FETE Model Solutions
Fall 2012

1. Learning Objectives:

1. Modern Corporate Financial Theory

Learning Outcomes:

(1b) Calculate the cost of capital for a venture or a firm using the most appropriate method for given circumstances and justify the choice of method.

(1c) Evaluate various profitability measures including IRR, NPV and ROE, etc.

Sources:

Financial Theory and Corporate Policy, Copeland, Weston, Shastri

- Chapter 2 Investment Decisions: The Certainty Case
- Chapter 15 Capital Structure and the Cost of Capital: Theory and Evidence

Commentary on Question:

This question tested the candidate's comprehension of cost of capital and profitability measures and the candidate's ability to determine whether a company should pursue the project given the NPV and Cost of Capital methods.

Candidates receiving maximum credit showed all of the formulas used, provided explanations of the NPV and cost of capital methods, and provided the rationale for whether each of the companies would accept or reject the project.

Solution:

(a)

(i) Explain why Colonial Creek and Johannesburg have the same systematic risk β_U of net operating cash flows;

(ii) Determine the value of β_U ;

(iii) Calculate the WACC for Johannesburg.

Commentary on Question:

Candidates did relatively poorly on this section. Many candidates did not adequately explain why the two companies had the same systematic risk. To do so they needed to explain that the cashflows of each company were proportional to each other. Most candidates failed to recognize that the weighted average cost of capital to use in the cost of capital method was to be calculated based upon the risk of the proposed project and the levels of leverage each company had.

1. Continued

However, many candidates correctly recognized that the β_U that was given was for the proposed project and was not the one that was requested in part (a)(ii). Candidates who had an incorrect part (a)(ii) answer were not penalized if they used it in part (a)(iii).

(i) Since Colonial Creek and Johannesburg have net operating cash flows which are proportional to each other, the cash flows differ only by a scale factor, so $CF_{cc} = \lambda * CF_J$
Therefore the expected returns for each company would be the same.
$R_{t,J} = (\lambda * CF_{t,J} - \lambda * CF_{t-1,J}) / (\lambda * CF_{j,J}) = (CF_{t,CC} - CF_{t-1,CC}) / CF_{j,CC} = R_{t,CC}$
$ß_U$ is the ratio of the covariance between the returns of the company and of the market to the variance of the market. $ß_U = Cov(R, R_m) / Var(R_m)$,
Given that the returns for each company are the same, the $ß_U$ must then also be the same.

(ii) Calculation of shared $ß_U$
Using $WACC = \rho (1 - \tau_c B\%) \Rightarrow \rho = WACC / (1 - \tau_c B\%)$ and
$\rho = R_f + (E(R_m) - R_f) ß_U \Rightarrow ß_U = (\rho - R_f) / (E(R_m) - R_f)$
Given was $\tau_c = 35\%$, B% (for CC) = 20%, WACC (for CC) = 9.96%
From the 1st equation $\rho = 9.96\% / (1 - 35\% * 20\%) = 10.71\%$
Substituted to the 2nd equation $ß_U = (10.71\% - 4\%)/(10\% - 4\%) = 1.12$

(iii) Calculation of weighted average cost of capital for Johannesburg:
Given B% (for J) = 40%, $R_f = 4\%$, $E(R_m) = 10\%$
Using $ß_U = 1.12$ from (ii)
CAPM Method:
The levered equity beta $ß_L = (1 + (1 - \tau_c) B\%/(1-B\%)) ß_U$
$= (1 + (1 - 35\%) (40/60)) (1.12) = 1.60$ or 1.61
Cost of levered equity $k_s = R_f + (E(R_m) - R_f) ß_L$
$= 4\% + (10\% - 4\%) (1.6) = 13.62\%$
$WACC = (1 - \tau_c) R_f B\% + k_s (1 - B\%)$
$= (1 - 35\%) (4\%)(40\%) + (13.62\%)(1 - 40\%) = 9.21\%$

M-M Method:
Given that companies have same $ß_U$, the ρ will be the same for both companies.
$\rho = 10.71\%$ from (ii)
$WACC = \rho (1 - \tau_c B\%)$
$= (10.71\%)(1 - 35\% * 40\%) = 9.21\%$

1. Continued

(b) Determine whether the project should be accepted by each of Colonial Creek and Johannesburg:

(i) By the NPV criterion;

(ii) By a cost of capital analysis.

Commentary on Question:
Candidates did well on this section overall. This section required the candidates to do a NPV calculation and then understand that negative NPV projects are not to be undertaken. Under the Cost of Capital approach the candidates simply had to know that a project's IRR had to exceed the project's WACC for each company to be undertaken. Candidates were required to use information from part (a) to solve part (b) however they were not penalized if the numbers carried forward were incorrect if the prior information was used correctly and correct conclusions were drawn off of that information. Candidates were penalized if they did not demonstrate knowledge of the methods.

(i) The NPV is calculated by discounting the projected cash flows of the project by the weighted average cost of capital for each company. Companies would then accept only projects when the NPV is positive.

For Colonial Creek:
$NPV = 2.5mm/(1+9.96\%) + 4.7mm/(1+9.96\%)^2 + 5.0mm/(1+9.96\%)^3 – 10.0mm = -78,647$
Since $–78,647 < 0$, the project is not acceptable to Colonial Creek

For Johannesburg:
$NPV = 2.5mm/(1+9.21\%) + 4.7mm/(1+9.21\%)^2 + 5.0mm/(1+9.21\%)^3 – 10.0mm = 68,488$
Since $68,488 > 0$, the project is acceptable to Johannesburg

(ii) Under the cost of capital method, the weighted average cost of capital for each company is determined as a function of the riskiness of the project and the amount of leverage of the company
This is then compared to the project's earn rate, which is the IRR, and the project is accepted only if the IRR is greater than the determined cost of capital.

The IRR was given as 9.56%

The required rate of return on the project if unlevered
$E(R_j) = R_f + (E(R_m) - R_f)\, ß_{U\,p}$
$ß_{U\,p} = 1.2$ was given

1. Continued

$$E(R_j) = 4\% + (10\% - 4\%)\,(1.2) = 11.2\% = \rho$$

To Colonial Creek:

The project's WACC = $\rho\,(1 - \tau_c\, B\%)$
= $(11.2\%)(1 - 35\% * 20\%) = 10.42\%$
Since IRR = 9.56% < WACC = 10.42%, the project is not acceptable to Colonial Creek

To Johannesburg:

The project's WACC = $\rho\,(1 - \tau_c\, B\%)$
= $(11.2\%)(1 - 35\% * 40\%) = 9.63\%$
Since IRR = 9.56% < WACC = 9.63%, the project is not acceptable to Johannesburg

2. Learning Objectives:

2. Corporate Financial Applications

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

Learning Outcomes:

(2a) Describe the steps necessary to obtain funds for a given project or firm from any specified source, and be able to recommend a specific approach to raising capital in a given situation.

(4e) Define principal-agency theory and explain how it affects capital structure, portfolio management and risk management.

Sources:

Doherty, Integrated Risk Management, Ch. 13: Contingent Leverage Strategies and Hybrid Debt, [FET-108-07]

Commentary on Question:

This question is designed to test student's understanding of principal-agency theory as well as its implications on different approaches to raising capital in a given situation.

Solution:

(a) As the new CFO you are concerned that two classic agency problems, underinvestment and asset substitution, could arise because of the planned bond issuance. Explain reasons for your concern.

Commentary on Question:

The candidates did well on this section. Most candidates demonstrated a good understanding of asset substitution however fewer candidates demonstrated a good understanding of underinvestment.

Shareholders obtain an embedded default put option from issuing regular corporate bond.

The increase in value of the default put option as the risk increases benefits the shareholders at the cost of bondholders.

Asset Substitution: With regular debt the firm will be tempted to ignore downside risk and pursue higher-risk investment projects after issuing bonds. Beyond the point at which shares become worthless, all downside risk falls on the bondholders, whereas shareholders will benefit from upside gains by pursuing higher-risk investment projects.

2. Continued

Underinvestment: Anticipating the ex-post asset substation actions from company/shareholders, potential investors will not command a sufficient price to buy the bond, which in turn prevent company from funding positive NPV projects.

Underinvestment also arose from the default put option on existing debt. Shareholders are cushioned from the full loss of value by defaulting on existing debt. Shareholders may reject some positive NPV project if value of the project accrues partly to creditors.

(b) You are considering three funding methods:

1. A bond with call option under which the issuer can buy back the bond at the face value (Callable bond)

2. A bond under which bondholders have option to convert bond to certain number of company shares (Convertible bond)

3. A bond under which company has option to convert bond to certain number of company shares (Reverse convertible bond)

Assess each funding method as it relates to underinvestment and asset substitution.

Commentary on Question:
The candidates did well on this section. Most candidates were able to illustrate the implications of agency problems on different approaches to raise capital to fund a project for both the convertible bond and the reverse-convertible bond. Some candidates were able to describe the implications on the callable bond.

Callable Bond:
Call option held by the company will mitigate asset substitution problem, which in turn will mitigate underinvestment problem.

Call option works in the opposite direction of the implicit default put option. Higher-risk projects will reduce the credit standing of the firm thereby reducing bond value, which in turn will reduce the value of the call option, and vice versa. Thus, the call option will reward (penalize) shareholders for decisions that reduce (increase) firm risk.

2. Continued

Convertible Bond:
Conversion option dampens risk sensitivity of both bondholders and shareholders. Risk effect from default option held by shareholders will offset conversion option held by bondholders. What bondholders lose on the default put option with an increase in risk that comes from asset substitution, they largely get back through an increase in the value of the conversion option. Resultantly, both asset substitution due to shareholders' incentive to take higher risk and underinvestment due to bondholders' unwillingness to commit to buy bonds are mitigated.

Reverse Convertible Bond:
Reverse conversion option is a contingent leverage instrument that is contingent upon the share price falling sufficiently that the company will choose to unlevel. In this way shareholders do not get to walk away from all downside risk; rather, equity is diluted by exercise of the option and the original shareholders share the downside with bondholders. Bondholders are also better off as default put option is replaced with smaller-value reverse conversion option, and bankruptcy costs are avoided as well. Asset substitution and underinvestment problems will be solved if conversion ratio is set appropriately.

(c) Recommend a Project and the appropriate funding method which maximizes the Ex Post value of the firm based on the above information.

Commentary on Question:
The candidates did relatively poorly on this section. Many candidates knew the steps to calculate the equity value under different capital raising approaches. Common mistakes included: 1) option prices were not properly implemented (wrong place, wrong direction, or not at all) into the calculation; 2) Equity value was not properly utilized as the reason for a recommendation; 3) Candidates did not provide a recommendation.

