMMB of this VA block	700 million
Time to Maturity of this VA block	3 Years
Assumed Risk Free Rate	5% per year
Assumed Volatility (based on S&P500 Historic data)	20%
Assumed VA Fund Management Fee Rate	Zero
Assumed Mortality and Lapse Rates	Zero

- (a) (2 points) Calculate the GMMB value by using the Black-Scholes model and the above assumptions.
- (b) (1 point) Calculate the vega of the GMMB and interpret the result.
- (c) (1 point) Describe potential problems with using the Black-Scholes model.
- (d) (1 point) Critique the volatility assumption that you were given.

6. (6 points) The Management team at Company B is reviewing different ways to distribute wealth to shareholders. The firm currently has 10 million shares outstanding and the following balance sheet:

Assets (millions)		Liabilities (millions)	
Cash	\$10	Debt	\$0
Other Assets	\$5	Equity	\$15
Total Assets	\$15	Total Liabilities	\$15

- (a) (2 points) List and describe the hypotheses associated with tender offers for share repurchase.
- (b) (3 points) Calculate the total wealth of an existing shareholder, who holds a single share, immediately after execution of each of the following:
- (i) Retain all earnings and no dividend
 - (ii) Use \$5 to pay out dividends
 - (iii) Issue \$5 worth of new ex-dividend shares to finance dividend payment to existing shareholders
 - (iv) Use \$5 to repurchase shares from shareholders in the market
- (c) (1 point) Describe how dividend payments may or may not appeal to different types of investors.

7. (5 points) ABank, a large bank, is considering whether to buy ALife, a small insurance company.

(a) (1 point) Identify situations under which a joint venture would be a better approach.

(b) (1 point) Explain three post-acquisition benefits of ABank's acquisition.

Consider the following additional information. ABank is considering buying ALife for 1,600 to be financed by issuing either debt or equity. ABank's board is particularly concerned about the impact on post-merger earnings per share (EPS).

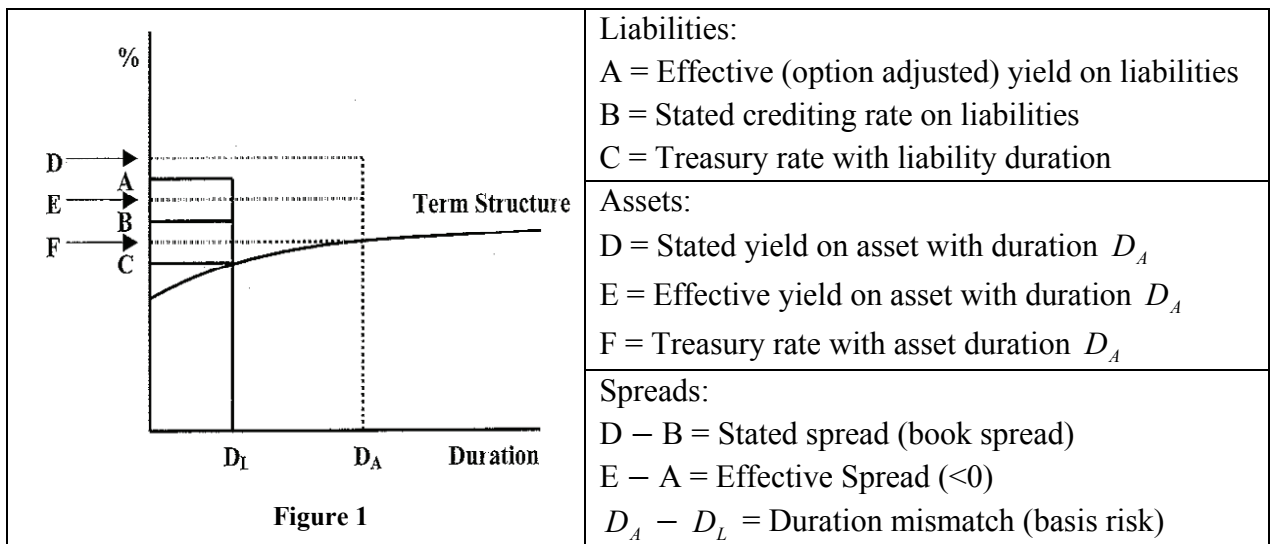
	ABank	ALife
Market Value	4,000	1,500
Net Income	400	100
Shares Outstanding	40	25
After Tax Revenue and Cost Synergies	20	
Fixed Deal Cost	10	
Cost of Issuing Equity	5	
Annual Interest	10%	
Goodwill Amortization Period (yrs)	10	
Stock Price	400	180

(c) (3 points) Determine the EPS under each funding method.

