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**SOCIETY OF ACTUARIES**  
**Enterprise Risk Management – Investment Extension**

**Exam ERM-INV**

**Date:** Wednesday, May 1, 2013

**Time:** 8:30 a.m. – 12:45 p.m.

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**INSTRUCTIONS TO CANDIDATES**

**General Instructions**

1. This examination has a total of 80 points.

This exam consists of 8 questions, numbered 1 through 8.

The points for each question are indicated at the beginning of the question. Questions 7 and 8 pertain to the Case Study, which is enclosed inside the front cover of this exam booklet.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

**Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam ERM-INV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.



## **CASE STUDY INSTRUCTIONS**

**The case study will be used as a basis for some examination questions. Be sure to answer the question asked by referring to the case study. For example, when asked for advantages of a particular plan design to a company referenced in the case study, your response should be limited to that company. Other advantages should not be listed, as they are extraneous to the question and will result in no additional credit. Further, if they conflict with the applicable advantages, no credit will be given.**



**\*\*BEGINNING OF EXAMINATION\*\***

- 1.** (6 points) You are an actuary working for Joint Life Co. You have fit a marginal distribution to each of the following two risk factors,  $X_1$  and  $X_2$ . You believe  $X_1$  and  $X_2$  have upper tail dependence but no lower tail dependence.

You are given:

$$\text{Clayton Copula: } C(x, y) = (x^{-\theta} + y^{-\theta} - 1)^{-1/\theta}$$

- (a) (1 point) Demonstrate that each marginal risk factor distribution is uniform on  $[0, 1]$  using the Clayton copula.
- (b) (1 point) The chief actuary at Joint Life is unfamiliar with copulas and prefers to use the variance-covariance approach when reflecting dependency between random variables. He recommends increasing the correlation factor in order to reflect the upper tail dependence that exists between  $X_1$  and  $X_2$ .

Provide advantages and disadvantages of the chief actuary's approach.

- (c) (2 points) Demonstrate that use of the Clayton copula results in an increasing level of lower tail dependence as the parameter  $\theta$  increases.
- (d) (2 points) One of your colleagues has developed a new copula, the FGM copula. You are given:

$$\text{FGM Copula: } C(x, y) = xy[1 + \theta(1-x)(1-y)]$$

Determine whether the use of the FGM copula is appropriate when modeling the joint distribution for  $X_1$  and  $X_2$ .

Show your work.

2. (13 points) Cramped Quarters Corporation (CQC) is evaluating its bond portfolio credit risk exposure. CQC uses the Merton model to calculate the probability of default for each holding in its portfolio.

CQC has exposure to a bond issued by Crow, a large publicly traded corporation.

You are given the following information about Crow:

Asset Book Value	\$800 Million
Asset Value Growth Per Year	8.4%
Asset Value Volatility Per Year	20%
Total Debt	\$500 Million
Debt Term	1 year

The Merton model gives the probability of default at time  $T$  as:

$$P(X_T \leq B) = \Phi \left( \frac{\ln \left( \frac{B}{X_0} \right) - \left( r_A - \frac{\sigma_A^2}{2} \right) T}{\sigma_A \sqrt{T}} \right)$$

- (a) (2 points)
- Define credit risk.
  - Describe the four components that should be considered when modeling credit risk.
- (b) (2 points) Calculate Crow's probability of default using the Merton model.
- (c) (2 points) Explain how each of the following would impact CQC's assessment of the probability of default for the Crow bond.
- Crow's asset book value decreases.
  - The risk free rate increases.
  - Crow's asset value volatility decreases.

## 2. Continued

- (d) (2 points) Explain why the formulation of the Merton model above uses  $r_A$  whereas the standard option pricing formula using Black-Scholes uses the risk free rate.

CQC also has exposure to sovereign debt.

- (e) (3 points) Describe the Merton model and evaluate its use with respect to modeling sovereign credit risk.
- (f) (2 points) Explain the discriminant analysis approach to modeling credit risk and describe how you might implement such an approach with respect to modeling sovereign credit risk.

3. (9 points) You are a newly hired risk manager for Hamsik Life Insurance Company. Hamsik has a block of term life insurance with \$25 billion in face amount. You have been asked to assist with the selection of a risk mitigation program aimed at protecting Hamsik against material variations in mortality.

