

ERM-INV Model Solutions

Spring 2013

1. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

Learning Outcomes:

- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.
- (2f) Analyze the importance of tails of distributions, tail correlations, and low frequency/high severity events.

Sources:

ERM-101-12: Measurement and Modeling of Dependencies in Economic Capital
Pages 22 (Section 5.6 Copulas) to 81

Financial Enterprise Risk Management, Sweeting, 2011 Chapter 10.4, Copulas
(Pages 195 to 219)

Commentary on Question:

The focus of this question was the concept of copulas as well as calculation of lower tail dependence and upper tail dependence using a copula formula. Successful candidates were expected to demonstrate their knowledge of the concept and formulas. As the tail dependence formulas use the concept of limits, some candidates lost partial points due to inability to perform the required calculus.

Solution:

- (a) Demonstrate that each marginal risk factor distribution is uniform on $[0,1]$ using the Clayton copula.

Commentary on Question:

This question required the candidates to recall that uniform means $C(x, 1) = x$ and $C(1, y) = y$, from the reading source.

Response:

$$C(x, 1) = (x^{-\theta} + 1^{-\theta} - 1)^{-1/\theta} = (x^{-\theta} + 1 - 1)^{-1/\theta} = (x^{-\theta})^{-1/\theta} = x$$
$$C(1, y) = (1^{-\theta} + y^{-\theta} - 1)^{-1/\theta} = (1 + y^{-\theta} - 1)^{-1/\theta} = (y^{-\theta})^{-1/\theta} = y$$

1. Continued

Therefore, each marginal risk factor distribution is uniform on $[0,1]$ using the Clayton copula.

- (b) The chief actuary at Joint Life is unfamiliar with copulas and prefers to use the variance-covariance approach when reflecting dependency between random variables. He recommends increasing the correlation factor in order to reflect the upper tail dependence that exists between X_1 and X_2 .

Provide advantages and disadvantages of the chief actuary's approach.

Commentary on Question:

Many candidates lost points in part (b) as they only answered from the general list of advantages and disadvantages of copulas relative to the variance-covariance approach, and didn't evaluate the chief actuary's approach of increasing the correlation factor in the context given.

Response:

Advantages of the chief actuary's approach:

- Current level of comfort with this approach
- Copulas are difficult to parameterize and more difficult to explain. Variance-covariance is simple and easy to communicate.
- Increasing the correlation factor will result in increased likelihood being assigned to extreme right tailed events. While this approach will not achieve the correct correlation across the entire distribution, Joint Life Co. is likely to be only concerned with the right tail.

Disadvantages of the chief actuary's approach:

- While increasing the correlation factor will result in increased likelihood being assigned to extreme right tailed events, it will also increase the likelihood being assigned to the rest of the distribution.
- The variance-covariance approach will not capture other than linear dependence structures, whereas the copula approach will enable Joint Life Co. to capture a more diverse set of correlations between its risk factors.
- Copula use is consistent with a typical actuarial and financial risk modeling process whereby marginal risk distributions for each risk are first determined and then one considers separately the aggregation process.

- (c) Demonstrate that use of the Clayton copula results in an increasing level of lower tail dependence as the parameter θ increases.

1. Continued

Commentary on Question:

Generally candidates did well in part (c). Some candidates lost partial points due to failure in calculating the limit. Candidates received partial points if they showed sample numerical results rather than providing a full demonstration.

Response:

Lower Tail Dependence:

$$\lim_{x \rightarrow 0} C(x, x) / x = \lim_{x \rightarrow 0} (x^{-\theta} + x^{-\theta} - 1)^{-1/\theta} / x = \lim_{x \rightarrow 0} (2x^{-\theta} - 1)^{-1/\theta} / x = \lim_{x \rightarrow 0} (2 - x^{\theta})^{-1/\theta} \\ = (2)^{-1/\theta}$$

As θ increases, $1/\theta$ decreases, $-1/\theta$ increases and $(2)^{-1/\theta}$ increases. Hence, lower tail dependence increases for increasing values of θ .

- (d) One of your colleagues has developed a new copula, the FGM copula. You are given:

$$\text{FGM Copula: } C(x, y) = xy[1 + \theta(1-x)(1-y)]$$

Determine whether the use of the FGM copula is appropriate when modeling the joint distribution for X_1 and X_2 .

Commentary on Question:

Many candidates had difficulties with the formula of upper tail dependence and calculation of the limit. However, if candidates understood the concept, but made errors in the calculation, they still received partial points.

Some candidates only tested for upper tail dependence as that is enough to show the FGM copula is not appropriate for the intended purpose.

Response:

Upper Tail Dependence:

$$\lim_{x \rightarrow 1} [1 - 2x + C(x, x)] / (1 - x) = \lim_{x \rightarrow 1} [1 - 2x + x^2 + x^2 \theta(1 - x)^2] / (1 - x) \\ = \lim_{x \rightarrow 1} [1 - 2x + x^2] / (1 - x) = \lim_{x \rightarrow 1} (1 - x)^2 / (1 - x) = \lim_{x \rightarrow 1} (1 - x) = 0$$

There is no upper tail dependence.

1. Continued

Lower Tail Dependence:

$$\lim_{x \rightarrow 0} C(x, x) / x = \lim_{x \rightarrow 0} x^2 [1 + \theta(1 - x)^2] (1 / x) = \lim_{x \rightarrow 0} x [1 + \theta(1 - x)^2] = 0$$

There is no lower tail dependence.

The use of the FGM copula is not appropriate as it has neither upper nor lower tail dependence.

2. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.
3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

Learning Outcomes:

- (1c) Identify and analyze risks faced by an entity, including but not limited to market risk, currency risk, credit risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk.
- (2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.
- (3e) Define and evaluate credit risk. Explain how to incorporate best practices in credit risk measurement, modeling and management.

Sources:

Financial Enterprise Risk Management, Sweeting, 2011

- Chapter 7, Definitions of Risk
- Chapter 14, Quantifying Particular Risks

ERM-106-12: Economic Capital-Practical Considerations – Milliman

Value-at-Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion, 2007

- Chapter 18, Credit Risk Management (excluding Appendices)

Commentary on Question:

The majority of candidates performed well on the first half of the question. The subsections were fairly straightforward and it allowed the candidates to demonstrate their understanding of credit risk, how to use the Merton model to calculate a default probability and how different parameters impact the default probability calculated by the Merton model.

The second half of the question required a much deeper level of understanding about the Merton model, including a theoretical question in part (d) and specific application questions with respect to sovereign debt in parts (e) and (f). It was clear from the responses that some candidates did not understand what sovereign debt was.

2. Continued

Sovereign debt refers to foreign government debt, and perhaps there was confusion over the use of the term “sovereign” in the exam. Our expectation had been that candidates would have had a basic understanding of sovereign debt at this stage of the exams. However, as this understanding was not evident and sovereign debt is only briefly mentioned in the ERM exam study material, the grading was adjusted to compensate for the lack of information on the subject matter.