8. (6 points) ABC Life issues Single Premium Deferred Annuities (SPDAs). ABC Life currently invests in low-grade bonds. It is currently engaged in a duration mismatch strategy to front-load earnings, where higher yields can be booked on assets than what would be the case if the assets were better matched to the liabilities.

Many insurers take numerous manipulative approaches to achieve value maximization.

- (a) (1 point) List approaches which trade away Franchise Value and Minimize Net Tangible Value.



- (b) (1 point) Identify and explain the options embedded in the liability that account for the difference between B and C.
- (c) (1 point) Identify and explain the risks that account for the difference in yield between D and F.
- (d) (1 point) Explain why when ABC Life writes liabilities and invests in assets to generate a stated spread of $D - B > 0$ the effective spread $E - A < 0$.
- (e) (2 points) Describe the problems associated with funding life insurance and annuities with mortgage-backed securities in varying interest rate environments.

Question 9(d) must be answered using the templates provided in your exam booklet. Please include these sheets in your Answer-Essay Envelope upon completion of the exam.

- 9.** (13 points) On January 1, your company, the Bank of Mom and Dad (BMD), offers a promotional loan to prospective college students. You are given the following:
- Loan amount is \$50,000.
 - The term of the loan is two-years.
 - The annual interest rate is 5%.
 - The rate is guaranteed for a 90-day offer period, at the end of which the student can either accept or reject the loan offer.
 - Interest on the loan is payable annually.
 - The loan balance including any accrued interest can be repaid at any time without penalty.

- (a) (1 point) Identify any embedded options in BMD's offer and conditions that might cause the options to be exercised.

BMD will fund the loan by borrowing at the 12-month LIBOR rate for two years. BMD would like to use interest rate derivatives to limit its interest rate risk. Two derivatives being considered are a cap and a swaption.

LIBOR/Swap zero curve: 4.0% flat
Volatility of all forward rates: 30%

- (b) (2 points) Calculate the price on January 1 of a derivative that caps the 12-month LIBOR rate, starting in 3 months, at 4.1% for 2 years, using the standard market model.
- (c) (2 points) Calculate the price on January 1 of a 3-month swaption giving BMD the right to enter a 2-year interest rate swap, paying 4.1% fixed interest in exchange for 12-month risk-free rate, using the standard market model.

You suspect that neither the cap, nor the swaption will be perfect hedges for BMD's loan promotion. You decide to test this hypothesis by evaluating the expected cashflows under a variety of scenarios, assuming that borrowers will behave rationally.

Assume that if the swaption is out of the money at the exercise date, BMD will not exercise the option and instead will enter into a swap at the prevailing interest rate at that time. Assume BMD can exercise their options instantaneously upon notification of loan acceptance.

Question 9 is continued on the next page

9. Continued

Your manager has provided you with the following template to assess the decision points of a rational player (student/bank) and expected cashflows for each instrument.

Time	0	0.25	1.25	2.25
Student Decision: Accept Loan Offer (Yes/No)				
A) Student Loan Cashflow				
B) Loan Funding Cashflow				
Net Loan Cashflow (A+B)				
BMD Decision: Exercise Swaption (Yes/No)				
Swaption/Swap Cashflow	Buy Swaption @ \$X			
Cap Cashflow	Buy Cap @ \$Y			
Student Decision: Prepay the Loan (Yes/No)				

She has completed the template for Scenario 1: Interest rates decrease 2.0% immediately after the loan offer.

Time	0	0.25	1.25	2.25
Student Decision: Accept Loan Offer (Yes/No)		No		
A) Student Loan Cashflow		n/a	n/a	n/a
B) Loan Funding Cashflow		n/a	n/a	n/a
Net Loan Cashflow (A+B)		n/a	n/a	n/a
BMD Decision: Exercise Swaption (Yes/No)		No		
Swaption/Swap Cashflow	Buy Swaption @ \$X		0	0
Cap Cashflow	Buy Cap @ \$Y		0	0
Student Decision: Prepay the Loan (Yes/No)		n/a	n/a	n/a

Justification of Decision Points:

Student Loan offer at not accepted because rates have decreased.

Swaption not exercised because there is no loan cashflow to swap to fixed rate of 4.1%.

9. Continued

- (d) (6 points) Complete the template, from the perspective of BMD for the following scenarios:
- (i) Scenario 2: Interest rates increase 2.0% immediately after the loan offer.
 - (ii) Scenario 3: Interest rates are unchanged until loan is accepted and then increase 2.0% immediately after loan acceptance.
 - (iii) Scenario 4: Interest rates are unchanged until loan is accepted and then decrease 2.0% immediately after loan acceptance.
- (e) (2 points) Describe the advantages and disadvantages to BMD of each interest rate derivative relative to the liability based on your observations above.

****END OF EXAMINATION****
Morning Session

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