

---

**SOCIETY OF ACTUARIES**  
**Enterprise Risk Management – Investment Extension**

**Exam ERM-INV**

**Date:** Tuesday, October 29, 2013

**Time:** 8:30 a.m. – 12:45 p.m.

---

**INSTRUCTIONS TO CANDIDATES**

**General Instructions**

1. This examination has a total of 80 points.

This exam consists of 8 questions, numbered 1 through 8.

The points for each question are indicated at the beginning of the question. Questions 7 and 8 pertain to the Case Study, which is enclosed inside the front cover of this exam booklet.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

**Written-Answer Instructions**

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam ERM-INV.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.



## **CASE STUDY INSTRUCTIONS**

**The case study will be used as a basis for some examination questions. Be sure to answer the question asked by referring to the case study. For example, when asked for advantages of a particular plan design to a company referenced in the case study, your response should be limited to that company. Other advantages should not be listed, as they are extraneous to the question and will result in no additional credit. Further, if they conflict with the applicable advantages, no credit will be given.**



**\*\*BEGINNING OF EXAMINATION\*\***

- 1.** (8 points) Blackhawk Life is a mid-size U.S. insurer that writes both deferred variable annuities with investment performance guarantees and deferred fixed annuities. The assets backing the fixed annuities are largely invested in U.S. Treasuries and corporate bonds. The variable annuity guarantees are dynamically hedged using a combination of equity options, equity futures, and interest rate swaps.

Blackhawk currently has a stress testing program that runs historical scenarios simulating major market crises that have occurred. Blackhawk is considering performing sensitivity tests as well.

- (a) (1 point) Evaluate the advantages of analyzing stress test results for the fixed and variable blocks separately versus in aggregate.
- (b) (2 points) Recommend four key unidimensional sensitivity tests that Blackhawk should run to analyze the risks of its business. Justify your response.
- (c) (3 points) Blackhawk is also considering performing multidimensional scenario analysis.
  - (i) Describe two types of multidimensional prospective scenarios.
  - (ii) Explain the drawbacks of using each of the two types of scenarios identified in (i).
  - (iii) Recommend one scenario of each type in (i) that would be meaningful to Blackhawk. Justify your response.
- (d) (2 points) Blackhawk reviews the stress testing results for the October 1987 Market Crash scenario and determines that these are unacceptable. As a result, Blackhawk proposes the following two approaches to manage the risks associated with severe equity market declines:
  - (i) De-risk the product portfolio by changing product design
  - (ii) Hold sufficient capital

Describe the consequences for each of (i) and (ii).

2. (11 points) Consider a portfolio comprised of two stocks, XYZ and ABC. Each stock price follows geometric Brownian motion with parameters:

Stock	Mean Annual Return ( $\mu$ )	Annual Volatility ( $\sigma$ )	Market Value ( $S_0$ )
XYZ	5%	15%	100
ABC	10%	20%	150

These two stocks are assumed to be uncorrelated during normal market conditions; however, when the market is performing poorly the stocks are positively correlated.

The variables  $n_1$  and  $n_2$  are independent and randomly simulated standard normal variables used to simulate the annual returns of holding companies XYZ and ABC, respectively.

The tail dependency of the returns is reflected as follows:

- When  $n_1 \leq -1.96$ , the correlation coefficient  $\rho = 0.8$
- When  $n_1 > -1.96$ , the correlation coefficient  $\rho = 0$

You have generated 21 simulations and ordered (highest portfolio value to lowest) the simulated values as follows:

**Table 1: Ordered Simulations**

Simulation	$n_1$	$n_2$
1	0.64	1.29
2	0.94	1.11
3	1.15	0.88
...	...	...
19	-1.12	-1.05
20	-1.65	-0.95
21	-2.71	-0.75

You are given:

$$S_t = S_0 e^{\left(\mu - \frac{\sigma^2}{2}\right)t + \varepsilon\sigma\sqrt{t}}$$

## 2. Continued

- (a) (1.5 points) Given the following formula:

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \rho & (1-\rho^2)^{1/2} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

- (i) Solve for  $\varepsilon_1$  and  $\varepsilon_2$ .
- (ii) Explain the purpose of this formula and how it works.
- (b) (3 points) Determine the one year VaR(95%) for the portfolio value. Show your work.
- (c) (2 points) Evaluate in qualitative terms (without performing additional calculations) the effect that each of the following changes would have on the VaR(95%) of the portfolio.
- (i) The correlation coefficient increases from 0.8 to 0.9 when  $n_1 \leq -1.96$
- (ii) Tail dependency is reflected when  $n_1 \leq -2.65$  rather than when  $n_1 \leq -1.96$
- (d) (3 points) Explain the steps required to determine VaR(95%) for the portfolio value using the antithetic variable reduction technique.
- (e) (1.5 points) Describe two additional variance reduction techniques and evaluate whether each is appropriate to use when modeling correlated equities.