When it came to applying the Merton model and Discriminant Analysis to sovereign credit risk modeling, there were only a few candidates who provided enough explanation and detail to show comprehension of how these methods and models work qualitatively.

Solution:

(a)

- (i) Define credit risk.
- (ii) Describe the four components that should be considered when modeling credit risk.

Commentary on Question:

Most candidates performed well on this question. Some candidates did not provide appropriate components in sub-part (ii), or they did not describe the components that they listed, but a majority of candidates received most of the points on this section of the question.

Response:

- (i) Risk of financial loss owing to a counterparty failure to perform its contractual obligations / make payment when payment is due.
- (ii)
 - 1. Probability of Default / Default risk
 - Likelihood of default by the counterparty
 - 2. Credit Exposure
 - Risk of fluctuations in the market value of the claim on the counterparty
 - At default, known as the exposure at default (EAD)
 - 3. Recovery Risk
 - Uncertainty about the fraction of the claim to be recovered after default
 - Known as $(1 - LGD)$, $LG D =$ loss given default
 - 4. Migration Risk
 - Adverse variations in transitions between credit ratings

2. Continued

- (b) Calculate Crow's probability of default using the Merton model.

Commentary on Question:

Most candidates performed extremely well on this question and were able to apply the Merton model correctly in the given situation.

Some candidates flipped the Asset Value and Total Debt. Given that the Merton Model defines a company as being in default when the debt is greater than a company's assets, the candidate should have recognized the variable assignment to be incorrect when equating the variables that way. There were also some candidates who confused the asset value volatility and variance.

Response:

Crow's Asset Value = $X_0 = \$800,000,000$

Crow's Asset Value Growth Per Year = $r_A = 8.4\%$

Crow's Asset Value Volatility Per Year = $\sigma_A = 20\%$

Crow's Total Debt = $B = \$500,000,000$

Crow's Debt Term = $T = 1$ year

$$\begin{aligned} P(X_1 \leq 500) &= \Phi((\ln(500/800) - (0.084 - (0.2^2)/2) * 1) / (0.2 \sqrt{1})) \\ &= \Phi((-0.47 - 0.064) / (0.2)) \\ &= 1 - \Phi(2.67) \\ &= 0.0038 \end{aligned}$$

- (c) Explain how each of the following would impact CQC's assessment of the probability of default for the Crow bond.
- (i) Crow's asset book value decreases.
 - (ii) The risk free rate increases.
 - (iii) Crow's asset value volatility decreases.

Commentary on Question:

Most candidates performed well on this part of the question. Partial credit was given to candidates who relied on more of a technical / numerical demonstration and full credit was assigned to candidates who could provide a more qualitative / theoretical explanation.

2. Continued

As the risk free rate was not given in the question nor linked to Crow's asset growth rate, sub-part (ii)'s answer could be based on an analysis of the Merton model or more general reasoning. Credit was given for any reasonable discussion of the impact of an increase in the risk free rate.

Response:

(i)

- This increases Crow's individual probability of default.
- Crow is closer to being insolvent because the Merton model defines insolvency when assets are insufficient to cover liabilities (assets fall below its debt).

(ii)

- This does not directly impact Crow's default probability.
- The Merton model uses the asset growth rate to calculate an asset's probability of default.
- Therefore, barring a change in the risk free rate and its impact on Crow's asset return, its default probability doesn't change.

(iii)

- This decreases Crow's likelihood of default
- The volatility of results and asset growth is now more stable than before.
- This makes it less likely there will be huge downswings in the firm's value resulting in insolvency as there is a lower probability of the assets going lower than liabilities.

- (d) Explain why the formulation of the Merton model above uses r_A whereas the standard option pricing formula using Black-Scholes uses the risk free rate.

Commentary on Question:

Some candidates struggled to provide an answer to this question and did little more than just restate the question. Others were more successful with relating the rates back to the purposes of the models. This was a higher-cognitive level question as there was not a direct response in the syllabus.

Response:

- Crow is interested in the probability of default in the real world, which the Merton model defines as the probability the value of the firm's assets X , at a fixed time T in the future, X_T , will be below the level of debt, B , at the same time T .

2. Continued

- The default probability and assets used are firm specific, thus the actual asset growth rate is needed for this determination.
 - Black-Scholes is used for risk-neutral option pricing where investors expect to earn the risk-free-rate.
- (e) Describe the Merton model and evaluate its use with respect to modeling sovereign credit risk.

Commentary on Question:

Some candidates struggled on this question due to the sovereign credit risk modeling component. Although candidates could describe the Merton model, most could not relate it well to sovereigns. There was not a lot on sovereign debt and credit risk in the syllabus, but it was expected that candidates would have general background knowledge on how sovereign debt would differ from a corporate bond. With an understanding of the Merton model, its strengths and weaknesses and a basic understanding of sovereign holdings, candidates should have been able to analyze how the Merton model would or would not work for sovereign debt holdings. The grading was adjusted to compensate for the evident lack of knowledge about sovereign debt.

Response:

Merton Model

- The Merton model is a structural model and the insolvency occurs when the value of the firm falls below the level of debt outstanding
- The value of the firm is assumed to follow a lognormal random walk process
- It is most appropriate for large borrowers with liquid, frequently traded equity stock since an accurate estimate for the volatility of corporate equity is needed

Model Adequacy for Sovereign Debt

- The Merton model is not appropriate for calculating the probability of default on the sovereign and government risks in CQC's portfolio based on a number of unresolved issues
 - 1) How to appropriately model the asset value (whatever it is) using a lognormal process
 - 2) How to incorporate foreign exchange rates into the calculation
- Sovereigns are not equity-based borrowers, making it difficult to assign a tangible asset value and volatility parameter utilizing the Merton model's principles

2. Continued

- It is difficult to determine what the insolvency level would be for a sovereign
 - 1) Nations may have more debt outstanding relative to their GDP levels and are not considered insolvent at that level
 - 2) Some nations may have mechanisms to avoid default that a corporation does not, such as raising taxes and printing money
- (f) Explain the discriminant analysis approach to modeling credit risk and describe how you might implement such an approach with respect to modeling sovereign credit risk.

Commentary on Question:

This question was not answered in a lot of detail by candidates and those candidates who performed better in (e) also tended to do better in this section. Most candidates were able to describe discriminant analysis, but did not explain in much detail how it could be used for sovereign debt credit risk modeling.

Response:

Discriminant Analysis

Discriminant analysis, as it pertains to credit modeling, is a quantitative approach that attempts to arrive at a credit score that represents the likelihood of insolvency for an entity.