2. Continued

callable bond						
					Ex Post	
value of the firm if project A is chosen		525.0		625.0	575.0	
senior debt		50.0		50.0	50.0	
new debt (will be called if value higher following project success)		157.5		157.5	157.5	
equity (existing shareholders)		317.5		417.5	367.5	
value of the firm if project B is chosen	345.0	575.0	445.0	675.0	510.0	
senior debt	50.0	50.0	50.0	50.0	50.0	
new debt (will be called if value higher following project success)	157.5	157.5	157.5	157.5	157.5	
equity	137.5	367.5	237.5	467.5	302.5	
Both project A and B are acceptable for Bondholders						
Shareholders will choose A over B to maximize equity value						
Convertible Bond	Convert if 1/3 of equity > 150			=>	firm value >	500
value of the firm if project A is chosen		525.0		625.0	575.0	
senior debt		50.0		50.0	50.0	
new debt (will be converted to equity if there is gain for bondholders)		153.3		186.7	170.0	
equity (existing shareholders)		321.7		388.3	355.0	
value of the firm if project B is chosen	345.0	575.0	445.0	675.0	510.0	
senior debt	50.0	50.0	50.0	50.0	50.0	
new debt (will be converted to equity if there is gain for bondholders)	145.0	170.0	145.0	203.3	165.8	
equity (existing shareholders)	150.0	355.0	250.0	421.7	294.2	
Shareholder will choose A, Bondholder, although perfer B, can accept A.						
Shareholders will choose A over B to maximize equity value						
Reverse Convertible Bond	Convert if 1/3 of equity < 150			=>	firm value <	500
value of the firm if project A is chosen		525.0		625.0	575.0	
senior debt		50.0		50.0	50.0	
new debt (will be converted to equity if there is gain for shareholders)		155.0		155.0	155.0	
equity (existing shareholders)		320.0		420.0	370.0	
value of the firm if project B is chosen	345.0	575.0	445.0	675.0	510.0	
senior debt	50.0	50.0	50.0	50.0	50.0	
new debt (will be converted to equity if there is gain for shareholders)	103.3	155.0	136.7	155.0	137.5	
equity (existing shareholders)	191.7	370.0	258.3	470.0	322.5	
Project B is not acceptable to bondholder, but knowing that shareholder will choose project A after issuing, bondholder is willing to commit.						
Shareholders will choose A over B to maximize equity value						
Conclusion:						
recommend reverse convertible bond as funding method. And project A will be choosen following bond issuance.						
It gives shareholders maximum gain while acceptable by bond investors.						

3. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3c) Identify embedded options in assets and liabilities.

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

(3g) Identify limitations of each option pricing technique.

Sources:

Hardy, Investment Guarantees

- Chapter 2 Modeling Long-Term Stock Returns
- Chapter 8 Dynamic Hedging for Separate Account Guarantees

Hull, OFOD, The Greek Letters, Chapter 17 (7^{th} Edition)

Commentary on Question:

This question tests understanding of dynamic hedging of a GMMB liability. The pricing of the GMMB liability is crucial to the pricing and hedging of the option which is very similar to the Black-Scholes pricing formula. Understanding the calculation of the hedge instrument position for hedging a GMMB contract and the limitations of delta hedging is important.

Solution:

(a) Calculate the total value of the Separate Account as of today.

Commentary on Question:

The candidates did relatively well on this section. A common mistake was the interpretation of the "separate account" as the value of each policyholder's portion of the aggregate account.

Separate Account = Principal * Survival(.25) * (1+ [rate of return]*.25) * (1- quarterly Fee)

= 10b x .98202 x (1- .03 x .25) x (1 - .005) = 9.698b

(b) Calculate the option value of the GMMB block as of today.

Commentary on Question:

The candidates did relatively well on this section. Common mistakes were:

1) The relationship between account value and guarantee at t=.25 were not consistent with respect to lapses.
2) Ignoring Policy holder survivability to the maturity of the contract.

3. Continued

F(0.25) =10b x (1- 0.03 * .25) x (1-0.005) = 9.875b
G = 0.9 x 10b = 9b
T = 5 - .25 = 4.75
r = .02
vol = .25
d1 = (ln(F(0.25) x (1-m)^(4 x T)/G) + (r + vol^2 / 2) x T) /(vol x T^0.5)
= (ln(9.875b x (.995)^(19) / 9b) + (.02 + .25^2 / 2) x 4.75) /(.25 x 4.75^0.5)
d2 = d1 – vol * T^0.5 = -0.10258
N(-d1) = 0.3291
N(-d2)= 0.5408

P(.25) = G x exp(-r x T) x N(-d2) – F(.25) x (1-m)^(4 x T) x N(-d1)
= 9b x exp(-.02 x 4.75) x .5408 – 9.875b x (1-0.0050)^(4 x 4.75) x .3291 = 1.47b

Adjust for lapses
= 1.47b * .69568 = 1.02b

(c) Calculate the number of three-month futures contracts required to delta hedge the GMMB.

Commentary on Question:
The candidates did poorly on this section. Many candidates were able to calculate delta of the future, however few were able to calculate the dollar delta of the liability. Few candidates declared the position should be short. Virtually all candidates ignored the survivability multiplier for the delta of the put.

Delta of the long put
–F(.25) x (1-m)^(4 x T) x N(-d1) * Surv Prob
= - 9.875b x (1-0.0050)^(4 x 4.75) x .3291 x .69568 = -2.06b
The insurance company is short the put and long $2.06b delta
The insurance company needs to short futures

The delta of a future is exp((r-q)*T)
T = 0.25, r = 2%, q =2.5%
Delta of a Future = exp((.02-.025) * .25) = 0.9988

Hence, insurance company needs to sell $2.1b/(1300*.9988*50)) = 31730.38
which is 31,730 contracts

3. Continued

(d) Calculate what the margin offset would be as of today if the guarantee were to be repriced.

Commentary on Question:
The candidates did poorly on this section. Most candidates put the formula down, but few were able to apply the formula.

alpha = B /(S(0.25) * Annuity Factor
Quarterly decrement = 1- (1-.07)^(1/4)) = 1.798%
SurvProb(t in quarters) = (1-quarterly decrement)^t = 0.9820^t
Annuity Factor = sum(t=0 to 4.75*4 -1 of (1-m)^t * SurvProb(t))=sum(t=0 to 18 of (1-m)^t * SurvProb(t))
Hence, annuity factor = sum(t=0 to 18 of (0.995*0.9820)^t) = sum(t=0 to 18 of (0.97709)^t)
This is geometric series = (1 - .97709^19)/(1-.97709) = 15.55

So, alpha = 1.02b / (9.652b *15.55) = 0.68%

(e) Discuss the sufficiency of the Quarterly Charges to finance the GMMB.

Commentary on Question:
The candidates did poorly on this section, which was correlated with the poor performance on (d). Many left this blank and did not attempt an answer.

The quarterly charge of 50 bps is not sufficient to cover 68 bps of the margin charge of the guarantee.

(f) In the context of a delta-hedging strategy:

(i) Define the concept of hedging error.

(ii) Explain the impact on the hedging error of frequency of rebalancing.

Commentary on Question:
The candidates did relatively well on this section. This is a straightforward list from the text.

The change in the stock part of the hedge is not the same as the change in the bond part of the hedge. This difference is hedging error.
With discrete time gaps, between which hedge is not adjusted, the hedge may not be self-financing.
Discrete hedging error is introduced when trading is not done continuously.

4. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

Sources:

Hull, Options Futures & Derivatives

- Chapter 13 The Black Scholes Merton Model
- Chapter 17 The Greek Letters

Commentary on Question:

This question tests hedging concepts using the Greek letters.

Solution:

(a) Describe the strategy and explain its pros and cons.

Commentary on Question:

The candidates did relatively well on this section. Many candidates stated easy implementation and buy high sell low feature of the strategy. Some candidates listed high transaction costs however they only got full credit if they explained why the costs were high.

- The aim of the option writer is to be fully covered whenever the option is in the money and naked whenever it is out of the money.
- The option writer buys the stock whenever the stock price reaches the exercise price of $150 from below, and sells whenever the stock price reaches $150 from above.
- In practice, the points of execution would be at points slightly above and below the exercise price, e.g., buy at $150.25, and sell at $149.75.
- This strategy is easy to implement.
- The cost of the hedge strategy could be zero if the option remains out of the money, but it can also be very high if the execution points are reached many times before the option matures.
- Also, every purchase and subsequent sale involves a cost (apart from transaction costs) since the strategy calls for buying high and selling low.

(b) Estimate the implied volatility given the above option price.

Commentary on Question:

The candidates did extremely well on this section. This is an easy calculation to test the candidate's ability to calculate an implied volatility. Those that received partial marks didn't show sufficient work on how the final answer was derived through iteration.

4. Continued

$c = S_0N(d_1) - Ke^{-rT}N(d_2) = 25$
$S_0 = 145$
$K = 150$
$r = 0.02$
$T = 1.5$
Using an iterative procedure, the candidate will get the call option value closest to 25 when variance = 0.1296.
Implied volatility = square root (variance) = $\sqrt{0.1296} = 0.36$

(c) Calculate

(i) The new delta of the option written by the company.

(ii) The number of shares of the stock that need to be bought or sold to make the combined position delta neutral.

Commentary on Question:
The candidates did relatively well on this section. This question tests the candidate's ability to calculate a delta on an option and how this will translate into a hedging strategy. In order to get full marks, the candidate needed to clearly state the negative delta of the portfolio and state the need to purchase shares to neutralize the negative delta.

Most candidates correctly calculated the delta in part (c)(i). However in part (c)(ii) most candidates answered to sell the shares without understanding that the company writes options and the portfolio has a negative delta. To neutralize the position, the company needs to do a purchase of shares. Also, many candidates missed the 100 contracts.

(i)

$$d1 = \frac{\left[\ln\left(\frac{S0}{K}\right) + \left(r + \frac{\sigma 2}{2}\right)T\right]}{\left[\sigma\sqrt{T}\right]}$$

$S_0 = 146$
$K = 150$
$r = 0.02$
$\sigma = 0.32$

$d_1 = 0.2035$

$N(d1) = 0.58$

4. Continued

(ii)

250 contracts = 250*100 = 25,000 shares/options
A short position in 25,000 options has a delta of -.58 * 25,000 = -14,500.
The position can be made delta neutral with the purchase of 14,500 shares.

(d) Calculate the rho of the option.

Commentary on Question:
The candidates did extremely well on this section. This question tests the candidates understanding of how to calculate a rho.

Rho (call) = $KTe^{-rT} N(d_2)$
K = 150
r = 0.02
T = 1.5
$N(d_2)$ = 0.418 (from the table in part b at 32% vol)
Alternatively, $d_2 = d_1 - \sigma * \text{sqr}(1.5)$ and numerical answer with linear interpolation can be either $N(d_2)$ = 0.4247 OR 0.4253
Rho (call) = 91.27 (calculation) if 0.418
Alternatively, Rho= 92.733 if use $N(d_2)$ = 0.4247 OR 92.87 if $N(d_2)$ = 0.4253 (using linear interpolation)
Interpretation of Rho, using Rho equals 91.27 as an example. Rho of 91.27 means that a 1% increase in the risk-free rate increases the value of the option by approximately 0.01 * 91.27 = 0.9127

(e) Estimate the profit or loss to be realized if the Company closes the open position.

Commentary on Question:
The candidates did poorly on this section. This question attempted to test if the candidates could apply their knowledge of the Greeks to a real hedging situation. Common mistakes included: 1) incorrect usage on the decimals, e.g. 0.5 was used as opposed to 0.5%; 2) misstating profit to the company as loss.

Also, candidates need to clearly state that the option cost <u>decreased</u> when risk free rate decreased by 0.5% in order to receive full marks.