**3.** (12 points) You are a risk management consultant and have been hired by Wrigley Financial to assist in the selection of a risk measure for Wrigley to use in its economic capital calculations. You are considering the following risk measures:

- $\rho_1(X) = 95^{\text{th}}$  percentile of  $X$
- $\rho_2(X) = E[X | X > \rho_1(X)]$
- $\rho_3(X) = \frac{1}{\alpha} \log E[e^{\alpha X}]$ , for  $0 < \alpha$

Where  $X$  represents a loss random variable.

- (a) (4 points) Explain the benefits of satisfying each of the following properties when selecting a risk measure in the context of setting economic capital:
- (i) Positive Homogeneity
  - (ii) Monotonicity
  - (iii) Translation Invariance
  - (iv) Subadditivity
- (b) (3 points) Demonstrate whether the following properties are satisfied by the associated risk measures:
- (i) Positive Homogeneity -  $\rho_1(X)$
  - (ii) Monotonicity -  $\rho_1(X)$
  - (iii) Translation Invariance -  $\rho_2(X)$
- (c) (4 points) Determine which of the criteria in (a) are satisfied by  $\rho_3(X)$ . Show your work.
- (d) (1 point) Recommend a risk measure for Wrigley to use in its economic capital calculations, from the three proposed. Justify your answer.



**THIS PAGE INTENTIONALLY LEFT BLANK**

4. (11 points) You are the marketing actuary for Hamsik Re. E&J Life Insurance Company is a client of Hamsik for traditional reinsurance but has not used financial reinsurance in the past.

E&J has an inforce block of traditional whole life policies issued between 1978 and 1984, having \$2 billion in face amount and \$512 million in statutory reserves (the “whole life block”). The assets backing this whole life block are duration matched to the liabilities and have an average maturity of 11 years. E&J manages the mortality risk on this block using YRT reinsurance.

Hamsik and E&J are contemplating entering into a reinsurance agreement effective January 1, 2014. Under this agreement Hamsik would assume the whole life block on a coinsurance with funds withheld basis.

The following projections are provided for the whole life block:

**Cash Flows and Inventory Items for the Whole Life Block Including YRT Reinsurance But Prior to Coinsurance with Funds Withheld Reinsurance**  
(in USD Millions)

Year	Premium (Net of YRT Premium)	Investment Income	Benefits	Increase in Statutory Reserve	Dividends*	Pre-tax Income	End of Year Statutory Reserves	Gross Face Amount	YRT Reinsured Face Amount
2014	50	31	23	(1)	20	39	511	1,968	1,669
2015	49	31	23	(2)	20	37	509	1,869	1,576
2016	47	31	23	(2)	20	36	507	1,776	1,499
2017	46	30	23	(2)	20	35	505	1,687	1,428
2018	44	30	23	(3)	20	34	502	1,603	1,348
2019	43	30	23	(3)	20	33	499	1,523	1,283
2020	42	30	22	(3)	20	32	496	1,446	1,223
2021	40	30	22	(3)	20	31	493	1,374	1,167
2022	39	30	22	(4)	20	30	489	1,305	1,106
2023	38	29	22	(4)	20	29	486	1,240	1,041

\* Dividends are paid annually and serve to return to the policyholders a portion of any positive variance in mortality or investment experience.

Under the proposed agreement, Hamsik would assess an annual risk charge of 100 basis points of reinsured reserves. This charge is scheduled to increase to 500 basis points on January 1, 2019, at which point it is assumed E&J will exercise its recapture rights as allowed for under the terms of the agreement.

Both Hamsik and E&J are domiciled in the same regulatory jurisdiction.

#### 4. Continued

- (a) (1 point) Provide arguments supporting the contention that the proposed agreement qualifies as a financial reinsurance transaction.
- (b) (1 point) Identify the characteristics of the whole life block that make it a good candidate for a financial reinsurance transaction.
- (c) (1 point) Identify and explain two benefits, one statutory and one economic, to E&J of entering into this transaction.
- (d) (1 point) Explain the benefits to both E&J and Hamsik of having this transaction be on a funds withheld basis as opposed to a coinsurance basis.
- (e) (4 points) Explain the importance of the following treaty provisions to E&J, and propose a formula to determine each for this proposed treaty.
  - (i) Outstanding Surplus Account (OSA)
  - (ii) Experience Refund (ER)
- (f) (2 points) E&J has indicated that it is interested in obtaining some amount of statutory reserve relief.