- The most familiar credit modeling approach using this technique is Altman's Z-score.
- The Z-score is determined by using financial ratios
- The Z-score is calculated by using a linear equation, which is parameterized by these financial ratios

In order to implement this approach for sovereign/government bond credit risk modeling:

- The Z-score can be modified to use other types of ratios based on economic, financial and socio-political data that are specific to sovereign nations – for example, the ratio of the present value of debt payments and/or debt interest payments to GDP levels, exports, unemployment levels, type of government (democratic, dictatorship, communist, etc.), and so on
- Note that creating these modified ratios may be difficult due to a lack of information
- The ratios may have highly skewed distributions (developing nations vs. established), which violate the underlying assumption that the variables used are independent and normally distributed

3. Learning Objectives:

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

- (4a) Evaluate the rationale for managing risk and demonstrate the selection of the appropriate risk retention level and hedging of risk.
- (4c) Demonstrate means for transferring risk to a third party, and estimate the costs and benefits of doing so.
- (4e) Demonstrate how derivatives, synthetic securities, and financial contracting may be used to reduce risk or to assign it to the party most able to bear it.
- (4k) Analyze methods of managing other risks (operational, strategic, legal and insurance) both pre-event and post-event.

Sources:

ERM 108-12: Tiller, Life, Health and Annuity Reinsurance, 3rd Edition, 2005 Ch. 5
Advanced Methods of Reinsurance

ERM 108-12: Tiller, Life, Health and Annuity Reinsurance, 3rd Edition, 2005 Ch. 17
Special Purpose Reinsurance Companies

Financial Enterprise Risk Management, Sweeting, 2011 Ch. 16 Responses to Risk

Commentary on Question:

This question is intended to test the candidate's ability to perform a cost-benefit analysis for a specific situation and present results to senior management in a well-organized and articulated manner. The analysis should be concise and clearly attempt to identify the key terms in a manner which is comparable across the options being considered. The request for additional information from management should demonstrate the candidate's insight from a risk management perspective.

Solution:

- (a) Management wants to evaluate the relative merits of the options listed above using a cost-benefit (including risks) analysis. Prepare a comparison of the three options considering at least five criteria that you consider key to making a decision.

Commentary on Question:

Stronger candidates provided a complete analysis (i.e. identifying all of the key criteria which should be considered in evaluating this decision) as well as a clear and concise presentation of the alternatives being considered. The mere listing of criteria without a comparison of the alternatives being considered did not satisfy the requirements for this part.

3. Continued

Response:

Option No.	I - Stop Loss	II - Cat Bond	III - Swap
Annual Cost	2.35M	2M	6.25M
Max Coverage	50M per annum	50M total	75M total
Deductible (above expected claims)	50M per annum	50M per annum	110M per annum
Term	Annual Renewable	5 Years	3 Years
Counterparty Risk?	Yes	No	Yes
Basis Risk?	No	No	Yes

For option II, the max coverage is \$50M for the entire 5 year period (as opposed to an annual limit).

For option III, the max coverage is \$75M for the entire 3 year period (as opposed to an annual limit).

- (b) Identify which of the three options is least favorable to Hamsik. Justify your response.

Response:

Option III - SWAP is the least favorable

- ⇒ Most expensive - \$6.25 M annual cost (and most expensive per unit of maximum benefit provided)
- ⇒ Largest Deductible -- covers mortality risk above 8.4% only
- ⇒ Lowest absolute amount of coverage
- ⇒ Introduces both counterparty and basis risks

- (c) You need more information in order to make a recommendation as to which option is most favorable to Hamsik.

Identify five key questions you would ask Hamsik senior management to enable a more informed recommendation.

3. Continued

Commentary on Question:

*This part asked for the **key** questions you would ask of senior management to get the information needed to make a decision. Therefore, the questions needed to reflect the candidate's understanding of how this decision fit into the broader objectives of Hamsik, specifically, ERM related objectives. Weaker candidates responded with very detail-oriented questions (such as counter-party credit standing, details on bond provisions or underwriting guidelines). While relevant, these were awarded fewer marks as they were not "key". For example, a number of candidates provided questions on better understanding expected mortality levels. This information was already provided in the question (i.e., both Hamsik and industry level expected mortality).*

A variety of possible questions are provided below. Only five appropriate questions were needed to receive full credit.

Response:

- What is Hamsik's objective in entering into this agreement?
 - ⇒ Does the company intend to manage or transfer catastrophe mortality risk?
 - ⇒ Does the company want to hedge this risk or immunize itself?
- What is Hamsik's mortality experience for this block of business; i.e., has it been volatile?
- What is Hamsik's annual budget for this program?
- What is Hamsik's risk appetite in general and for catastrophic mortality events specifically?
- How much capital does Hamsik have available to cover catastrophic mortality losses?
- How much does management appreciate the benefit of locked in coverage? Is management comfortable with the risk of renewing annual coverage at a potentially higher cost or are they willing to pay more for coverage extending over a number of years?

4. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.
5. The candidate will understand the concept of economic capital, risk measures in economic capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:

- (1c) Identify and analyze risks faced by an entity, including but not limited to market risk, currency risk, credit risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk.
- (4c) Demonstrate means for transferring risk to a third party, and estimate the costs and benefits of doing so.
- (4h) Define credit risk as related to derivatives, define credit risk as related to reinsurance ceded, define counterparty risk and demonstrate the use of comprehensive due diligence and aggregate counterparty exposure limits.
- (5a) Describe the concept of economic measures of value and demonstrate their uses in the risk management and corporate decision-making processes.
- (5b) Define the basic elements and explain the uses of economic capital.
- (5d) Apply risk measures and demonstrate how to use them in economic capital assessment. Contrast and understand regulatory, accounting, statutory and economic capital.

Sources:

ERM-106-12: Economic Capital-Practical Considerations – Milliman

ERM-108-12: Tiller, Life, health and Annuity Reinsurance, 3rd Edition, 2005, Chapter 5, Advanced Methods of Reinsurance

Commentary on Question:

The purpose of this question was to test the candidates' knowledge of economic capital concepts, and how economic capital (EC) is impacted by different scenarios and business circumstances (Examples: Reinsurance, market shocks, new business etc.). Many candidates did well demonstrating their knowledge of how Required EC and Excess EC are impacted under the different prescribed situations.

4. Continued

Some candidates confused statutory capital with economic capital, and correspondingly confused statutory implications with economic implications, particularly in sections (b) and (d). Many candidates did not do as well on the Available EC parts of the question, as this required the candidates to rationalize through the change to both MVA and MVL. Often, only one part of the rationalization was done correctly. It is also noted that many candidates did not understand what market capitalization is.