You are provided with the following:

	Expected Mortality Rate Per Thousand
Hamsik	4.0
U.S. Industry Claims index (USIC)	6.0

You have been provided the following three options by your insurance broker:

- I. Hamsik purchases annual renewable stop loss reinsurance from a monoline insurer that specializes in providing such coverage. Hamsik pays an annual premium of \$2.35 million, retains claims up to 150% of expected claims, and receives reimbursement for aggregate claims exceeding 150% of expected claims up to 200% of expected claims.
  - II. Hamsik issues \$50 million in mortality cat-bonds at an annual net cost of 400 bps of bond face amount. The bonds provide reimbursement to Hamsik once aggregate annual claims exceed \$150 million. The bonds mature at the end of a 5-year period and require that principal be repaid at maturity in the amount of the original \$50 million reduced by any reimbursement amounts paid to Hamsik.
  - III. Hamsik enters into a 3-year swap agreement whereby it agrees to pay an annual fixed payment of 0.025% of Notional in exchange for floating payments which equal 50% of the claims in excess of 140% of USIC Expected Mortality, up to an aggregate maximum floating payment amount of \$75 million. The Notional amount equals \$25 billion.
- (a) (4 points) Management wants to evaluate the relative merits of the options listed above using a cost-benefit analysis. Prepare a comparison of the three options considering at least five criteria that you consider key to making a decision.
  - (b) (2 points) Identify which of the three options is least favorable to Hamsik. Justify your response.
  - (c) (3 points) You need more information in order to make a recommendation as to which option is most favorable to Hamsik.

Identify five key questions you would ask Hamsik senior management to enable a more informed recommendation.



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4. (13 points) You are a consultant helping to implement the Economic Capital (EC) model for your client, Pier Life, a publicly-traded life insurance company.

Pier Life offers insurance products that allow for the accumulation of significant account value balances. The products offer interest rate crediting guarantees that were competitive when the products were sold. Pier Life also offers protection-oriented products that provide a death benefit but do not result in the accumulation of significant account values.

Pier Life's statutory liabilities are prescribed by formula and are primarily backed by high quality fixed income assets. These assets are duration and convexity matched to the statutory liabilities. Statutory risk-based capital is also prescribed by formula. Statutory surplus is backed by a mix of 70% high-quality fixed income assets and 30% equity assets.

The EC model you propose is stochastic and includes real world scenarios that measure underwriting, credit, and market risks, all of which are shown to be material based on your preliminary analysis. The scenarios are a function of the current yield curve as of the valuation date.

Economic capital is set using a VaR 95% measure.

- (a) (2 points) Describe the following terms. Include in your description the equations that define these items and how they relate to one another.
- (i) Available Economic Capital (AEC)
  - (ii) Required Economic Capital (REC)
  - (iii) Excess Economic Capital
  - (iv) Market Capitalization
- (b) (4 points) Your analysis will independently consider the following three potential events:
- (i) A significant increase in interest rates
  - (ii) A significant drop in the stock market
  - (iii) A reduction from VaR 95% to VaR 90%

Explain the probable implications in terms of direction and significance to all four items in part (a) for each of the events. Justify your response and identify any additional assumptions you made in your analysis.

#### 4. Continued

- (c) (1 point) Explain how the addition of one year of new business to the existing EC model would be expected to impact AEC and REC.
- (d) (6 points) Your client is considering entering into one of two reinsurance arrangements.
- 30% Quota Share Yearly Renewable Term (YRT) for the purpose of insurance risk mitigation
  - 30% First Dollar Quota Share Coinsurance for the purpose of providing statutory capital relief

Assume that both reinsurance arrangements are fairly priced and cover substantial amounts of inforce business.

- (i) Compare and contrast coinsurance and YRT reinsurance, including the impact each is expected to have on both statutory reserves and capital.
- (ii) Explain the probable implications for each of the following, in terms of direction and relative magnitude, of entering into the YRT arrangement.
- I. Available Economic Capital (AEC)
  - II. Required Economic Capital (REC)
  - III. Excess Economic Capital
- (iii) Explain the probable implications for each of the following, in terms of direction and relative magnitude, of entering into the coinsurance arrangement.
- I. Available Economic Capital (AEC)
  - II. Required Economic Capital (REC)
  - III. Excess Economic Capital
- (iv) Describe how you would assess which reinsurance arrangement results in a better EC position for the company.