The risk free interest rate decreased by 0.5%.
Using the rho from part (d), this implies that option cost DECREASED by 0.005 * 91.27 = 0.4564
# of options = 250 * 100 = 25,000
The total cost of the option decreased by 0.4564 * 25,000 = 11,408.8 OR
11,591.6 if part c answer was 92.73 OR
11,608.75 if part c answer was 92.87

4. Continued

By closing the open position, the Unlimited Options Company will realize a profit of $11,408.8 OR 11,591.6 OR 11,608.75 depending on how part (d) was answered.

5. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3m) Describe issues and best practices in the estimation or calibration of financial models.

Sources:

Hull, Options Futures and Other derivatives, 8th Edition

- Chapter 22 Estimating Volatilities and Correlations

Hardy, Investment Guarantees

- Chapter 2 Modeling Long-Term Stock Returns

Commentary on Question:

This question tests understanding of the stochastic nature of autoregressive models particularly GARCH(1,1) mathematically.

Solution:

(a)

(i) Estimate the daily volatility, if S&P 500 index closed at 1440 today.

Commentary on Question:

Candidates generally did extremely well in this section. A small number of candidates confused ω with the long-term volatility V_L. Note that the daily volatility refers to the standard deviation (σ_n), not the variance (σ_n^2).

We are given that the S&P 500 was closed at 1400 yesterday.

$\mu_{n-1} = (1440 - 1400) / 1400$
$= 0.028571429 \ldots$

Then we have

$\sigma_n^2 = \omega + \alpha \mu_{n-1}^2 + \beta\sigma_{n-1}^2$
$= 0.00002 + 0.05 * (0.028571429 \ldots)^2 + 0.93 * (0.01)^2$
$= 0.000153816 \ldots$

Hence the daily volatility is

$\sigma_n = SQRT(0.000153816 \ldots) = 1.2402\%$ per day

5. Continued

(ii) Calculate the expected value of σ_n^2 at the end of five days from today.

Commentary on Question
Candidates generally did relatively well in this section. The who did not do well generally did not use the formula for $E[\sigma_{n+t}^2]$ correctly from the formula sheet. A frequent error was a confusion between V_L and ω. An occasional error was to use σ_n (as calculated in part (i)) instead of σ_n^2.

Recall that
$E[\sigma_{n+t}^2] = V_L + (\alpha + \beta)^t (\sigma_n^2 - V_L)$

We have
$V_L = \omega / (1 - \alpha - \beta)$
$= 0.00002 / (1 - 0.05 - 0.93)$
$= 0.001$

With t = 5, we get
$E[\sigma_{n+t}^2] = 0.001 + (0.05 + 0.93)^5 (0.000153816 - 0.001)$
$= 0.00023512$

(iii) Interpret parameter β in the GARCH(1,1) model.

Commentary on Question:
The candidates did well on this section. Most candidates recognized that β reflects historical volatility in the current volatility. Only a handful of candidates identified β as the "decay rate." Most candidates omitted the fact that the weight applied to μ_{n-i}^2 is $\alpha\beta^{i-1}$, which is declining exponentially at rateβ. Another common error was stating that β is the weight applied to the historical "stock return" rather than saying the weight is associated with historical volatility of the stock return.

The parameter β is the "decay rate". The weight applied to μ_{n-i}^2 is the weight $\alpha\beta^{i-1}$, which is declining exponentially at rate β.

Also, the β defines the relative importance of the observations on the μ's in determining the current variance rate. Showing the importance of the past volatility for the current volatility.

(iv) Determine whether the given GARCH(1,1) model is a stable model and justify your answer.

5. Continued

Commentary on Question:
The candidates did relatively well on this section. Most candidates correctly stated the model was stable, however many candidates did not justify why the model is a stable model. In order to obtain full credit, a valid justification must be given. Additionally, a few candidates assumed the incorrect condition $\alpha + \beta \leq 1$. The inequality should be "strictly less than 1" (< 1).

Yes, it is stable, because $\alpha + \beta = 0.05 + 0.93 = 0.98 < 1$.

(b) Compare autoregressive models (ARCH and GARCH) to models from the stable distribution family and recommend one for modeling stock returns.

Commentary on Question:
The candidates did relatively poorly on this section. Most candidates mixed up the pros and cons of the autoregressive models and stable family distributions models. In some cases they even contradicted themselves, suggesting that they did not fully understand the models. Some candidates confused the terms "autoregressive" and "autocorrelation." In addition, some candidates neglected the fact that the autocorrelation can be incorporated into ARCH and GARCH models by modifying the formula as shown below.

Autoregressive model: deviations from the long-term mean influence the distribution of subsequent values.

<u>ARCH</u>
It assumes volatility is not constant (or stochastic volatility).

It is designed to model volatility clustering (bunching).

At the beginning of each period, volatility is fixed but then it is determined again for following periods:

$Y_t = \mu + \sigma_t * \varepsilon_t$
$\sigma_t^2 = a_0 + a_1 * (Y_{t-1} - \mu)^2$

To allow for both volatility bunch and autocorrelation, the model can be modified to:

$Y_t = \mu + a * (Y_{t-1} - \mu) + \sigma_t * \varepsilon_t$
$\sigma_t^2 = a_0 + \alpha * (Y_{t-1} - \mu)^2$

5. Continued

GARCH(1, 1)
It is more flexible and better fit for many econometric applications than ARCH. It recognizes over time that the variance tends to get pulled back to a long-run average level of V_L (mean reverting).

$Y_t = \mu + \sigma_t * \varepsilon_t$
$\sigma_t^2 = \alpha_0 + \alpha_1 * (Y_{t-1} - \mu)^2 + \beta * \sigma_{t-1}^2$

Autocorrelation can be captured by using the following formula:

$Y_t = \mu + a * (Y_{t-1} - \mu) + \sigma_t * \varepsilon_t$
$\sigma_t^2 = \alpha_0 + \alpha_1 * (Y_{t-1} - \mu)^2 + \beta * \sigma_{t-1}^2$

Stable Family Distribution
It can be summarized by their characteristic function.

Pros and Cons of Stable Family Distribution
Pros:
Fat tails
Easy to combine as the sum of stable family distributions is another stable distribution (can convolute the distribution)

Cons:
Generally, it is not possible to describe in terms of distribution functions
(Parameter) estimation requires advanced techniques
Not easy to simulate (or to use)

Recommendations for modeling stock return (sample)
GARCH:
More flexible and provide a better model fitting than ARCH
Capture auto-correlation
More practical / easy to use than stable distribution family

6. Learning Objectives:

1. Modern Corporate Financial Theory

2. Corporate Financial Applications

Learning Outcomes:

(1c) Evaluate various profitability measures including IRR, NPV and ROE, etc.

(2e) Apply real options analysis to recommend and evaluate firm decisions on capital utilization.

Sources:

Financial Theory and Corporate Policy, Copeland, Weston, Shastri

- Chapter 9 Multi-period Capital Budgeting under Uncertainty: real Options Analysis

Commentary on Question:

This question tested the candidate's understanding of the differences between the NPV method and the ROA method to calculate the value of an option, both theoretically and practically.

Overall, candidates did relatively well on this question, and were consistent across all subsections.

Solution:

(a)

(i) Describe how NPV and ROA respond to the resolution of uncertainty in different ways;

(ii) Describe how DTA and ROA assume discount rates in different ways.

Commentary on Question:

This section attempted to test the candidate's knowledge of key differences of several option valuation methods. Overall the candidates did relatively well on this section.

For (a)(i), candidates received full credit if they correctly identified the treatment of cash flows, if decisions could be made in the future, and the law of one price (i.e., no-arbitrage principle). Most candidates mentioned that constant rates were applied in NPV/DTA and that the discount rate is adjusted for ROA. Also, most candidates outlined the difference in ability to make decisions in the future. However a smaller number of candidates went on to discuss the Law of One Price, No Arbitrage Principle, or the use of replicating portfolios for ROA.

6. Continued

For (a)(ii), candidates were given full credit if they specified which rate was used, and whether it changed at different nodes of the tree. Most candidates did well to describe the difference between the constant discount rate, i.e., the WACC used in DTA, and the risk-adjusted rate applied in ROA. A common mistake made was some candidates answered this from the NPV perspective, whereas the question referred to DTA

(i)

- NPA uses a constant rate to discount the expected cash flows
- NPA assumes no decisions can be made in the future, i.e., all cash flows are pre-committed
- ROA applies decision trees to model the optimal cash flows in the future
- ROA is consistent with the Law of One Price

(ii)

- DTA applies a constant discount rate throughout the tree
- ROA uses a risk-adjusted rate at each branch of the tree
- ROA applies replicating portfolios

(b) Calculate the value of

(i) Project 1 using DTA

(ii) Project 2 using ROA

Commentary on Question:
This section attempted to test the candidate's understanding of DTA and ROA through a calculation. Overall, the candidates did relatively well on this section. Most candidates seemed to have a better understanding of the ROA method and calculations. It was noted that there were several calculations required for part (b) and rounding differences may have caused answers to differ slightly from the solution below. Candidates were not penalized for these rounding differences and were given credit for correct formula usage, i.e., entering the correct amounts in the formulas, as well as correct answers.

For both (b)(i), and (b)(ii), the majority of candidates did not apply the possibility of exercising the contraction option at the point of initial investment. Additionally, some candidates incorrectly applied the WACC in (b)(ii) and the risk-free rate in (i), so were penalized more heavily for this error, which represented a fundamental difference between the two methods.

6. Continued

In Section (b)(i) many candidates did not show an understanding of the method and calculation. Many candidates were able to calculate the upside real-world probability correctly, however some incorrectly applied a time period of two years instead of one, and applied this throughout the nodes of the tree. Candidates would have received nearly full credit if their application of this rate was correct.

In section (b)(ii), most candidates answered this section sufficiently and were able to calculate the upside risk-neutral probability correctly. A common mistake made was to not apply the option to contract at the initial investment node.