Solution:

- (a) Describe the following terms. Include in your description the equations that define these items and how they relate to one another.
- (i) Available Economic Capital (AEC)
 - (ii) Required Economic Capital (REC)
 - (iii) Excess Economic Capital
 - (iv) Market Capitalization

Commentary on Question:

Part (a) was trying to test the candidates understanding of the definitions of economic capital. Candidates did fairly well with AEC, REC, and Excess EC. The majority of candidates did not provide a proper definition for market capitalization. Overall the scores on part (a) were good.

Response:

AEC: The excess of assets over liabilities on a realistic/market consistent basis.
(A-L)

REC: The amount of capital needed to support a business within a certain probability of default.

Excess EC: Represents the amount of capital held over the required amount.
 $\text{Excess EC} = \text{AEC} - \text{REC}$

Market Capitalization: The total value of the issued shares of a publicly traded company. It is $\text{AEC} + \text{Economic Franchise Value}$.

- (b) Your analysis will independently consider the following three potential events:
- (i) A significant increase in interest rates
 - (ii) A significant drop in the stock market
 - (iii) A reduction from VaR 95% to VaR 90%

Explain the probable implications in terms of direction and significance to all four items in part (a) for each of the events. Justify your response and identify any additional assumptions you made in your analysis.

4. Continued

Commentary on Question:

For subpart (i) many candidates stated that “Due to duration matching, assets and liabilities decrease by the same amount.” This was inaccurate because, while the statutory values of assets and liabilities were duration matched, it does not necessarily follow that the economic values of these items were also matched. There were a surprising number of candidates who did not understand the impact of interest rates on existing fixed income assets, and thus incorrectly thought an increase in interest rates increased the market value of fixed-income assets.

For subpart (ii), some candidates incorrectly believed the drop in the equity market had an impact on the market value of the liabilities because of the guarantee rate, despite the fact that these liabilities are tied to fixed income investments. Few candidates understood that liability values were stable (with the exception of possible changes in surrenders), and thus REC would remain stable.

For subpart (iii), candidates who knew the definitions of the items in part (a) did well on this subpart.

Overall the first two subparts of part (b) were poorly done, and the third subpart was well done.

Response:

- (i) Increase in interest rates: Since assets and liabilities are duration matched on a statutory basis, a change in interest rate will be of a different magnitude on an economic basis.

Fixed income assets decrease in value with an increase in interest rates, so MVA decreases. Assuming that the liability discount rate increases with an increase in rates, the MVL decreases. Assuming that the MVA decreases more than the MVL, AEC decreases.

Since the MVL decreases, the REC also decreases.

Assuming REC decreases more than AEC decreases, then the excess EC increases.

It is unclear how market capitalization is affected by a change in interest rates, if at all. Thus, credit was given for any rational response arguing either a correlation or independence between the equity and interest rate markets.

4. Continued

- (ii) Drop in stock market:

Value of equity assets declines, but this is a small proportion of total assets since only statutory surplus is backed by equity. MVA decreases.

MVL is unaffected by change in equity. So AEC decreases and REC remains relatively constant. Excess EC decreases.

Assuming that the company's stock has a positive beta, the market capitalization decreases.

- (iii) AEC is unchanged by the VaR metric.
REC decreases because $VaR(90) < VaR(95)$.
Excess EC increases as a result of the above.
Market capitalization is unaffected because the REC metric is assumed to be an internal metric that would not change investor sentiments.

- (c) Explain how the addition of one year of new business to the existing EC model would be expected to impact AEC and REC.

Commentary on Question:

Overall this part of the question was poorly done. Many candidates understood that adding new business would increase REC, but there was a lack of understanding as to how AEC would change.

Response:

MVA is unaffected, as the new business is added after the valuation date. Assuming that the new business is priced profitably, then MVL should decrease, so AEC increases.

New business is likely to increase the risks as liabilities have increased. So REC is likely to increase. The increase in REC may not be dollar for dollar with a calculation performed assuming only the new business cohort, because there may be diversification benefits from new business being combined with existing business.

- (d)
- (i) Compare and contrast coinsurance and YRT reinsurance, including the impact each is expected to have on both statutory reserves and capital.

Commentary on Question:

Many candidates did well on this subpart. Some candidates did not have a firm understanding of YRT reinsurance.

4. Continued

Response:

Coinsurance:

In Coinsurance, the insurer pays an initial premium to the reinsurer. The insurer pays 30% of all the block's net cash flows to the reinsurer (possibly adjusted by an allowance) and effectively eliminates 30% of the liability from its book. Assets are also transferred.

In YRT, 30% of Net Amount of Risk is ceded at prescribed premium rates. Unlike coinsurance, assets are not transferred with YRT, so the investment risk is not covered.

They are similar in the sense that the insurer is covered for its mortality risks.

With respect to statutory reserves, YRT does not provide large reserve credits whereas with Coinsurance statutory reserves are reduced 30%.

With respect to capital relief, YRT provides some statutory capital credit, whereas coinsurance would likely have more of an impact as required capital and reserves are further reduced.

- (ii) Explain the probable implications for each of the following, in terms of direction and relative magnitude, of entering into the YRT arrangement.
- I. Available Economic Capital (AEC)
 - II. Required Economic Capital (REC)
 - III. Excess Economic Capital

Commentary on Question:

Many candidates did not correctly explain the implications to AEC. Very few candidates recognized that YRT is likely priced at a profit to the reinsurer, and thus AEC should decline.

Response:

The MVA is unchanged, unless the first year reinsurance premium is reflected. Reinsurer likely priced the YRT at a profit, so MVL should increase. Thus AEC decreases slightly.

REC decreases because risk is being transferred to the reinsurer.

Presumably, the YRT should have more of an impact in tail risk scenarios, so the REC should decline more than the AEC, and the Excess EC increases.

4. Continued

- (iii) Explain the probable implications for each of the following, in terms of direction and relative magnitude, of entering into the coinsurance arrangement.

- I. Available Economic Capital (AEC)
- II. Required Economic Capital (REC)
- III. Excess Economic Capital

Commentary on Question:

Many candidates did not understand what would happen to AEC. Many assumed it would drop without giving justification.

Response:

MVA decreases because assets are transferred to the reinsurer, but offset by an initial ceding allowance. MVL decreases as liabilities are ceded to the reinsurer, offset by future allowances. This decrease is somewhat offset by the fact that the coinsurance deal is priced at a profit by the reinsurer. Impact on AEC depends on the relative decreases of the MVA and the MVL, so AEC could increase or decrease, but likely not by large amounts.

REC decreases because risk is transferred to the reinsurer.

Since REC decreases and change in AEC is likely not significant, Excess EC probably increases.

- (iv) Describe how you would assess which reinsurance arrangement results in a better EC position for the company.

Commentary on Question:

Many candidates proposed assessing the reinsurance arrangement purely based on the impact on Economic Capital. Very few considered the return aspect of the reinsurance arrangements. Risk and return need to be assessed together, not separately.