5. (8 points) You are an actuary working on interest-sensitive products at Saddle Peak Life. You have been asked to model the term structure of interest rates in order to calculate VaR and test the reserve and asset adequacy of these product lines.

The selection of arbitrage-free versus equilibrium and risk neutral versus realistic attributes creates four classes of model forms for potential use as follows:

Model Classification	Risk Neutral	Realistic
Arbitrage-free	A	B
Equilibrium	C	D

You have received a copy of the model that has been successfully used by the derivatives team for Saddle Peak's interest rate swap hedging program. The derivative team's model is a form of the Black-Karasinski Model, expressed as follows:

$$du = \kappa(t)(\theta(t) - u)dt + \sigma(t)dz$$

- (a) (2 points) Compare and contrast arbitrage-free versus equilibrium interest rate models.
- (b) (2 points) Compare and contrast the use of risk neutral probabilities versus realistic probabilities in the parameterization of interest rate models.
- (c) (1 point) Identify the model classification letter of the derivative team's model and explain how the parameters would be set.
- (d) (2 points) Identify the model classification letter of the model that would be most appropriate for your analysis of interest-sensitive products. Justify your choice.
- (e) (1 point) Propose a modified form of the Black-Karasinski model to use in your work and explain how the parameters will be set.

6. (11 points) Palisades is a mutual fund company which gained popularity by offering a competitively priced suite of passively managed mutual funds. In response to market demand, Palisades has recently begun offering actively managed funds.

Palisades is considering the launch of a product which would allow clients to purchase protection against declines in specific Palisades mutual funds. In exchange for this protection, clients would be periodically assessed a charge equal to a percentage of their fund balances, with the percentage varying by mutual fund.

In order to price this product, Palisades has taken the following steps:

- Two equity return models have been developed, one using the normal distribution and the other using a lognormal distribution
- The mean return parameter for both of these models has been estimated using mutual fund specific return data from 2007 – 2011
- The volatility parameter for both of these models has been estimated using S&P 500 data from 2007 – 2011
- The mean fund management fee (assessed by the mutual fund manager) has been determined using the average of the fees assessed on the ten most common mutual funds held across all Palisades accounts as follows:

$$\text{Average Management Fee} = \left[ \sum \sum \text{fee}(i, j) \times \text{Fund Balance}(i, j) \right] / \left[ \sum \sum \text{Fund Balance}(i, j) \right]$$

where the summations are over all ten funds selected ( $i = 1, 2, \dots, 10$ ) and calendar years  $j = 2011, 2010, \dots, 2007$  (and where the ten funds selected for each calendar year are specific to that calendar year.)

Palisades has hired you to provide consulting services to support the launch of this product.

- (a) (2 points) Define parameter risk, sampling risk and data bias, and explain how these three items relate to one another.
- (b) (2 points) Explain to Palisades its potential exposure to sampling risk using examples which are specific to its situation.
- (c) (2 points) Explain to Palisades its potential exposure to data bias using examples which are specific to its situation.
- (d) (2 points) Explain how parameter risk can be either systemic or diversified across a portfolio of risks. Provide examples which are specific to the Palisades context.
- (e) (3 points) Recommend improvements Palisades could implement to reduce parameter risk, sampling risk and data bias.

**Questions 7 and 8 pertain to the Case Study.**  
**Each question should be answered independently.**

7. (10 points) SLIC is concerned about the risk stemming from the exposure to U.S. equities in its Defined Benefit pension plan (DB Plan). The portfolio holds over 400 different U.S. stocks, with the Top 10 Holdings comprising about 40% of the total portfolio market value.

You are considering four methods to aggregate equity risk exposure:

- I. The Full Variance Covariance method
- II. The Beta Model method
- III. The Diagonal Model method
- IV. The Undiversified VaR method

Your analysis considers the Plan's top three equity holdings – General Electric (GE), Citigroup (Citi), and Exxon Mobil (ExxM). Your analysis will assume a \$100 million aggregate portfolio value with one-third invested in each of the three stocks.