(i)

p = the up side real-world probability
= $[(1+WACC)\ PV_{0,0} - PV_{1,0})] / (PV_{1,1} – PV_{1,0})$
= (1.12*125 - 88.086) / (177.383 - 88.086)
= 0.581; $1-p_1 = 0.419$

DTA value = Max (value with and without exercising the contraction option)

At maturity node (2, j): DTA2, j = max(PV2,j, PV2,j (1 - b) + c),
At other nodes (i, j): DTAi, j = max(DVi,j, PVi,j(1 - b) + c), where
where DVi,j = (p DTAi+1, j+1 + (1-p) DTAi+1, j) / (1 + WACC)

For Project 1
At maturity node (2, j):
DTA2, 2 = max(PV2,2, 0.6*PV2,2 + 50) = max(251.719, 0.6*251.719 + 50) = 251.719
DTA2, 1 = max(PV2,1, 0.6*PV2,1 + 50) = max(125, 0.6*125 + 50) = 125
DTA2, 0 = max(PV2,0, 0.6*PV2,0 + 50) = max(62.073, 0.6*62.073 + 50) = 87.244

At node (1, j):
DV1, 1 = (p DTA2, 2 + (1-p) DTA2,1) / (1 + WACC) = (0.581*251.719+ 0.419*125) / 1.12 = 177.343
DTA1, 1 = max(DV1,1, 0.6*PV1,1 + 50) = max(177.343, 0.6*177.383 + 50) = 177.343
DV1, 0= (p DTA2, 1 + (1-p) DTA2,0) / (1 + WACC) = (0.581*125+ 0.419*87.244) / 1.12 = 97.842
DTA1, 0 = max(DV1,0, 0.6*PV1,0 + 50) = max(97.842, 0.6*88.086 + 50) = 102.852

At node (0, 0):
DV0, 0 = (p DTA1, 1 + (1-p) DTA1,0) / (1 + WACC) = (0.581*177.343+ 0.419*102.852) / 1.12 = 130.474

6. Continued

DTA0, 0 = max(DV0,0, 0.6*PV0,0 + 50) = max(130.474, 0.6*125 + 50) = 130.474

Less the cost of the investment, the value is 130.474 – 100 = 30.474

(ii)
q = the up side risk-neutral probability
= $[(1+r)\ PV_{0,0} - PV_{1,0})] / (PV_{1,1} – PV_{1,0})$
= (1.02*125 - 97.35) / (160.503 - 97.35)
= 0.477; 1-q = 0.523

ROA value = Max (value with and without exercising the contraction option)

At maturity node (2, j): (similar to (b)(i))
ROA2, 2 = max(PV2,2, 0.6*PV2,2 + 54) = max(206.09, 0.6*206.09 + 54) = 206.090
ROA2, 1 = max(PV2,1, 0.6*PV2,1 + 54) = max(125, 0.6*125 + 54) = 129
ROA2, 0 = max(PV2,0, 0.6*PV2,0 + 54) = max(75.816, 0.6*75.816 + 54) = 99.490

At node (1, j):
RV1, 1 = (q ROA2, 2 + (1-q) ROA2,1) / (1 + r) = (0.477*206.09+ 0.523*129) / 1.02 = 162.522
ROA1, 1 = max(RV1,1, 0.6*PV1,1 + 54) = max(162.522, 0.6*160.503 + 54) = 162.522
RV1, 0 = (q ROA2, 1 + (1-q) ROA2,0) / (1 + r) = (0.477*129 + 0.523*99.49) / 1.02 = 111.399
ROA1, 0 = max(RV1,0, 0.6*PV1,0 + 54) = max(111.399, 0.6*97.35 + 54) = 112.410

At node (0, 0):
RV0, 0 = (q ROA1, 1 + (1-q) ROA1,0) / (1 + r) = (0.477*162.522 + 0.523*112.41) / 1.02 = 133.641
ROA0, 0 = max(RV0,0, 0.6*PV0,0 + 54) = max(133.641, 0.6*125 + 54) = 133.641

Less the cost of the investment, the value is 130.474 – 100 = 33.641

(c) Recommend and justify which project the company should invest in.

6. Continued

Commentary on Question:
The candidates did relatively poorly on this section. Nearly all candidates made a recommendation on a project, but few provided sufficient justification to attain full credit. Most candidates were able to recommend ROA over NPV analysis, but many did not describe the use of real-world vs. risk-neutral probabilities. If the candidates had a better understanding of this concept it would have enabled them to answer part (b)(i) more successfully.

The company should invest in the project with the greater ROA value because:

- The NPV valuation does not recognize the value of the contraction option.
- The DTA assumes the constant WACC of 12% throughout the tree.
- The DTA uses real-world probability in assessing the option value.
- The ROA uses risk-neutral probability in assessing the option value.

So, Project Value using ROA:

DTA(project 1) = 33.039
DTA(project 2) = 33.641

Project 2 is the winner project using the ROA valuation since it has greater va

7. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3m) Describe issues and best practices in the estimation or calibration of financial models.

Sources:

Variance of the CTE Estimator by Manistre and Hancock

Commentary on Question:

The question tests the candidates understanding of the variance of CTE, the variance reduction technique, and its practical application.

Solution:

(a) Calculate the CTE(99.5%) of the given results.

Commentary on Question:

The candidates did relatively well on this section. Full points were given for selecting the correct formula, determining the value for k (the number of scenario results to include in the arithmetic average), and performing the substitution.

Select appropriate CTE formula which is the arithmetic weighted average of the worst 5 scenarios. Determine k = 5 based on 1000 scenarios at CTE 99.5% (1000 X (1-0.995)) and substitute into the formula to obtain 1,370 as the CTE(99.5%).

$$CTE(\alpha) = \frac{1}{k}\sum_{i=1}^{k} x_i$$

CTE(99.5%) = (2,000 + 1,500 + 1,200 + 1,100 + 1,050)/5 =1,370

(b) Estimate the standard error of CTE(99.5%).

Commentary on Question:

Candidates did relatively well on this section. Full points were given for selecting the correct formula, determining $x_{(k)}$ = 1050, which is the best of the k=5 results from part (a) and performing the substitution. This part of the question was relatively straight forward. Candidates generally knew how to calculate the standard error but ended up many mistakes while getting to the end result. Common mistakes included: 1) using $x_{(k)}$ =1050 as CTE(99.5%) instead of the result obtained in part (a) which is 1370; 2) using 4 as the divisor instead of 5 or vice versa; 3) leaving off parts of the formula when the substitution was performed.

7. Continued

Select the variance formula to estimate the standard error of CTE and then calculate the variance of the CTE estimate.

$$VAR(CTE) \approx \frac{VAR(x_{(1)}, x_{(2)}, \ldots x_{(k)}) + \alpha.(CTE - x_{(k)})^2}{k}$$

Where $x_{(k)} = 1050$ and

$$VAR(x_{(1)}, x_{(2)} \ldots x_{(k)}) = \frac{(2,600-1,370)^2 + (1,500-1,370)^2 + (1,200-1,370)^2 + (1,100-1,370)^2 + (1,050-1,370)^2}{4} = 164,500$$

and then the variance of the estimate is

$$VAR(CTE99.5\%) \approx \frac{154,500 + 0.995 * (1370-1050)^2}{5} = 51,278$$

Finally, the standard error of the CTE is the square root of the VAR(CTE99.5%) or
STD (CTE99.5%) = (51,278)^0.5 =226

(c) Describe the Importance Sampling Technique and show how the CTE formula needs to be adjusted to account for this method.

Commentary on Question:
The candidates did poorly on this section. Most candidates indicated that this technique focuses on what is important in the distribution, meaning the worst case scenarios, but nothing further. Only a handful of candidates mentioned the change to the formula needed to reflect the new distribution and then calculating the new CTE estimate (a conditional probability type calculation).

The importance sampling technique focuses on the elements of the distribution that are worthy of study, in this case, the worst case scenarios or the tail of the distribution. The actual distribution of results F is replaced by a new distribution G so that more weight is placed on the tail. F and G are related as follows, dF = W*dG for an everywhere positive weight W. A random sample of size n is then drawn from the new distribution G.

The estimate for CTE is then determined by replacing dF with W*dG in the CTE estimate formula:

$$1 - \alpha = \frac{\int_{x_{(k)}}^{\infty} W d\hat{G}_n}{\int_{-\infty}^{\infty} W d\hat{G}_n}$$

7. Continued

$$\widehat{CTE}_n = \frac{\int_{(x_k)}^{\infty} x * Wd\hat{G}_n}{\int_{-\infty}^{\infty} Wd\hat{G}_n}$$

where x_k is the α-quantile.

(d) Assess the appropriateness of Importance Sampling Technique for this VA block modeling based on results in (a) and (b) above.

Commentary on Question:
Candidates did poorly on this section. The candidates were mixed with about 50% saying they thought this method was appropriate and the remaining either not knowing or saying it was inappropriate. The study note describes this technique as being able to improve the CTE estimate for all cases with varying degrees of success depending on the fatness of the tail. Also, very few candidates noted a practical point that the results could be improved further with a technique called "Importance Sampling with Stratification."

By looking to the results in (a) and (b), we can see that the first term of the variance estimate, 154,500/5 = 30,900 is much greater than the second term, 0.995* {(1,370-1,050)^2}/5=20,378. This is a simple way of determining if the distribution is heavy tailed (i.e. the option is in the money). Because of this heavy-tailed distribution, the importance sampling technique is less effective but still provides a result that is improved over the CTE estimate using the formula in (b). Furthermore, the approach can be improved by using Importance Sampling with Stratification.

(e) Describe the steps to implement the Control Portfolio approach.

Commentary on Question:
The candidates did poorly on this section. Most candidates confused this question with the control-variate technique. The study note referenced a very practical solution to limit the number of scenarios needed and this is called the control portfolio approach. Candidates received full credit if they simply listed the items in the study note.

There are 6 main steps to implementing this approach described below:

1) Construct a "Control Portfolio," which is a representative, suitably small portfolio that approximates the characteristics of the actual inforce business.
2) Value the Control Portfolio over a large number of scenarios N to get the best estimate of the true CTE, call it $\widehat{CTE}^{*}$

7. Continued

3) Suppose α_1 is the lowest level for which a CTE estimate is required and M is the maximum/target number of scenarios that the company wants to use (reasonable computation time and practical).
4) Select M scenarios from the N * (1 - α_1) scenarios for $\widehat{CTE}^{*}$, the average of the subset should then closely approximate $\widehat{CTE}^{I}(\alpha_k)$ calculated from the full set.
5) Then for each $\alpha_k \geq \alpha_1$ define an "adjustment factor,"

$$\varphi_k = \frac{\widehat{CTE}^{*}(\alpha_k)}{\widehat{CTE}^{I}(\alpha_k)}$$

$\widehat{CTE}^{I}(\alpha_k)$ is calculated by averaging the "worst" **control** results from the M scenarios $M \times \left(\frac{1-\alpha_k}{1-\alpha_1}\right)$ such that $\alpha_k \geq \alpha_1$ for all k.
6) Once you run the portfolio over the subset of M scenarios, the final CTE is determined as

$\widehat{CTE}(\alpha_k) = \varphi_k * \widehat{CTE}^{II}(\alpha_k)$ where $\widehat{CTE}^{II}(\alpha_k)$ is calculated by averaging the "worst" results from the M scenarios $M \times \left(\frac{1-\alpha_k}{1-\alpha_1}\right)$ such that $\alpha_k \geq \alpha_1$ for all k.

8. Learning Objectives:

1. Modern Corporate Financial Theory

Learning Outcomes:

(1i) Identify sources of agency costs and explain methods to address them.

Sources:

Financial Theory and Corporate Policy, Copeland, Weston, Shastri, 4th Edition, 2005

- Chapter 18 Acquisitions Divestitures Restructurings and Corporate governance, Pages 648-650

FET-166-09: Megginson, W. L., Corporate Finance Theory, Ch. 2: Ownership, Control, and Compensation, Page 76

Commentary on Question:

This question tested the candidates understanding of manager compensation plans. The candidates should be able to explain the weaknesses and strengths of different approaches and recommend the best approach for the situation described in the question.

Solution:

(a) Describe the strengths and weaknesses of each Plan:

- Plan 1 bases compensation solely on the company's growth in earnings during the last 12 months.

- Plan 2 bases compensation on the company's stock price performance relative to other companies in the industry.

Commentary on Question:

The candidates did well on this section. Most candidates stated that earnings based compensation was easy to implement because earnings are easily available. However, very few noted that it makes it easy to judge performance of business units because earnings are available by business unit. Most candidates also missed the idea that investor expectations are not a problem for earnings based compensation.

- Earnings results are generally available for individual business units of a company, allowing managers to be compensated on the performance of their unit.
- Earnings based compensation avoids the problem (faced by stock performance based compensation) of compensation being affected by investors expectations of management performance.
- Basing compensation on total earnings gives the manager incentive to increase the scale of the corporation even if doing so causes the company to take on negative net present value projects.