Response:

The firm could choose a return on capital metric such as RAROC – the arrangement with the higher return is better. The AEC and REC metrics alone do not do an adequate job of determining the reinsurance arrangement because the return aspect is not considered.

5. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

Learning Outcomes:

- (2a) Demonstrate how each of the financial risks faced by an entity can be amendable to quantitative analysis including an explanation of the advantages and disadvantages of various techniques such as Value at Risk (VaR), stochastic analysis, scenario analysis.
- (2d) Apply and analyze scenario and stress testing in the risk measurement process.
- (2h) Construct approaches to managing various risks and evaluate how an entity makes decisions about techniques to model, measure and aggregate risks including but not limited to stochastic processes.

Sources:

ERM-602-12: *Investment Management for Insurers*, Babbel and Fabozzi, Chapter 11, The Four Faces of an Interest Model

ERM-106-12: Economic Capital-Practical Considerations – Milliman

Commentary on Question:

This question tested candidates' ability to understand and apply various interest rate models. Most candidates did well on comparing/contrasting equilibrium and arbitrage-free models (part (a)) and in identifying the appropriate models for various situations (parts (c) through (e)). Few candidates did well on comparing/contrasting risk neutral and realistic probabilities (part (b)).

Solution:

- (a) Compare and contrast arbitrage-free versus equilibrium interest rate models.

Commentary on Question:

Most candidates did well on this part.

Response:

Arbitrage-free model:

- Takes the current market prices as given and backs into the parameters such that the model fits the current prices
- Does not model the dynamics of the term structure
- Includes time-dependent parameters with at least one parameter for each market price used
- Is basically an interpolation system

5. Continued

Equilibrium models:

- Unlike arbitrage-free, these models do model the dynamics of the term structure over time
- They do not match the current market prices
- They utilize a statistical approach
- Do not utilize time-dependent parameters

- (b) Compare and contrast the use of risk neutral probabilities versus realistic probabilities in the parameterization of interest rate models.

Commentary on Question:

Few candidates did well on this part. Many didn't seem to realize that the risk neutral process requires adjustments to the probabilities and was equivalent to the realistic scenario except it used a risk-free rate. We note that there were two sources in the syllabus that described risk neutral valuation and they took different approaches to adjusting rates or cash flows in risk-neutral valuation (ERM 602-12 and ERM 106-12). Candidates received full credit for describing either approach if it was explained accurately.

Response:

Risk-neutral:

- Stipulates that no matter how risk averse an investor is, we can identify a set of spot rates that value the bonds correctly relative to the market.
- This is done by changing the probability distribution of the short rate such that the spot rate of every term is equal to the expected return from investing at the short rate over the same term.
- Risk-neutral is used for current pricing. When market prices are reliable they are used in an arbitrage-free setting. When prices are not reliable they would be used with an equilibrium model.

Realistic

- Recognizes that there is normally an upward sloping yield curve, reflecting term premium
- Used in stress testing and asset/liability strategies under adverse movements in interest rates.

- (c) Identify the model classification letter of the derivative team's model and explain how the parameters would be set.

Commentary on Question:

Most candidates were able to correctly identify the correct model. Many understood that the parameters were set by matching them to the bond and swap prices.

5. Continued

Response:

The correct model is A, Risk-Neutral (no term risk premium) and Arbitrage-free (parameters are a function of time).

u_0 and $\theta(t)$ are matched to the bond prices.

$\kappa(t)$ and $\sigma(t)$ are matched to the swap prices.

- (d) Identify the model classification letter of the model that would be most appropriate for your analysis of interest-sensitive products. Justify your choice.

Commentary on Question:

Most candidates were able to correctly identify the model, but many did not justify their answer sufficiently.

Response:

Model D, equilibrium and with realistic probabilities, is required.

It is important to model the behavior of the term structure over time (equilibrium).

We are just interpolating to get the derivative prices (equilibrium).

It is important to capture the risk premium (realistic) in order to complete a real-world analysis of asset adequacy and VaR calcs.

- (e) Propose a modified form of the Black-Karasinski model to use in your work and explain how the parameters will be set.

Commentary on Question:

Answers were mixed on this section. Some candidates correctly removed the time dependent parameters or included the term premium, but few made both of the changes required.

Response:

$$du = \kappa(\theta - \lambda(u) - u) dt + \sigma dz$$

u_0 is statistically fit to bond prices

$\kappa, \theta, \sigma, \lambda(u)$ are historically estimated

6. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

Learning Outcomes:

- (2g) Analyze and evaluate model and parameter risk.

Sources:

Financial Enterprise Risk Management, Sweeting, 2011

- Chapter 14, Quantifying Particular Risks, pgs. 311 – 313

ERM-104-12: Study Note on Parameter Risk, Venter and Sahasrabudde

Commentary on Question:

This question was focused on the application of parameter risk. Candidates did well on the recall part of this question but struggled to apply the concepts to the Palisades case.

Solution:

- (a) Define parameter risk, sampling risk and data bias, and explain how these three items relate to one another.

Commentary on Question:

Most candidates did well on this part of the question, which was primarily recall.

Response:

Parameter risk – uncertainty as to whether the parameters are appropriate for the phenomenon being modeled

Sampling risk – uncertainty that the sample of data upon which our parameter estimate is based differs from the estimate derived based on the entire population

Data bias – uncertainty that is introduced when observed experience is adjusted / needs to be adjusted in order to render it homogeneous and comparable over different periods of time

Sampling risk and data bias are the primary sources of parameter risk

- (b) Explain to Palisades its potential exposure to sampling risk using examples which are specific to its situation.

Commentary on Question:

Since the question asked for examples (plural), two examples were required for full credit. Many candidates only provided one example of sampling risk. Most focused on identifying the use of the 2007-2011 data set to estimate the mean and volatility return parameters as an example of sampling risk.

6. Continued

To obtain full credit, the answers needed to both identify and explain how the example applied to Palisades specifically. For example, it was not sufficient to say that using 10 funds to estimate fund management fee levels is a source of sampling risk. The candidate needed to explain that using 10 funds may lead to sampling risk as 10 funds constitute a small sample. The smaller the sample size, the greater the likelihood that the sample does not reflect the characteristics of the population. In this case, the fund management fee level for these 10 funds may not adequately represent the overall fee level especially since there is no assurance that these 10 funds represent the bulk of assets managed by Palisades.

To be clear, this explanation is required because the use of 10 funds may also lead to data bias if one bases the response on the explanation that these 10 funds are passively managed and the fund management fee levels Palisades wants to estimate relate to actively managed funds.

Response:

Sampling risk can arise from the estimation of the mean return and volatility parameters, which are estimated using data from 2007-2011. To the extent that this period of time is not representative of the mean returns / volatilities over the period of time for which Palisades is projecting, this may introduce error. The sample period may be unrepresentative of expected future mean returns / volatilities because the period might be too short.