You are provided the following data:

	$\sigma_i$	Variance/Covariance			Correlations			$V(\varepsilon_i)$	$\beta_i$
		GE	Citi	ExxM	GE	Citi	ExxM		
GE	9.4%	0.9%	0.2%	0.1%	1.0	0.3	0.1	0.4%	1.60
Citi	7.7%	0.2%	0.6%	-0.1%	0.3	1.0	-0.2	0.4%	0.99
ExxM	8.6%	0.1%	-0.1%	0.7%	0.1	-0.2	1.0	0.7%	0.49

The  $V(\varepsilon_i)$  values were derived from a regression of each stock's monthly returns on the S&P 500. The market monthly volatility is 4.34%.

- (a) (5 points) Calculate the monthly VaR for the \$100 million portfolio at the 95% confidence level (to the nearest thousand dollars) using each of the four methods above.

Show your work.

- (b) (2 points) Explain why methods II, III and IV resulted in a different estimate of portfolio VaR (relative to method I).
- (c) (3 points) Recommend the most suitable method of the four in (a) for assessing the total U.S. Equity portfolio monthly VaR in SLIC's DB Plan. Justify your recommended method over the other three methods.

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**Questions 7 and 8 pertain to the Case Study.**  
**Each question should be answered independently.**

- 8.** (10 points) You have been asked to assess the interest rate risk exposure in SLIC's Defined Benefit pension plan (DB Plan). The actuarial value of plan liabilities is \$736,325 thousand with duration of ten years.

To simplify your analysis, you have selected three key rates: Short, Medium, and Long. The corresponding key rate durations (KRDs) are as follow:

	Normal Allocation	Market Value (\$000)	Key Rate Durations		
			Short	Medium	Long
U.S. Equity	40%	\$257,690			
International Equity	20%	\$128,845			
U.S. Fixed Income	30%	\$193,267	1.75	1.85	1.40
Real Estate	4%	\$25,769	1.10	5.30	8.60
Private Equity	4%	\$25,769			
Cash	2%	\$12,884			
Total Assets	100%	\$644,224			

- (a) (2 points)
- (i) Calculate the DB Plan's current funding ratio (ratio of the value of assets to the value of liabilities) on the actuarial valuation basis assuming the Normal Allocation of assets above.
  - (ii) Evaluate the impact on the funding ratio of a 1% downward yield curve shift.

Show your work.

- (b) (3 points) You want to use a 10-year interest rate swap with duration of 8.5 years in order to hedge the DB Plan's overall interest rate risk exposure under the Normal Allocation assumption.

Compute the notional principal of the swap required to mitigate the DB Plan's interest rate risk exposure, recognizing the DB Plan's current funded status. Show your work.



## 8. Continued

- (c) (2 points) You are assessing the Plan's interest rate exposures on a key rate 100 (KR100) basis, measuring the dollar value impact of a 100 basis point shift in a key rate.

Determine the net KR100 exposure by calculating the missing KR100 values in the table below (to the nearest thousand dollars):

SLIC DB Plan	Market Value (\$000s)	KR100 (\$000s)		
		Short	Medium	Long
U.S. Fixed Income	\$193,267			
Real Estate	\$25,769			
Other Assets	\$425,188	0	0	0
<b>Total Assets</b>	<b>\$644,224</b>			
<b>Total Liability</b>	<b>\$736,325</b>	<b>\$13,401</b>	<b>\$34,902</b>	<b>\$25,330</b>
<b>Net</b>	<b>-\$92,101</b>			

Show your work.

- (d) (3 points) You are given the following KR100 values for the following three Interest Rate Swaps:

Swap Tenor	Notional	Short KR100	Medium KR100	Long KR100
30-Year	\$1,000	\$18	\$38	\$72
10-Year	\$1,000	\$25	\$60	\$0
2-Year	\$1,000	\$18	\$0	\$0

Determine the Notional Principal amount and the direction (receive fixed or pay fixed) of a basket of the above three swaps that would hedge the net KR100 exposure determined in (c).

Show your work.

**\*\*END OF EXAMINATION\*\***

**USE THIS PAGE FOR YOUR SCRATCH WORK**