8. Continued

- Earnings that can be pulled from income statements are accounting numbers which include various adjustments and therefore might not provide a good indicator of performance.
- Stock based compensation motivates the manager to improve stock prices.
- Stock based compensation rewards managers on the company's performance relative to investor expectations.
- If investors had low expectations of the manager's performance, then mediocre performance will raise the stock price and increase compensation.
- However, if investor expectations are high and management performs at a high level, the stock price may not change, resulting in low compensation despite high management performance.
- Basing performance on stock price relative to other companies in the industry will prevent managers from being punished or rewarded for factors that are outside of their control.
- However relative performance based pay can also give a manager incentives to compete more aggressively for business since this will hurt other firms in the industry and improve his compensation.
- Both proposals do not include a base salary component.
- Lack of a fixed amount / base salary compensation component, could leave managers very fearful that a few bad quarters would lead to their financial ruin. It also may make the company's employment offer uncompetitive in the marketplace.
- Both proposals also fail to defer any compensation beyond the current year which is important to force managers to focus on longer term performance.

(b) Explain how a value based management plan could provide better incentives to the managers.

Commentary on Question:
The candidates did well on this section. Most candidates understood that the value based plan subtracts the cost of capital from the value of the cashflows, however most did not specify the cashflows should be economic as opposed to accounting cashflows. Very few candidates noted that value based compensation allows business units to be evaluated separately.

- Value based management transforms accounting cash flows to economic cash flows, which better measure performance and value created.
- The economic cash flows for a business unit are identified and used to judge managers of that unit.
- Value created by the managers is determined by subtracting from the economic cash flows a charge for the amount of capital employed by the unit.

8. Continued

- By appropriately adjusting for cost of capital and using economic cash flows, managers are compensated based on the value they create, aligning their goal with that of stockholders.

(c) Calculate each of Lloyd's and Harry's compensation for 2012 under both Plan 1 and Plan 3.

Commentary on Question:
The candidates did extremely well on this section. Candidates who did not get full credit generally did not show the earnings for Lloyd and Harry separately.

- In the first proposed compensation plan, both Harry and Lloyd are paid 7% * $10k = $70k
- Lloyd's life segment has economic cash flows of 1,000,000 and a cost of capital charge of 1,000,000. He has created 0 value, and received 0 pay.
- Harry's P&C segment has economic cash flows of $2M and his COC is $1M. He has created $1M of value and his pay is equal to 20% of this amount, $200k.

(d) Recommend and justify which compensation plan should be implemented.

Commentary on Question:
The candidates did relatively well on this section. Candidates that did not receive full credit generally only discussed the benefits of plan 2 without comparing them to the relative benefits and/or downsides of plan 1.

- The first proposed compensation plan rewards Lloyd and Harry equally despite the fact that Harry's unit performed better.
- The value based management plan is superior because it rewards Harry, who created $1 million dollars of value and does not reward Lloyd, who created 0 value.

9. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3j) Define and apply the concepts of martingale, market price of risk and measures in single and multiple state variable contexts.

(3o) Use numerical methods to effectively model complex assets or liabilities.

Sources:

Options Futures & Other Derivatives, Hull, J.C. 8th Edition, 2012

- Chapter 13 Wiener Processes and Ito's Lemma (Appendix, exclude multivariate material)
- Chapter 27 Martingales and Measures

Commentary on Question:

The first part of the question asks the candidate to demonstrate understanding of Ito's Lemma, the classical option pricing model, the properties of the lognormal distribution, and model risk. This is considered core material for this examination.

The second part of the question asks the candidate to demonstrate understanding of the martingale concept.

Overall candidates did extremely well on this question.

Solution:

(a) Show that S_t is lognormally distributed, by using Ito's Lemma.

Commentary on Question:

Candidates did extremely well on this section. The most common mistake was a failure to precisely specify the mean and standard deviation of dB_t and dG.

Let $G = G(S,t) = \ln St$

$dG = (\partial G/\partial S_t\, \mu S_t + \partial G/\partial t + \frac{1}{2}\, \partial^2 G/\partial S_t^2\, \delta^2\, S_t^2)dt + \partial G/\partial S_t\, \delta S_t\, dB_t$

$\partial G/\partial S_t = d\ln S_t / dS_t = 1/S_t$

$\partial G/\partial t = 0$

$\partial^2 G/\partial S_t^2 = -1/(S_t)^2$

$dG = (\mu - \delta^2/2)dt + (\delta)dB_t$

dB_t follows Normal $(0, (dt)^{0.5})$ Since B_t is a Wiener process

⇨ δdBt follows $N(0, (\delta^2 dt)^{0.5})$

⇨ dG follows $N((\mu - \delta^2/2)dt, \delta * (dt)^{0.5})$

Since G follows normal distribution, S will follow log normal distribution

9. Continued

(b) Show that V_t is not a martingale.

Commentary on Question:
Candidates generally did relatively well on this part.
The most common mistakes were:

- Draw the conclusion that V_t has a nonzero drift without explaining why
- In order to show a non-zero drift, the following 3 conditions need to hold

$\mu > r$, $r > 0$, $\theta_t > 0$. Most candidates did not mention all 3 conditions.

$$dV_t/V_t = (1-\theta_t)\, dP_t/P_t + \theta_t\, dS_t/S_t$$
$$= (1-\theta_t) r dt + \theta_t (\mu dt + \delta dB_t)$$
$$dV_t = ((\mu-r)\,\theta_t + r)\, V_t\, dt + \delta\, \theta_t\, V_t\, dB_t$$

For V_t to be a martingale, drift = 0
$\Rightarrow (\mu-r)^* \theta_t + r = 0 \Rightarrow \theta_t = -r/(\mu - r)$
$\mu > r \Rightarrow (\mu - r) > 0$, $r > 0 \Rightarrow \theta_t < 0$

But θ_t is positive given from the question $\Rightarrow V_t$ cannot be a martingale

Updated 2/26/2013

10. Learning Objectives:

1. Modern Corporate Financial Theory

3. Derivatives and Pricing

Learning Outcomes:

(1d) Define and compare risk metrics used to quantify economic capital and describe their limitations.

(3j) Define and apply the concepts of martingale, market price of risk and measures in single and multiple state variable contexts.

(3o) Use numerical methods to effectively model complex assets or liabilities.

Sources:

Options Futures & Other Derivatives, Hull, J.C., 8th Edition, 2012

- Chapter 12 Wiener Processes and Ito's Lemma (Appendix, exclude multivariate material)
- Chapter 20 Basic Binomial Trees
- Chapter 27 Martingales and Measures

Chapter 9 CSFB Handbook, Risk Measures: How Long is a Risky Piece of String?

Commentary on Question:

This question tested the candidate's understanding of stock price movement under the Geometric Brownian motion model (including mean and Value-at-Risk). Also, the question tested the candidate's basic understanding of put-call parity as it relates to hedging strategies.

Solution:

(a) Calculate the amount Y you need to invest in the stock at time zero so that the expected value is $250,000 in 15 years.

Commentary on Question:
Candidates did relatively well on this section.

$S_t = S_0 \exp(\mu * t)$
$\$250,000 = S_0 \exp(10\% * 15)$
$S_0 = \$250,000 / \exp(10\% * 15)$
$S_0 = \$250,000 / 4.4817$
$S_0 = \$ 55,782$

10. Continued

(b) Calculate 99% value-at-risk applied to the difference between the value of your stock investment and $250,000 after 15 years, if you invest amount Y in the stock at time zero.

Commentary on Question:
Candidates did poorly on this section. Most candidates failed to show that they understood the basic principle of value-at-risk, specifically that it is related to the worst 1% of possible outcomes.
Some candidates provided an answer greater than $250,000. A reasonableness check would have indicated to those candidates that there was a significant calculation error.

The 99^{th} percentile of normal distribution is Z = -2.3263
from the normal distribution table
So the 99^{th} worst outcome for the investment at t = 15
$= S_0 * \exp((\mu - \delta^2/2) * t) + Z * \delta * t^{1/2}$
$= 55{,}782 * \exp(0.08 * 15 - (20\% * 2.3263 * 15^{1/2}))$
$= 55{,}782 * 0.54775 = 30{,}554$
VAR relative to $250,000 = 250,000 -30,554 = 219,446

(c) Critique the suggestion from your colleague.

Commentary on Question:
The candidates did relatively poorly overall on this section. Some candidates showed a good understanding of how to apply put-call parity and relate it to the recommended strategy. However, many candidates answered the question in a non-specific manner, making general comments about using puts and calls that were not necessarily relevant to the strategy in question. One key oversight was that the strategy involved owning the stock in addition to the derivative positions taken; a key part of the ability to eliminate the risk.

Put-call parity
$Ke^{-rt} + C = S + P$
Re-arranging gives
Ke-rt = S + P – C
the right side of this equation is the suggested strategy.
This shows that the strategy is equivalent to earning the risk-free rate.
Marks were also given for other general comments on the strategy such as: discussing the bid-ask spread, the ability to purchase 15 year derivatives, etc

11. Learning Objectives:

1. Modern Corporate Financial Theory

Learning Outcomes:

(1e) Apply the concept of economic capital and describe methodologies for allocating capital within a financial organization.

Sources:

FET-178-12: Economic Capital Modeling: Practical Considerations

Commentary on Question:

This question tests candidates' knowledge of economic capital and demonstration of their understanding by solving a numerical problem. Overall candidates did poorly.

Solution:

(a) Define Required Economic Capital and identify four components associated with defining Required Economic Capital.

Commentary on Question:

Candidates did relatively poorly on this section. Many candidates were able to define the concept of the Required Economic Capital. However, most candidates did not identify the components. The components were a direct list from the study note, but often other lists were provided.

"Required Economic Capital" is the capital amount required to support the business with a certain probability of default. It includes four key components:

- The type of risks under consideration
- The probability of ruin to be accepted
- The time period over which the probability of ruin is to be assessed
- Inclusion/Exclusion of future new business

(b) Determine the company's Required Economic Capital on 12/31/2012 based on the shocks above.

Commentary on Question:

This question asks candidates to demonstrate their understanding of the Required EC concept by solving a straight-forward numerical problem. Candidates did relatively poorly in this part due to misunderstanding of a few key concepts: "Market value" vs. "Book value" vs. "Face value," "Available EC" vs. "Required EC". Confusion of these key concepts led to many errors in the calculation. This is an indication that candidates need to truly understand the concept of required economic capital (rather than just memorizing the phrase).

11. Continued

Partial credit was given as long as the candidate's answer sheets showed an attempt to solve for the market values. For example, candidates could use "change of market value" = "initial market value" * duration * "change of rate," even though the candidates confused "initial market value" with "initial book value."

Since all bonds and GIC are default-free, the market values of the assets and liabilities as of 12/31/2012 are

	Pre-shock	Post-shock
a. The market value of cash =	15	15
b. The market value of bond1 =	100/(1+0.7%) = 99.3	100/(1+1.7%) = 98.33
c. The market value of bond2 =	200/(1+2%)^5 = 181.15	200/(1+4%)^5 = 164.39
d. The market value of GIC =	290/(1+1%)^2 = 284.29	290/(1+2%)^2 = 278.74
Available EC = a + b + c – d =	11.16	-1.02

Required EC on 12/31/2012 = 11.16 – (-1.02) = 12.18

(c) Determine as of 12/31/2013 the maximum shareholder dividend the company can pay and remain solvent.