Provide an estimated range for the amount of statutory reserve relief Hamsik would be willing to provide under the terms of this agreement. Justify your estimate and explain the process that Hamsik might follow in order to determine how much relief it is comfortable providing under the terms of this agreement.

- (g) (1 point) The pricing manager at Hamsik proposes increasing the risk charge to 1,000 basis points (instead of 500 basis points) at the end of five years to afford Hamsik increased protection against extremely adverse experience.

Propose a response to the pricing manager.

**5.** (8 points) Millennium Life is a publically traded U.S. life insurance company that has performed consistently since its inception in 1913. Millennium set up an ERM department in 2003 and developed the following Risk Appetite Statement (RAS) at that time:

- I. Excess Capital (measured on a statutory basis) shall not decrease by more than 20% in a 1 in 200 year event.
- II. Return on statutory capital (ROC) shall not fall below 5% per annum in any year.
- III. S&P financial strength rating shall not decrease below current AA level.

Millennium applies components I and II of this RAS to each line of business (LOB) to ensure that Millennium meets these goals in aggregate. For this purpose, all investable assets are allocated to the lines of business.

- (a) (1 point) Define Risk Appetite.
- (b) (2 points) Explain the rationale for including each of the metrics in Millenium's RAS and how they individually and collectively affect business decisions.
- (c) (2 points) Explain how the following stakeholders may view the existing RAS:
  - (i) Millennium's Pricing Department
  - (ii) Regulators
  - (iii) Shareholders
  - (iv) Debt Holders
- (d) (1 point) Explain the implications of Millennium's approach to managing to its RAS by LOB.
- (e) (2 points) During the 2008 financial crisis, Excess Capital dropped by 25% and ROC was -1%, but the risk limits have been consistently met since 2010.

Explain how Millennium's ERM department should have viewed this failure to satisfy the RAS and what actions, if any, should have been taken.

**THIS PAGE INTENTIONALLY LEFT BLANK**

- 6.** (10 points) Dearbourne Corporation is an innovative electronics corporation, bringing customers the latest in today's best computerized gadgets. Dearbourne has suffered losses recently due to both operational failures and emerging competition.

Dearbourne has only two divisions:

Division A: Manufacturing and quality control of product

Division B: Research and Development of new electronic technologies

- (a) (2 points) Currently Dearbourne uses the following practices to identify its top operational risks:

- I. Survey managers
- II. Use external data – i.e., Expert/Industry surveys
- III. Use internal data

Identify the potential shortcomings with each of these practices.

Dearbourne is in the process of completing its next manager operational risk survey. These surveys are completed by individually interviewing selected managers.

- (b) (1.5 points) Outline the objectives you intend to achieve in conducting these surveys.

- (c) (3 points) The following managers are among those selected to be surveyed.

- I. Building Security Manager
- II. Chief Information Officer
- III. Chief Officer of Quality Control

Propose key survey questions specific to each manager.

## 6. Continued

Dearbourne thinks environmental scanning would help identify emerging competitive risks and opportunities. One evolving issue Dearbourne is monitoring is the emergence of wearable technology (clothing and accessories incorporating computer and other electronic technologies).

- (d) (2.5 points)
  - (i) Explain how a conditional viewing on the evolution of wearable technology could help identify Dearbourne's competitive risks.
  - (ii) Provide four specific examples of information that Dearbourne might obtain by using this approach.
- (e) (1 point) Explain how undirected viewing of the current technology environment could help identify Dearbourne's competitive risks.

**Questions 7 – 8 pertain to the Case Study.**  
**Each question should be answered independently.**

7. (12 points) You are developing a VaR model to measure the interest rate risk exposure in SLIC’s bond portfolios. As the full variance-covariance approach is impracticable, you are assessing the following two multivariate model alternatives:

- A one-factor duration model, whereby:

$$(dP / P) = -D^* (dy)$$

$$VaR(dP / P) = |D^*| VaR(dy)$$

- A two-factor Principal Component Analysis (PCA) model, whereby:

$$\beta_{i_p} = x' \beta_i$$

$$\sigma^2(R_p) = \sum (\beta_{i_p})^2 \sigma^2(Z_i)$$

In order to evaluate the two models, you prepare an analysis using a simplified \$100 million bond portfolio invested in 1-year, 10-year and 30-year zero coupon Treasuries. To develop the portfolio weights, you have mapped SLIC’s actual bond holdings to each of the three Treasury maturities.