Sampling risk can arise if the 10 mutual funds upon which Palisades estimates the fund management fee are not representative of the population average. Funds are selected based on being common to most accounts which may not represent the majority of assets invested. Therefore, it may provide a poor estimate of the true average fund management fee.

- (c) Explain to Palisades its potential exposure to data bias using examples which are specific to its situation.

Commentary on Question:

Similar to part (b), two examples were required to receive full credit. Candidates struggled with this part of the question. Most of the examples given were actually sampling risk rather than data bias. Again, certain aspects of the data could be considered an example of data bias or of sampling risk (for example, the use of 10 funds as further explained in (b) above) – to receive credit, the candidate needed to include an explanation that showed an understanding of the difference between the two. The best answers also explained how the example applied to Palisades specifically.

6. Continued

Response:

Data bias can arise from using the calendar years 2007-2011 to estimate mean and volatility, since that period included the financial crisis and thus may not be comparable to other periods unless adjusted.

If different fund types (with different fund management fee levels) were prevalent during different portions of this time period, then the average fund management fee estimate will fail to account for this.

Similarly if the average level of fund management fees has been trending lower because of increased competition or increased prevalence of indexed funds, or trending higher because of increased prevalence of actively managed funds, then the estimation of an average fund management fee will be biased by this trend.

Since the new product only offers protection against declines in specific Palisades mutual funds, the most popular funds may not be representative of the ones allowed in the new product.

- (d) Explain how parameter risk can be either systemic or diversified across a portfolio of risks. Provide examples which are specific to the Palisades context.

Commentary on Question:

Some candidates were able to provide definitions of systemic risk and diversification but many did not explain how these applied to Palisades. Partial credit was given for definitions only. For full credit for this part, the candidate needed to explain how the concepts applied to Palisades specifically. Some candidates focused on the diversification of adding more data instead of how diversification impacted Palisades' parameter risk.

Response:

Systemic risk is created when the model's parameters are faulty because the model is biased consistently by the error. For example, if Palisades mis-estimates the volatility parameter, then the Palisades model will systematically over or under price this new product.

A diversification benefit may be created if more than one parameter is mis-estimated or if a particular parameter is misstated for a variety of funds. For example, if Palisades has a number of parameter estimation errors, the resulting errors may be offsetting and thus the risk will be diversified.

- (e) Recommend improvements Palisades could implement to reduce parameter risk, sampling risk and data bias.

6. Continued

Commentary on Question:

Many candidates were able to provide 2 or 3 recommendations for reducing parameter risk, sampling risk, or data bias; in some cases a single recommendation addressed more than one of the risks. The strongest candidates provided recommendations specific to Palisades, linked these directly to the risk that would be reduced and then explained how that risk would be reduced. Weaker candidates only provided a list and did not explain how they applied in Palisades' case specifically or what specific risk they would reduce.

Several possible answers are presented below; to get full credit, a candidate needed to address all three risks within the recommendations.

Response:

Parameter risk

- Palisades could back-test the model in various time frames to determine how well the model would have held up in the past.

Sampling risk

- Palisades should use a longer time frame of data to establish the mean and volatility estimates. Using a longer period will help to reduce the impacts of extreme market conditions (e.g., the financial crisis).
- Palisades could use more funds to establish the fund management fees and focus specifically on funds which are expected to reflect the characteristics of the funds it intends to model (i.e., select sample from the population of funds being modeled).
- Palisades could use funds that represent the largest portion of its account values rather than the most common funds.

Data bias

- Palisades should base its fund management fee assumption on the funds that are included in the protection product.
- Palisades should examine trends in the fund management fees that have occurred over this time period to determine if adjustments need to be made, based on the types of funds that were popular during the prior time period as compared to those that they allow the client to choose from under the new product.
- Palisades should consider using a different period to base its mean and volatility parameters on, given that the 2007-2011 timeframe was subject to the financial crisis. Alternately, Palisades could place weights on the data from this time period to reflect its assessment of the likelihood that such market conditions occur again.

7. Learning Objectives:

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

Learning Outcomes:

- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (3b) Analyze and evaluate the properties of risk measures (e.g. Delta, volatility, duration, VaR, TVaR, etc.) and their limitations.
- (3c) Analyze quantitative financial data and construct measures from insurance data using modern statistical methods (including asset prices, credit spreads and defaults, interest rates, incidence, causes and losses). Contrast the available range of methods with respect to scope, coverage and application.

Sources:

Value-at-Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion, 2007

- Chapter 7, Portfolio Risk: Analytical Methods
- Chapter 8, Multivariate Models

Commentary on Question:

Purpose of the question:

The purpose of the question was to test the understanding and applicability of different approaches to calculating Value-at-Risk (VAR) for an investment portfolio. In addition, the question tested the capacity to select and comment on the appropriateness of a chosen VaR approach based on the inner working and limitations of each methodology.

Important considerations for receiving maximum points

Since this question tested comprehension of the material, analysis based on required calculations, and knowledge utilization, a candidate received maximum points if he demonstrated mastery of each of these goals respectively.

For part (a), a candidate had to correctly calculate using each methodology in order to obtain maximum points, showing intermediary steps leading to the final answer. Partial credit was given for only showing the development process for each methodology.

For part (b), a candidate had to explain briefly why each VaR methodology provided a slightly different value compared to method I by explaining the underlying basis of each methodology.

7. Continued

For part (c), a candidate had to recommend the most suitable methodology by providing relevant information about that method as well as by explaining why other methodologies were not part of his recommendation.

Areas where some candidates did well/Areas where some candidates had trouble

In general, part (a) was well answered, with methods IV and I being the easiest for candidates. The calculations using the Beta and Diagonal methodologies were harder for some candidates.

For part (b), giving an explanation involved providing some insight as to the operation of each methodology. The Undiversified methodology (IV) was straightforward and was well explained by most candidates. Many candidates provided valid comments about both methodologies II and III. However, many candidates did not provide a comparison back to methodology I.

Part (c) was not as well answered as the earlier parts. Many candidates wrote some disparate statements about each methodology, for which some credit was given, but the purpose of the question was to test if the candidate was able to provide a valid and specific recommendation and justify it. Some candidates were able to make a valid recommendation and provide satisfying arguments for and against the other methodologies.

Solution:

- (a) Calculate the monthly VaR for the \$100 million portfolio at the 95% confidence level (to the nearest thousand dollars) using each of the four methods above.

Commentary on Question:

The first calculation consisted of estimating the portfolio VaR using the Variance/Covariance matrix from the question and then applying it to the weights that represented the relative importance of each stock in the portfolio. Then, the candidate had to obtain the required factor corresponding to the desired confidence level of 95% -- 1.65 -- and complete the calculation.