Commentary on Question:
This question is an application of the Required EC concept that further requires the candidates to understand the difference between the statutory balance sheet (book value-based) and economic balance sheet. Candidates did poorly on this section, which was highly correlated to them doing relatively poorly on part (b). About 5% of the candidates correctly solved for the maximum dividend based on either statutory considerations or EC considerations, but none looked at the issue from both perspectives.

Partial credit was given as long as the answer sheets showed any of the correct values (as in the model solution) regardless of candidate's final conclusion.

On 12/31/2013, the maximum dividend payable is such that the company remains solvent after the dividend payout.

(i) From EC perspective, the maximum dividend is the excess of Available EC over the Required EC, i.e., the post-shock Available EC.
(ii) From statutory perspective, the maximum dividend is the statutory capital.
(iii) Thus, the maximum dividend the company can afford to pay is the lesser of (i) and (ii) above.

11. Continued

	Post-Shock Available EC (Market Value Basis)	Statutory Capital (Book Value Basis)
a.	Cash (after spending \$5) =15 – 5 = 10	10
b.	Bond1 value (matured) = 100	100
c.	Bond2 value = $200/(1+3\%)^4$ = 177.70	$170*(200/170)^{(1/5)}$ = 175.62
d.	GIC value = 290/(1+1.7%) = 285.15	$279*(290/279)^{(1/2)}$ = 284.45
	Post-shock Avail. EC =a+b+c–d = 2.55	Stat Capital = a+b+c–d = 1.17

The maximum dividend the company can pay = 1.17, even though the company could pay up to 2.55 to remain solvent on EC basis.

12. Learning Objectives:

2. Corporate Financial Applications

3. Derivatives and Pricing

Learning Outcomes:

(2g) Recommend an optimal capital structure and how to implement it for a given business or strategy and be able to justify the recommendation.

(2h) Describe how behavioral characteristics and biases of users and providers of capital affect the capital structure.

(3c) Identify embedded options in assets and liabilities.

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

Sources:

FET-108-07: Integrated Risk Management, Doherty, Chapter 13 Contingent Leverage Strategies and Hybrid Debt, Pages 474 – 478

FET-176-11: A Survey of Behavioral Finance - Barberis and Thaler

Options Futures & Other Derivatives, Hull, J.C. 7th Edition, 2008

- Chapter 11 Binominal Trees, Pages 237 – 246
- Chapter 12 wiener Processes and Ito's Lemma
- Chapter 24 Exotic Options, Pages 558 - 561

Commentary on Question:

This question tests whether the candidate has an integrated understanding of the concepts and methodologies related to contingent securities. The question integrates concepts from contingent capital/hybrid debt, option theory, binominal trees, and behavioral finance.

Solution:

(a) List the reasons why investors might buy RCD.

Commentary on Question:

The candidates did relatively well on this section. Most candidates were able to recall the three main points. Candidates receiving full credit were able to demonstrate an understanding with further explanation.

1) With RCD, probability of bankruptcy will fall.
   - The fall in stock price triggers the conversion to deleverage the firm, resulting in lower bankruptcy probability.

12. Continued

- Lower bankruptcy probability, in turn, lowers the probability that debt holders would pay the bankruptcy costs.

2) With RCD, incentives of the firm to undertake new investment change.
   - Shareholders share the downside risk with debt holders who are forced to become joint share holders when equity drops below the trigger level.
   - This aligns the interest of both debt and share holders and helps improve the efficiency of project selection.

3) RCD will unlever the firm after a loss and will reduce the post-loss external financing.

(b) Explain in words each of the 3 terms in the formula.

$$V(S_0,H,T)=1000e^{-(r_f+D)T}+1000\sum_{t=1}^{T}C_t e^{-(r_f+D)t}-\frac{1000}{S_0}e^{-DT}p_{di}(S_0;H;S_0;T)$$

Commentary on Question:
The candidates did relatively well on this section. Most candidates observed that the first two terms relate to straight bond. Some candidates did not understand or explain the third term as a down-in barrier put option.

1) The first term is the present value of the principal or face value, discounted at risk-free rate plus a credit spread.
2) The second term is the present value of all coupons discounted at risk free rate plus a credit spread.
3) The third term is the value of the down-and-in (barrier) put option written by the bondholder to the firm.

(c) Calculate the value of the RCD using a Binomial Tree.

Commentary on Question:
The candidates did well on this section. Most candidates correctly identified that the option would trigger in the down-down scenario. However, very few candidates received full credit due to a variety of reasons. The most common error candidates made was to incorrectly use the formula provided in part (b), instead of using a binomial tree. Another common error some candidates made was not using the risk neutral valuation approach. The last common mistake some candidates made was missing the coupons, particularly in the down-down scenario.

Candidates who used exponential discounting also received full credit.

12. Continued

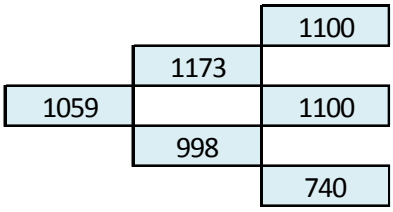
First, calculate the risk neutral probability:

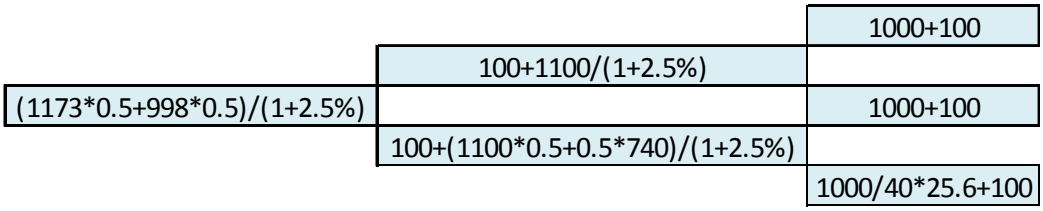
$$u=\frac{50}{40}=1.25$$

$$d=\frac{32}{40}=0.8$$

$$q=\frac{(1+r_f)-d}{u-d}=\frac{(1+2.5\%)-0.8}{1.25-0.8}=0.5$$

Then, calculate the cash flows backwards:





For the Up-Up and Up-Down scenarios, stock price is over the trigger price. No RCD conversion to stock. RCD principal gets paid back in full and receives 10% coupon per year.

For the Down-Down scenario, stock price drops below the trigger price and thus leads to the RCD converted to shares (# of shares converted equals face value over the initial stock price, i.e., 1000/40) and valued at $25.6.

Discount all the cash flows (principal plus coupons) back using risk-neutral probability and risk-free rates.

The PV of the RCD is $1,059.

(d) Explain investors' overvaluation by applying Kahneman and Tversky's *Prospect Theory* to:

- Narrow framing
- Mental accounting

12. Continued

Commentary on Question:
The candidates did relatively poorly on this section. Most candidates were able to recall the concepts in the behavioral finance context however only a few were able to apply them to this specific situation.

Framing effect: A reverse convertible bond is framed by investors as a strategy of buying a high-yield bond, not as buying a bond and writing a put option on the equity.

Mental accounting: investors count the high-yield bond separately from writing the put option to the issuing firm.

13. Learning Objectives:

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

Learning Outcomes:

(4f) Explain the implications of incomplete markets for financial theory

Sources:

FET-175-10: Models by Derman

FET-176-11: Economics of Finance, Chapter 18 A Survey of Behavioral Finance

Commentary on Question:

This question tests the implication of incomplete markets for financial theory and the weakness in financial models and their implications.

Solution:

(a) Identify and explain four ways psychologists have learned about how people appear to form beliefs.

Commentary on Question:

This question was designed to test the candidates knowledge of how people form beliefs. The candidates did relatively poorly on this section. Most candidates were able to identify at least four ways people form belief expectations. However, many candidates failed to fully explain the beliefs to demonstrate their understanding of the belief expectations. The source material from this question can be found in study note FET-176-11, pages 1065-1068.

The list below includes all belief expectations. However, the candidate was only asked to provide four. Thus if a candidate provided more than four, the four best answers were counted.

- Overconfidence. Extensive evidence shows that people are overconfident in their judgments. First, the confidence intervals people assign to their estimates of quantities are far too narrow. Second, people are poorly calibrated when estimating probabilities.
- Optimism and wishful thinking. Most people display unrealistically rosy views of their abilities and prospects. They think they are above average in common domains (e.g. Driving, getting along with others, humor). They also display a systematic planning fallacy: they predict that tasks will be completed much sooner than they actually are.

13. Continued

- Representativeness. When people try to determine the probability that a data set A was generated by a model B, or that an object A belongs to a class B, they often use the representativeness heuristic. This means that they evaluate the probability by the degree to which A reflects the essential characteristics of B. Representativeness can generate severe biases: base rate neglect and sample size neglect.
- Conservatism - base rates are over-emphasized relative to sample evidence. If data sample is representative of an underlying model, people overweigh the data. If data sample is not representative of any salient model, people rely too much on their priors.
- Belief perseverance. Once people have formed any opinion, they cling to it too tightly for too long. People are reluctant to search for evidence that contradicts their beliefs. Even if they find such evidence, they treat it with excessive skepticism.
- Anchoring. When forming estimates, people often start with some initial, possibly arbitrary value, and then adjust away from it. The adjustment is often insufficient.
- Availability biases. When judging the probability of an event, people often search their memories for relevant information. But not all memories are equally retrievable or available. More recent events and more salient events will weigh more heavily and distort the estimate.

(b)

(i) Map each of the behaviors to the action to which it is associated.

(ii) Explain, for each of the four actions above, why one of the mapped behaviors is associated with it.

Commentary on Question:
Part (i) asked the students to match each behavior on a list to one of the actions that was associated with it. Part (ii) asked the students to choose one of the behaviors for each action and explain why it was associated with it. Most candidates did relatively well on this section as they were able to map the behaviors to the appropriate actions for part (i).

Full explanations for part (ii) for all mappings are provided below. Candidates that provided explanations for more than one mapping behavior for actions 3 and 4 were only given credit for the best explanation. Additionally, candidates were only given credit for the behavioral explanation if it was mapped to the correct action.

The source material from this question can be found in study note FET-176-11, pages 1102-1110.

13. Continued

1) Ambiguity and familiarity. Ambiguity and familiarity offer a simple way of understanding the different examples of insufficient diversification. Investors may find their national stock markets more familiar (or less ambiguous) than foreign stock indices, firms geographically closer, employers stock. Since familiar assets are attractive, people invest heavily in those, and invest little or nothing at all in ambiguous assets.

2) Overconfidence. The most prominent behavioral explanation of such excessive trading is overconfidence. People believe that they have information strong enough to justify a trade, whereas in fact the information is too weak to warrant any action.

3) Investors may have an irrational belief in mean-reversion. This is a phenomenon labeled "disposition effect." Investors are more likely to sell stocks that have increased in price relative to purchase than they are to sell stocks that have gone down.

3) Prospect theory and narrow framing. The concavity (convexity) of the value function in the region of gains (losses) is central. The investor is gambling that the stock will eventually break even, saving him or her from having to experience a painful loss.

4) Self-control. Many people exhibit self-control problems. To deal with them, people often set rules. A natural rule people might create to prevent themselves from consuming their wealth is "only consume the dividend, but don't touch the portfolio capital."