You have gathered the following data for your simplified portfolio:

Maturity	Portfolio Weight	Yield	Yield Volatility
1	45%	0.8%	0.31%
10	50%	2.3%	0.27%
30	5%	2.0%	0.24%

- (a) (1 point) Describe the simplifying assumptions inherent in SLIC’s use of the one-factor duration model for measuring interest rate risk.
- (b) (2 points) Assume a normal distribution of bond yields for your \$100 million portfolio and a portfolio yield volatility of 0.28%.

Calculate the following using the one-factor duration model:

- (i) Yield VaR(95%)
- (ii) Returns VaR(95%)

Show your work.



**Questions 7 – 8 pertain to the Case Study.**  
**Each question should be answered independently.**

**7. Continued**

You have derived the following preliminary results for the simplified portfolio in your two-factor Principal Component analysis:

Maturity	Eigenvectors	
	$\beta_1$	$\beta_2$
1	0.56	0.83
10	0.59	-0.34
30	0.58	-0.45
Eigenvalue	2.832	0.163

The correlation matrices fitted by the first and second component, respectively, are as follows:

1st Component	1	10	30
1	0.889		
10	0.932	0.978	
30	0.926	0.971	0.965

2nd Component	1	10	30
1	0.111		
10	-0.046	0.019	
30	-0.060	0.025	0.033

- (c) (1 point)
- (i) Determine the percentage of variation explained by each of the two principal component factors for each of the zero coupon Treasuries.
  - (ii) Determine the percentage of variation explained in total for each of the zero coupon Treasuries.
- (d) (2 points) Explain what aspect of interest rate risk exposure appears to be captured by each of the two principal components in your analysis.

*Question 7 is continued on the next page*

*Questions 7 – 8 pertain to the Case Study.  
Each question should be answered independently.*

**7. Continued**

- (e) (2 points) You have calculated the following dollar exposures for the three zero-coupon Treasuries:

Maturity	Exposure (in USD Millions)
1	0.14
10	1.32
30	0.35

Determine the portfolio's exposure to each of the two Principal Components. Show your work.

- (f) (2 points) Determine the portfolio Return VaR(95%) under your two-factor PCA model. Show your work.
- (g) (2 points) Compare and explain the difference in VaR results between the one-factor duration model and the two-factor PCA model.

**Questions 7 – 8 pertain to the Case Study.**  
**Each question should be answered independently.**

- 8.** (8 points) As SLIC's CFO role is still vacant, Janis Baer, VP, Human Resources is currently the only trustee managing the SLIC defined benefit pension plan (DB Plan) on a day-to-day basis. The DB Plan's investment strategy is currently due for a review.

You are an Investment Actuary supporting a wide range of SLIC's ALM efforts. Janis has asked for your assistance with the DB Plan review.

Plan advisors are recommending a change from the Plan's current 60% equity / 40% fixed income allocation to an 80% equity / 20% fixed income allocation, just below the policy equity allocation maximum.

You suggest that increasing the allocation to domestic fixed income is a more appropriate strategy for SLIC as a taxable corporate sponsor of this DB Plan.

- (a) (2 points) Identify three key stakeholders with an interest in the investment management policy for SLIC's DB Plan and explain each of their perspectives.
- (b) (1 point) Explain the traditional arguments for large allocations to equities in DB plans that would support the Plan advisors' recommendation.
- (c) (2 points) Counter the traditional arguments in support of high equity allocations in DB plans by identifying and explaining two specific aspects of SLIC's DB Plan which are not consistent with these arguments.
- (d) (3 points) You argue that SLIC should take full advantage of the tax-exempt status of the DB Plan, as SLIC's shareholders prefer investing in financial assets themselves, and having SLIC management focus on investing in SLIC's business.

To illustrate the shareholder perspective, you assume the following:

- 35% corporate tax rate,  $t_c$
- 40% personal income tax rate on bonds,  $t_{pb}$
- 15% personal income tax rate on stocks,  $t_{ps}$
- 7% return on bonds,  $r_b$
- 12% return on stocks,  $r_s$

Demonstrate and explain the Tepper tax arbitrage if SLIC's DB Plan shifts \$1 of assets from equities to bonds while SLIC's shareholders shift their own portfolios to maintain their equity exposure. Show your work.

**\*\*END OF EXAMINATION\*\***

**USE THIS PAGE FOR YOUR SCRATCH WORK**