Response:

- (i) Full Variance/Covariance Method:

Σ is defined as the variance-covariance matrix.

$$VAR_p = \alpha \sigma pW = \alpha \sqrt{x' \Sigma x}$$

$$VaR = \$100 \text{ million} \times 1.65 \times \sqrt{w' \Sigma w} \text{ where } w = (1/3, 1/3, 1/3)$$

$$= \$100M \times 1.65 \times \sqrt{0.289\%} = \$100m \times 1.65 \times 0.05376 = \$8.87 \text{ million}$$

7. Continued

- (ii) The second calculation consisted of estimating the Variance/Covariance matrix by a different methodology based on the Betas of each stock as represented here:

$$\text{Beta Model Variance-Covariance Matrix: } \Sigma = (w' \beta \beta' w) \sigma_m^2$$

Then, the calculation was done as follows:

$$\begin{aligned} \text{VaR} &= \$100 \text{ million} \times 1.65 \sqrt{w' \Sigma w} \quad \text{where } w = (1/3, 1/3, 1/3) \\ &= \$100\text{M} \times 1.65 \times \sqrt{0.20\%} \\ &= \$100\text{M} \times 1.65 \times 0.04472 = \$7.79\text{million} \end{aligned}$$

- (iii) The third calculation was based on the Diagonal methodology. The estimated Variance/Covariance matrix was modified to account for the error terms obtained from the regression that had estimated the market Betas of each stock as used in calculation (ii):

$$\text{Diagonal Model Variance-Covariance Matrix: } \Sigma = w' (\beta \beta' \sigma_m^2 + D_\varepsilon) w$$

Then, the calculation was:

$$\begin{aligned} \text{VaR} &= \$100 \text{ million} \times 1.65 \sqrt{w' \Sigma w} \quad \text{where } w = (1/3, 1/3, 1/3) \\ &= \$100\text{M} \times 1.65 \times \sqrt{0.37\%} \\ &= \$100\text{M} \times 1.65 \times 0.0608 = \$10.032 \text{ million} \end{aligned}$$

- (iv) The last calculation consisted of estimating the portfolio Undiversified VaR methodology by summing individual VaR calculations for each of the three stocks as follows:

Undiversified VaR = sum of individual VaRs:

$$\text{GE: } 1.65 \times 9.4\% = 15.51\%$$

$$\text{Citi: } 1.65 \times 7.7\% = 12.705\%$$

$$\text{ExxM: } 1.65 \times 8.6\% = 14.19\%$$

$$\begin{aligned} \text{Undiversified VaR} &= \$100 \text{ million} \times (1/3)(\text{VaR}(\text{GE}) + \text{VaR}(\text{Citi}) + \\ &\text{VaR}(\text{ExxM})) \\ &= \$100\text{m} \times (15.51\% + 12.705\% + 14.19\%)/3 = \$14.135 \text{ million} \end{aligned}$$

- (b) Explain why methods II, III and IV resulted in a different estimate of portfolio VaR (relative to method I).

7. Continued

Commentary on Question:

The answer below is an example of an appropriate explanation for the question. Specific wording varied from one candidate to another and other responses were accepted if relevant and appropriate to each methodology. In all answers, we looked for a reference back to methodology I and some valid elements that explained the difference in the VaR values obtained. Full credit was granted if there were appropriate comments as required by the question. Partial credits were given for a more limited response.

Response:

The Full Variance/Covariance Matrix Method uses the full covariance matrix of individual stocks and provides an exact portfolio VaR measure over the required period based on the portfolio's composition.

The portfolio VaR based on the Beta Method is lower than the Full Variance/Covariance method because it estimates the VAR/COV matrix, not based on individual stocks' variability/covariability, but from their Betas obtained from a regression equation with the market. These Betas are averages over time, which minimizes the overall variability with the market data.

The portfolio VaR based on the Diagonal Method provides a closer approximation to the Full Variance/Covariance method because it corrects the previous estimate of VAR/COV of the Beta Method by adding back the error terms of the regression, i.e., the residuals, which provides a closer approximation to the portfolio VaR.

Finally, the portfolio VaR based on the Undiversified Method generates the highest VaR compared to the Full VAR/COV method since it assumes that all correlation coefficients between the stock assets are 1. The method does not recognize the benefits of diversification as integrated in the Full VAR/COV method. It represents an upper bound of the portfolio VaR.

- (c) Recommend the most suitable method of the four in (a) for assessing the total U.S. Equity portfolio monthly VaR in SLIC's DB Plan. Justify your recommended method over the other three methods.

Commentary on Question:

A valid answer to the question consisted in making first a recommendation with relevant justifications followed by other statements as to why the other methodologies were less appropriate based on the context of the question. Partial credits were given for relevant justifications for each method without a valid overall recommendation.

7. Continued

Response:

The most suitable method to calculate the portfolio VaR would be based on the Diagonal Method because the value obtained, \$10M, comes closest to the “exact VaR” of \$ 8.9M obtained by the VAR/COV method. It also allows for a simplification of the calculations by reducing the number of parameters to be used in the calculation, particularly the number of VAR/COV factors. The number of parameters required is $2N+1$, or 801 for a portfolio of 400 stocks.

The Beta Method approach, although similar to the Diagonal Method, could be a second choice in terms of simplification. However, in this case it substantially underestimates the portfolio VaR due to ignoring the residual terms of the regression that generates the Betas. In this case the underestimation is substantial due to the highly concentrated portfolio of SLIC's DB pension plan.

The Full VAR/COV method would be less appropriate because of the large number of different stocks held and the resulting difficulty in calculating the matrix. For example, for a portfolio of 400 stocks, more than $(400 * (400+1))/2 = 80,200$ different variance and covariance terms would need to be estimated. Also, correlations between stocks may not be estimated correctly nor be stable during periods of stress.

Finally, the Undiversified Method, while easy and efficient to calculate, overstates market risks of the portfolio by ignoring the diversification benefits. At \$14.1 million, it is too high compared to the Full VAR/COV value but constitutes an upper bound.

8. Learning Objectives:

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.
4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

- (3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.
- (4e) Demonstrate how derivatives, synthetic securities, and financial contracting may be used to reduce risk or to assign it to the party most able to bear it.
- (4i) Analyze funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Explain the concepts of immunization including modern refinements and practical limitations. Contrast the various risk measures and be able to apply these risk measures to various entities.

Sources:

ERM-304-12: Mind the Gap: Using Derivatives Overlays to Hedge Pension Duration

ERM-606-12: Fixed Income Securities, Tuckman, Second Edition, Chapter 7, Key Rate and Bucket Exposures

Commentary on Question:

The overall performance on this question was above expectations. The question is focused on understanding and hedging interest rate risks. Candidates showed good understanding of the topics. Most mistakes were of a numerical rather than conceptual variety.