4) Mental accounting. By designating an explicit dividend payment, firms make it easier for investors to segregate gains from losses and hence to increase their utility.

4) Regret. By paying dividends, firms help investors avoid regret. Regret is frustration that people feel when they imagine having taken an action that would have led to a more desirable outcome. It is stronger for errors of commission than for errors of omission. If the firm had paid a dividend and the investor was able to finance his consumption out of it, a rise in the stock price would be an error of omission. To be better off, the investor would have had to reinvest the dividend.

0) Attention Effect is not paired with any of the four examples.

13. Continued

(c) Explain the general obstacles faced by rational approaches to cross-sectional evidence.

Commentary on Question:
Candidates did poorly on this section, in fact no candidate actually answered this question correctly. Many candidates referenced other study material, which was focused on more general issues such as efficient markets. However, the wording of the question was such that it addressed a specific topic of the study material and did not allow for deviations. The source material from this question can be found in study note FET-176-11, pages 1090-1092.

- Data mining. If we sort and rank stocks in enough different ways, we are bound to discover patterns.
- Rational models typically measure risk as the covariance of returns with marginal utility of consumption. Stocks are risky if they fail to pay out at times of high marginal utility. The problem is that there is little evidence that the portfolios with anomalously high average returns do poorly in bad times.
- Some of the portfolios earn average returns below the risk-free rate. It is not easy to explain why a rational investor would willingly accept a lower return than the T-bill rate on a volatile portfolio.
- Large percentage of the high (low) average returns is earned over a very small number of days around earning announcements. It is hard to tell a rational story for why the premia should be concentrated in this way.
- The outperformance of a portfolio over another is present in almost every period. It is not easy to see any risk that might justify the outperformance.

(d) Describe the steps you would take to obtain a price for this asset under your chosen method.

Commentary on Question:
Candidates did relatively poorly on this section. Candidates that identified the correct methodology of pricing the debt were generally able to explain the steps taken to price the asset. Candidates that identified specific models or processes were also given credit if appropriate.

The source material from this question can be found in study note FET-175-10, page 31.

- The speculative equilibrium hypothesis does not apply since there are likely no observable prices since the instrument is new and privately-issued.
- Identify an asset or portfolio that has the same estimated future payouts as the new asset.

13. Continued

- To prove, one must develop a model to demonstrate that the two items have identical estimated future payouts.
- Then demonstrate payoff identity through:
  - Specify the universe of circumstances
  - Determine strategy for identifying the payoffs in each circumstance
- The new asset's value is therefore the replicating portfolio's value.

14. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3o) Use numerical methods to effectively model complex assets or liabilities.

Sources:

Options Futures & Other Derivatives, Hull, J. C., 8th Edition, 2012

- Chapter 19 Volatility Smiles

Commentary on Question:

This question tests general understanding of variance reduction techniques and analysis of model results.

Solution:

(a) Estimate a 95% percent confidence interval for the value of:

(i) the insurance liability, and

(ii) the European put option.

Commentary on Question:

The candidates did relatively well on this section. Most candidates were able to calculate the variance correctly. However most candidates did not take into account the number of trials when calculating the confidence level thereby considerably widening the confidence interval.

(i) As the three variables were independent, the Var of (A+2*B +0.75*C) = Var(A) + 2^2*Var(B) + 0.75^2*Var(C) = 2^2 + 4*1.5^2 + 9/16*(0.5)^2 = 13.141
So the standard deviation = 13.141^0.5 = 3.625.

The 95% confidence level is the average plus minus 1.96 * standard deviation / square root of the number of trials.

For (i): 3.05+/-1.96*3.625/100 = 3.05+/-.071=2.979to3.121
For (ii): 3 +/- 1.96*1/100 = 3-.0196 to 3.0196 which is 2.9804 to 3.0196

(b) Estimate the value of the insurance liability using the control variate technique.

Commentary on Question:

Most candidates did extremely well on this section. There were two common mistakes. 1) A few candidates did not describe the formula in general and just applied it. As a result it was difficult to give partial credit if done incorrectly. 2) A few candidates swapped signs for the put options when applying the formula. This was accepted as long as it was clear that was the approach being used.

14. Continued

" Estimate of value of liability, F: a +2b +0.75c" = 6.5 - 2x2.1 + 0.75x1 = 3.05
(This was given in the problem chart as well)
New Estimate: F - Fe + Fbs = 3.05 - 3.0 + 2.95 = 3.00 As a reasonableness check, the model produced a higher value than the Black-Scholes formula, this implies that we need to reduce the liability value to bring it closer in line with the known (Black-Scholes) model.

(c) Describe briefly three other variance reduction techniques that may be applied to Monte Carlo models.

Commentary on Question:
The candidates did extremely well on this section. Those that didn't get full credit just wrote the names without an explanation. Candidates were given credit for the best 3 methods however most candidates only provided 3 as the question asked.

(i) Antithetic Variable Technique:
Create a trial using random variable ei, second trial uses –ei for each random variable.

(ii) Importance Sampling:
Only paths that are important are calculated to estimate the derivative.

(iii) Stratified Sampling:
Sample representative values rather than random variables from a probability distribution.

(iv) Moment Matching:
Involves adjusting the samples from a standardized normal distribution so the moments (usually first, second, and sometimes higher) are matched.

(v) Quasi Random Sequences (low discrepancy sequences):
A sequence of representative samples from a probability distribution. Has property that standard error being proportional to 1/m rather than 1/(m^1/2). Similar to stratified sampling.

(d) Recommend whether or not to use the proposed valuation system and justify your recommendation.

Commentary on Question:
The candidates did relatively poorly on this section. Because very little background information was given about the liability, it was impossible to evaluate the model for appropriateness. This question asked the candidates to evaluate the model. The key item to recognize is that you also have a plain put modeled by the system and you calculated a confidence interval in part (a). Since most candidates had the confidence interval wrong, if this comparison was made to the answer in part (a), most credit was earned even though the conclusion would be different.

14. Continued

Those that commented that the confidence interval was too big demonstrated some understanding of the concepts and were awarded some partial credit.

Many candidates made a comment that the liability could not be negative even though no discussion of liability rules was made.

The 95% confidence interval in part (a)(ii) was 2.98 to 3.02 yet the put option under B-S was 2.95. This means the model should be rejected if you feel the B-S model value should have been replicated.

15. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2b) Describe the process, methods and uses of financial reinsurance (surplus relief) and recommend a structure that is appropriate for a given set of circumstances.

Sources:

FET-161-08: Tiller & Tiller, Life, Health and Annuity Reinsurance, Chapter 5: Advanced Methods of Reinsurance

Commentary on Question:

This question tests the process, methods and uses of financial reinsurance for a given set of circumstances.

Solution:

(a) Outline the uses of reinsurance in financial planning.

Commentary on Question:

The candidates did relatively well on this section. Candidates who did not score full credit often gave a more general answer.

Facilitate a strategic business decision/planning objective
Improve capital management
Improve returns
Optimize the use of tax gains and losses
Create administrative savings

(b) Outline the features of a simple co-insurance transaction.

Commentary on Question:

The candidates did well on this section. This question asked for specifics about a co-insurance transaction however, the candidates tended to give general descriptions of reinsurance. A common mistake is that candidates tended to repeat the same answer in several different ways. Often these answers overlapped with those needed in part c, in which case we only gave them credit in part c.

- Transferring assets is required
  - Simple, can be used on any product
  - Insurer holds reserve, pays a fee
- It requires the reinsurer to manage the assets
- Transfer assets will incur capital gain and loses
  - Reinsurer effectively assumes a percentage of the business
- Impossible to purchase assets needed given the short timeframe for execution

15. Continued

(c) Explain why Kakabeka Life will prefer to retain the assets in this situation.

Commentary on Question:
The candidates did well on this section. Most candidates got points for the licensure, capital G/L, credit risk. Few candidates noted the investment risk or control of interest rates to the reinsurer as reasons.

- Transferring assets is required.
  - Ceding company must transfer control of the assets equal to reserves to the reinsurer.
  - Control over the interest rate determination may be transferred to the reinsurer.
- It requires the reinsurer to manage the assets. Reinsurer may not want to bear additional investment risk.
- Capital gains or losses occur in asset transfer.
- Ceding company may not be able to take reserve credit since the reinsurer is not licensed in the ceding company's' state of domicile.
- Ceding company exposes to an additional credit risk

(d) Calculate the first-year and second-year Mod-Co adjustment for both reinsurance options.

Commentary on Question:
The candidates did relatively poorly on this section. Most candidates got some points on the mod-co adjustment. On the funds withheld allowance, most candidates either did not include the funds withheld allowance or left it blank.

The Mod-Co adjustment is equal to the change in reserve minus investment credit. This is $1,000 in year one ($1,000 – 0 – 0 x 10%) and -$30 in year two ($1,070 – $1,000 – $1,000 x 10%). The funds withheld adjustment is the same in year one, and -$20 in year two after the allowance is removed ($1,070 – $1,000 - ($1,000-$100) x 10%).

(e) Prepare Kakabeka Life's first-year and second-year Mod-Co Reinsurance Report.

Commentary on Question:
The candidates did relatively poorly on this section. This section was derived directly out of the source material. Many students provided statement balances as opposed to a reinsurance report. Those that scored some points generally wrote down the components.

15. Continued

Year 1
Ceding Premium = 1000
Allowance = 100
Benefit = 0
Mod Co Adjustment =1000
Experience Fund = 0
Due to XYZ Re = 1000 -100-0-1000 = -100

Year 2
Ceding Premium = 0
Allowance = 0
Benefit = 5
Mod Co Adjustment =-30
Experience Fund = 1
Due to XYZ Re = 0-0-5-(-30)-1 =24

16. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3j) Define and apply the concepts of martingale, market price of risk and measures in single and multiple state variable contexts.

Sources:

Options Futures & Other Derivatives, Hull, J. C., 8th Edition, 2012

- Chapter 13 Weiner Processes and Ito's Lemma (Appendix, exclude multivariate material)
- Chapter 27 Martingales and Measures

Commentary on Question:

This question tested the candidates understanding of the market price of risk and the principles of the risk neutral framework.

Solution:

(a) Explain the market price of risk.

Commentary on Question:

The candidates did well on this section. Most candidates defined the market price of risk either through formula or words. Some candidates explained the market price of risk is the same for all securities with the same underlying source of uncertainty. Very few candidates described the trade-off between risk and reward. No one commented on the underlying stochastic process and time.

- Excess return over risk-free rate per unit of volatility: $\frac{(\mu - r)}{\sigma}$
- Measure of trade-off between risk and reward
- Same for all securities with same underlying source of uncertainty
- Can only depend on underlying stochastic process w and time but nothing else

(b) Calculate the expected return of U at time $t = 10$, if S has a constant expected return of 6% per annum and if $k = 0.04$.

Commentary on Question:

The candidates did extremely well on this section. Most candidates either got correct answer or made small calculator typo (e.g. switched sign) but clearly understood concept.

At t=10: volatility of U = exp(-0.4) x volatility of S

However, market price of risk at t=10 must be the same for U and S,

$$\rightarrow \frac{(\mu - r)}{\sigma} = \frac{(v - r)}{\sigma 0.67032} \Leftrightarrow \frac{(0.06 - 0.04)}{\sigma} = \frac{(v - 0.04)}{\sigma 0.67032}$$

Solve for v = 0.053406

16. Continued

(c) Determine the process for G in the traditional risk-neutral world.

Commentary on Question:
The candidates did poorly on this section. Full credit was given if they were able to derive the risk neutral return. However, most candidates scored 0 and didn't appear to know where to begin.

$$\frac{dG}{G} = \frac{b(G_0 - G)}{G}dt + k\sqrt{G}dW$$

Expected growth rate in G = $\frac{b(G_0-G)}{G}$

Volatility of G = $k\sqrt{G}$

In risk-neutral world: $\lambda = \frac{(\mu - r)}{\sigma} = \frac{\frac{b(G_0-G)}{G} - r}{k\sqrt{G}} \rightarrow r = \frac{b(G_0-G)}{G} - \lambda k\sqrt{G}$

$\rightarrow \frac{dG}{G} = [\frac{b(G_0-G)}{G} - \lambda k\sqrt{G}]dt + k\sqrt{G}dW$

17. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2a) Describe the steps necessary to obtain funds for a given project or firm from any specified source, and be able to recommend a specific approach to raising capital in a given situation.

Sources:

Doherty, Integrated Risk Management, Ch 13.

FET-165-08: Integrated Risk Management, Doherty, Chapter 16: A Case Study: the Securitization of Catastrophic Risk

Commentary on Question:

The purpose of this question was to test the candidates' understanding of different debt financing strategies and their impact on the firm, debt holders and shareholders.

In general, candidates did extremely well for the calculation of values on part (a) and (c) of this question but most struggled in part (b) to explain the conflicting interest of the debtholders and shareholders.

Solution:

(a) Calculate the expected Value of the Firm, Senior Debt, Junior Debt, and Equity Value for both projects.

Commentary on Question:

The candidates did extremely well on this section with most candidates gaining full points. Some candidates incorrectly deducted the initial cost from the payoff, appearing to confuse firm value with initial equity. This had the effect of reducing the firm value and all components related to it as equity value and the action for the conversion of junior convertible debt. Also many candidates provided a matrix of values with very few details of calculations or explanations.

Project A:

State X			
Firm Value =	**150 (Project A) +300 (Existing operations with scenario 1)**	=	450
Senior Debt =	minimum (Firm Value , Senior Debt) = min.(450,100)	=	100
Junior Debt =	minimum ((Firm Value minus Senior Debt) , Junior Debt) = min.(450-100,100)	=	100
Equity =	Firm Value - Senior Debt - Junior Debt = 450 - 100 – 100	=	250

17. Continued

State Y			
Firm Value =	**150 (Project A) +500 (Existing operations with scenario 2)**	=	650
Senior Debt =	minimum (Firm Value , Senior Debt) = min.(650,100)	=	100
Junior Debt =	minimum ((Firm Value minus Senior Debt) , Junior Debt) = min.(650-100,100)	=	100
Equity =	Firm Value – Senior Debt - Junior Debt Value = 650 - 100 - 100	=	450

Total expected for Project A

Firm Value=	50% of Firm Value State X + 50% of Firm Value State Y = 0.5*(450+650)	=	**550**
Senior Debt =	50% of Senior Debt State X + 50% of Senior Debt State Y = 0.5*(100+100)	=	**100**
Junior Debt =	50% of Junior Debt State X + 50% of Junior Debt State Y = 0.5*(100+100)	=	**100**
Equity =	50% of Equity State X + 50 % of Equity State Y = 0.5*(250+450)	=	**350**

Project B:

State A-X

Firm Value =	**260 (Project B with scenario 1) +300 (Existing operations with scenario 1)**	=	560
Senior Debt =	minimum (Firm Value , Senior Debt) = min.(560,100)	=	100
Junior Debt =	minimum ((Firm Value minus Senior Debt) , Junior Debt) = min.(560-100,100)	=	100
Equity =	Firm Value - Senior Debt - Junior Debt = 560 - 100 - 100	=	360

State A-Y

Firm Value =	**260 (Project B with scenario 1) +500 (Existing operations with scenario 2)**	=	760
Senior Debt =	minimum (Firm Value , Senior Debt) = min.(760,100)	=	100
Junior Debt =	minimum ((Firm Value minus Senior Debt) , Junior Debt) = min.(760-100,100)	=	100
Equity =	Firm Value - Senior Debt- Junior Debt Value = 760 - 100 - 100	=	560

17. Continued

State B-X

Firm Value =	**50 (Project B with scenario 2) +300 (Existing operations with scenario 1)**	=	350
Senior Debt =	minimum (Firm Value , Senior Debt) = min.(350,100)	=	100
Junior Debt =	minimum ((Firm Value minus Senior Debt) , Junior Debt) = min.(350-100,100)	=	100
Equity =	Firm Value - Senior Debt - Junior Debt = 350 – 100 - 100	=	150

State B-Y

Firm Value =	**50 (Project B with scenario 2) +500 (Existing operations with scenario 2)**	=	550
Senior Debt =	minimum (Firm Value , Senior Debt) = min.(550,100)	=	100
Junior Debt =	minimum ((Firm Value minus Senior Debt) , Junior Debt) = min.(550-100,100)	=	100
Equity =	Firm Value – Senior Debt - Junior Debt Value = 550 - 100 - 100	=	350

Total expected for Project B

Firm Value=	25% State A-X + 25% State A-Y + 25% State B-X + 25% State B-Y = 0.25*(560+ 760 + 350 + 550)	=	**550**
Senior Debt =	25% State A-X + 25% State A-Y + 25% State B-X + 25% State B-Y = 0.255*(100 + 100 + 100 + 100)	=	**100**
Junior Debt =	25% State A-X + 25% State A-Y + 25% State B-X + 25% State B-Y = 0.25*(100 +100 + 100 + 100)	=	**100**
Equity =	25% State A-X + 25% State A-Y + 25% State B-X + 25% State B-Y = 0.25*(360 + 560 + 150 + 350)	=	**355**

17. Continued

(b) Explain why the company may have difficulty securing Debt financing for either project.

Commentary on Question:
The candidates did relatively poorly on this section. To get full points, the candidates needed to highlight the different motivations of each group of investors and how those were conflicting by answering the following questions:
1) What is the risk for shareholders?
2) What is the risk for bondholders?
3) Which project the shareholders are more likely to select and why it conflicts with the interest of bondholders.

Many candidates mentioned covenants, however the question didn't mention covenants. Additionally, those covenants wouldn't be a deterrent to the possible bondholders.

The Value of equity is higher for Project B.

Then it would be difficult for equity holders to choose Project A since Project B has higher Value to them.

Shareholders enjoy the potential of higher upside with limited downside.

For Debt holders Project B provide larger downside risk and hence lower Value of Debt for this project.

Project B may require use of existing operation payoff.

Therefore, Debt holders would not be willing to issue Debt for either project.

(c) Calculate the expected Value of the Firm, Senior Debt, Junior Debt, and Equity Value for both projects.

Commentary on Question:
The candidate did extremely well on this section with most candidates gaining full points. Some candidates again incorrectly deducted the initial cost from the payoff, appearing to confuse firm value with initial equity. Also many candidates provided a matrix of values with very few details of calculations or explanations.

17. Continued

Junior Debt will convert if:

ratio * (Firm Value - Senior Debt) > Junior Debt Value
ratio= Increase in number of share (200) because conversion /
Total number of shares after conversion (500) = 200 / 500 = 0.4

When 0.4*(Firm Value - 100 **(senior Debt)**) > 100 **(Value of junior Debt)**
or when Firm Value >350

Project A:

State X

Firm Value =	**150 (Project A) +300 (Existing operations with scenario 1)**	=	450
Senior Debt =	minimum (Firm Value , Senior Debt) = min. (450,100)	=	100
Junior Debt =	Since Firm Value > 350, will convert (0.4*(Firm Value minus Senior Debt) = 0.4*(450-100)	=	140
Equity =	Firm Value - Senior Debt - Junior Debt = 450 - 100 – 140	=	210

State Y

Firm Value =	**150 (Project A) +500 (Existing operations with scenario 2)**	=	650
Senior Debt =	minimum (Firm Value , Senior Debt) = min. (650,100)	=	100
Junior Debt =	Since Firm Value > 350, will convert (0.4*(Firm Value minus Senior Debt) = 0.4*(650-100)	=	220
Equity =	Firm Value - Senior - Junior Debt Value = 650 - 100 – 220	=	330

17. Continued

Total expected for Project A

Firm Value=	50% of Firm Value Sate X + 50% of Firm Value state Y = 0.5*(450+650)	=	**550**
Senior Debt =	50% of Firm Value Sate X + 50% of Firm Value state Y = 0.5*(100+100)	=	**100**
Junior Debt =	50% of Firm Value Sate X + 50% of Firm Value state Y = 0.5*(140+220)	=	**180**
Equity =	50% of Firm Value Sate X + 50 % of Firm Value state Y = 0.5*(210+330)	=	**270**

Project B:

State A-X

Firm Value =	**260 (Project B scenario 1) +300 (Existing operations with scenario 1)**	=	560
Senior Debt =	minimum (Firm Value , Senior Debt) = min. (560,100)	=	100
Junior Debt =	Since Firm Value > 350, will convert (0.4*(Firm Value minus Senior Debt) = 0.4*(560-100)	=	184
Equity =	Firm Value - Senior Debt - Junior Debt = 560 - 100 – 184	=	276

State A-Y

Firm Value =	**260 (Project B scenario 1) +500 (Existing operations with scenario 2)**	=	760
Senior Debt =	minimum (Firm Value , Senior Debt) = min. (760,100)	=	100
Junior Debt =	Since Firm Value > 350, will convert (0.4*(Firm Value minus Senior Debt) = 0.4*(760-100)	=	264
Equity =	Firm Value - Senior - Junior Debt Value = 760 - 100 – 264	=	396

17. Continued

State B-X			
Firm Value =	**50 (Project B scenario 2) +300 (Existing operations with scenario 1)**	=	350
Senior Debt =	minimum (Firm Value , Senior Debt) = min. (350,100)	=	100
Junior Debt =	Since Firm Value not > 350, will not convert minimum ((Firm Value minus Senior Debt) , Junior Debt) = min.(350-100,100)	=	100
Equity =	Firm Value - Senior Debt - Junior Debt = 350 - 100 – 100	=	150
State B-Y			
Firm Value =	**50 (Project B scenario 2) +500 (Existing operations with scenario 2)**	=	550
Senior Debt =	minimum (Firm Value , Senior Debt) = min. (550,100)	=	100
Junior Debt =	Since Firm Value > 350, will convert (0.4*(Firm Value minus Senior Debt) = 0.4*(550-100)	=	180
Equity =	Firm Value - Senior - Junior Debt Value = 550 - 100 – 180	=	270

Total expected for Project B

Firm Value=	25% State A-X + 25% State A-Y + 25% State B-X + 25% State B-Y = 0.25*(560+ 760 + 350 + 550)	=	**555**
Senior Debt =	25% State A-X + 25% State A-Y + 25% State B-X + 25% State B-Y = 0.255*(100 + 100 + 100 + 100)	=	**100**
Junior Debt =	25% State A-X + 25% State A-Y + 25% State B-X + 25% State B-Y = 0.25*(184 +264 + 100 + 180)	=	**182**
Equity =	25% State A-X + 25% State A-Y + 25% State B-X + 25% State B-Y = 0.25*(276 + 396 + 150 + 270)	=	**273**