Solution:

- (a)
 - (i) Calculate the DB Plan's current funding ratio (ratio of the value of assets to the value of liabilities) on the actuarial valuation basis assuming the Normal Allocation of assets above.
 - (ii) Evaluate the impact on the funding ratio of a 1% downward yield curve shift.

8. Continued

Commentary on Question:

Most candidates did well on this part. Some candidates simply calculated the change in the MVA directly, instead of calculating asset duration and then the change in the MVA; they received full credit. Common errors were mistakes in calculating the asset duration or not recognizing that the MVL changes as well.

Response:

- (i) Funding Ratio (FR) = $A/L = \$644.2/\$736.3 = 0.875$ or 87.5%
- (ii) Total Asset duration = $(0.30 \times 5) + (0.04 \times 15) + (0.66 \times 0) = 2.1$ years
- -1% shock: Asset impact = $+ 2.1 \times 1\% \times \$644.2 = \$13.5282$ million
 - Duration of Liabilities = 10.0 years
 - -1% shock: Liability impact = $+ 10 \times 1\% \times \$736.3 = \73.63 million
 - -1% shock: FR impact = $(\$644.2 + \$13.5)/(\$736.3 + \$73.63) = 0.812$ or 81.2%
- Thus the Funding Ratio drops by 6.3% (87.5% - 81.2%) under a -1% yield curve shift.
- (b) You want to use a 10-year interest rate swap with duration of 8.5 years in order to hedge the DB Plan's overall interest rate risk exposure under the Normal Allocation assumption.

Compute the notional principal of the swap required to mitigate the DB Plan's interest rate risk exposure, recognizing the DB Plan's current funded status. Show your work.

Commentary on Question:

Most candidates did well on this part. Some used an alternative formula with target duration instead of liability duration, which if used correctly is an equivalent approach. However, a common mistake for those using the alternative formula was to use the liability duration as given, rather than calculating the target duration.

Response:

Actual Asset MV = \$644.2

Liability MV = \$736.3

Notional Principal = $[(MVL \times \text{liability duration}) - (MVA \times \text{asset duration})] / \text{Swap duration}$

$NP = \{(\$736.3 \times 10) - (\$644.2 \times 2.1)\} / 8.5 = \707.08 million

Thus the Notional Principal has to be larger to cover the funding shortfall.

8. Continued

Alternative Formula:

$$NP = MVA \times (\text{target duration} - \text{assets duration}) / \text{Swap Duration}$$

Where:

$$\text{target duration} = MVL \times \text{liability duration} / MVA = 11.43$$

- (c) You are assessing the Plan's interest rate exposures on a key rate 100 (KR100) basis, measuring the dollar value impact of a 100 basis point shift in a key rate.

Determine the net KR100 exposure by calculating the missing KR100 values in the table below (to the nearest thousand dollars):

SLIC DB Plan	Market Value (\$000s)	KR100 (\$000s)		
		Short	Medium	Long
U.S. Fixed Income	\$193,267			
Real Estate	\$25,769			
Other Assets	\$425,188	0	0	0
Total Assets	\$644,224			
Total Liability	\$736,325	\$13,401	\$34,902	\$25,330
Net	-\$92,101			

Commentary on Question:

Most candidates did well on this part. Some candidates mistakenly included the "Other Assets" value when calculating the various exposures.

Response:

Multiply the different asset class \$MVs by each KR100 x 1%:

$$\text{Fixed Income: Short KR100} = \$193,267 \times 0.0175 = \$3,382$$

$$\text{Fixed Income: Mid KR100} = \$193,267 \times 0.0185 = \$3,575$$

$$\text{Fixed Income: Long KR100} = \$193,267 \times 0.0140 = \$2,706$$

$$\text{Real Estate: Short KR100} = \$25,769 \times 0.011 = \$283$$

$$\text{Real Estate: Mid KR100} = \$25,769 \times 0.053 = \$1,366$$

$$\text{Real Estate: Long KR100} = \$25,769 \times 0.086 = \$2,216$$

Other Assets: all KR100's are \$0

Total Asset KR100s: (sum of individual asset class KR100's)

$$\text{Total Asset: Short KR100: } \$3,382 + \$283 + \$0 = \$3,665$$

$$\text{Total Asset: Mid KR100: } \$3,575 + \$1,366 + \$0 = \$4,941$$

$$\text{Total Asset: Long KR100: } \$2,706 + \$2,216 + \$0 = \$4,922$$

8. Continued

Net: Short KR100 = $-\$13,401 + \$3,665 = -\$9,736$

Net: Mid KR100 = $-\$34,901 + \$4,941 = -\$29,960$

Net: Long KR100 = $-\$25,329 + \$4,922 = -\$20,407$

- (d) Determine the Notional Principal amount and the direction (receive fixed or pay fixed) of a basket of the above three swaps that would hedge the net KR100 exposure determined in (c).

Commentary on Question:

Most candidates did well on this part, though some candidates omitted it entirely. Some candidates did not recognize the correct direction of the hedge they calculated. The candidates who calculated incorrect exposures in part (c) but applied the correct formulae in this part of the question received full credit for part (d).

Response:

Start with hedging the Long KR100 exposure with 30-Yr Interest Rate Swap:

30-Yr IRS Notional = $\text{Net Long KR100} / 30\text{-yr IRS Long KR100} = 20,407 / 72 = 283.431$ or \$283,431 Notional Receive Fixed 30-Yr IRS

Netting the \$283,431 30-YR IRS swap KR100's from the Net KR100's:

New Net KR100 Long: $-\$20,407 + (\$283.431 \times 72) = \$0$ (check)

New Net KR100 Mid: $-\$29,960 + (\$283.431 \times 38) = -\$19,190$

New Net KR100 Short: $-\$9,736 + (\$283.431 \times 18) = -\$4,634$

Next hedge the New Net Mid KR100 with 10-yr IRS Mid KR100:

10-Yr IRS Notional = $\text{New Net Mid KR100} / 10\text{-yr IRS Mid KR100} = 19,190 / 60 = 319.833$ or \$319,833 Notional Receive Fixed 10-Yr IRS

Netting the \$319,833 10-YR IRS swap KR100's from the New Net KR100's:

New Net KR100 Mid: $-\$19,190 + (\$319.833 \times 60) = \$0$ (check)

New Net KR100 Short: $-\$4,634 + (\$319.833 \times 25) = \$3,362$

Next hedge the last New Net Short KR100 with 2-yr IRS Short KR100:

2-Yr IRS Notional = $\text{New Net Short KR100} / 2\text{-yr IRS Short KR100} = (3,362) / 18 = (186.778)$ or \$186,778 Notional PAY Fixed 2-Yr IRS

Netting the (\$186,778) 2-YR IRS swap Short KR100's from the New Net Short KR100's:

New Net KR100 Short: $\$3,362 + (-\$186.778 \times 18) = 0$ (check)