SOCIETY OF ACTUARIES
Exam FETE
Financial Economic Theory and Engineering Exam (Finance/ERM/Investment)

Exam FETE

AFTERNOON SESSION

Date: Thursday, November 1, 2012
Time: 1:30 p.m. – 4:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This afternoon session consists of 9 questions numbered 9 through 17 for a total of 60 points. The points for each question are indicated at the beginning of the question.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam FETE.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.

9. (*5 points*) You are assessing how to invest your retirement savings. You are choosing from two possible investments:

1. a money market fund with price P_t and which grows at a constant risk-free interest rate r, and

2. a stock S_t whose price fluctuates randomly.

P_t satisfies the ordinary differential equation:

$dP_t = rP_t dt$

The price S_t of one unit of stock at time t satisfies the stochastic differential equation:

$dS_t = \mu S_t dt + \delta S_t dB_t$

μ and δ are positive constants
B_t is a Wiener process
$\mu > r$

You have capital V_0 at time 0 and your investment strategy at time t is to invest a proportion θ_t of your wealth in the stock and $1 - \theta_t$ in the bond. $\theta_t > 0$ for all t.

The value of your portfolio V_t satisfies:

$$\frac{dV_t}{V_t} = (1 - \theta_t)\frac{dP_t}{P_t} + \theta_t \frac{dS_t}{S_t}$$

(a) (*2 points*) Show that S_t is lognormally distributed, by using Ito's Lemma.

(b) (*3 points*) Show that V_t is not a martingale.

10. (*4 points*) You are given the following information on a stock:

μ	10%
r	3%
δ	20%
Stock price	100
Dividend rate	0%

Assume that the stock price S follows Geometric Brownian Motion:

$$dS_t = \mu S_t dt + \delta S_t dB_t$$

where B_t is a Wiener process

(a)   (*1 point*) Calculate the amount Y you need to invest in the stock at time zero so that the expected value is $250,000 in 15 years.

(b)   (*1 point*) Calculate 99% value-at-risk applied to the difference between the value of your stock investment and $250,000 after 15 years, if you invest amount Y in the stock at time zero.

A colleague suggests the following strategy: sell a call and then purchase a put with the proceeds in order to remove all risk from the portfolio.

(c)   (*2 points*) Critique the suggestion from your colleague.

11. (*7 points*) Vinnufossen Life sells Guaranteed Investment Contracts (GIC). The company's statutory balance sheet on valuation date 12/31/2012 is shown below:

Statutory balance sheet on 12/31/2012 ($Millions)

Assets		Liability & Equity	
Cash	15	GIC	279
Zero-coupon bond1	98		
Zero-coupon bond2	170	Equity	4
Total:	283	Total:	283

Additional information is given below:
- All bonds and GIC are default-free. Their face values and maturities are shown below.

Par Value ($Millions)		Time-to-Maturity as of 12/31/2012
Zero-coupon bond1	100	1 year
Zero-coupon bond2	200	5 years
GIC	290	2 years

- Cash earns 0% interest.
- GIC cannot be surrendered before maturity.
- All assets and liabilities on the statutory balance sheet are reported on book value basis.
- The company spent $5 million cash on 6/30/2013 to resolve an unexpected problem.
- The company issued no new business between 12/31/2012 and 12/31/2013.
- The company's Required Economic Capital is determined from the interest rate stress-shock analysis.
- The Required Economic Capital is also subject to maintaining statutory solvency.
- The default-free spot interest rates (zero-coupon rates, expressed as effective annual rates) on 12/31/2012 and on 12/31/2013 are the same, as shown below:

Term:	1-year	2-year	3-year	4-year	5-year
Pre-Shock Spot Rates as of 12/31/2012 and 12/31/2013	0.7%	1.0%	1.2%	1.5%	2.0%
Post-Shock Spot Rates as of 12/31/2012 and 12/31/2013	1.7%	2.0%	2.5%	3.0%	4.0%

(a) (*2 points*) Define Required Economic Capital and identify four components associated with defining Required Economic Capital.

(b) (*2 points*) Determine the company's Required Economic Capital on 12/31/2012 based on the shocks above.

(c) (*3 points*) Determine as of 12/31/2013 the maximum shareholder dividend the company can pay and remain solvent.

12. (*8 points*) During the financial crisis of 2008-2009 Yumbilla Bank issued reverse convertible debt (RCD) to forestall potential financial distress. The RCD is convertible to 25 shares of Yumbilla stock at the maturity date of the bond if the stock price at that time is below the conversion trigger price.

You are given the following:

Current Yumbilla Stock Price (S_0)	$40
RCD Face Value (B)	$1000
RCD Maturity (T)	2 Years
RCD Annual Coupon Rate (C)	10%
Conversion Trigger Price (H)	$35
Yumbilla Credit Default Spread (D)	300 basis points
Risk-free Rate (r_f)	2.5%
Implied Volatility (σ)	20%

(a)   (*2 points*) List the reasons why investors might buy RCD.

You are given the following formula for the value of the RCD:

$$V(S_0, H, T) = 1000e^{-(r_f + D)T} + 1000\sum_{t=1}^{T} C_t e^{-(r_f + D)t} - \frac{1000}{S_0} e^{-DT} p_{di}(S_0; H; S_0; T)$$

(b)   (*2 points*) Explain in words each of the 3 terms in the formula above.

12. Continued

Suppose that Yumbilla's stock price has the following projection:

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                                62.5
                    50
        40                      40
                    32
                                25.6

        T = 0       T = 1       T = 2
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(c)   (*3 points*) Calculate the value of the RCD using a Binomial Tree.

You have observed that the calculated model price for the RCD is consistently far below the market price.

(d)   (*1 point*) Explain investors' overvaluation by applying Kahneman and Tversky's *Prospect Theory* to:

- Narrow framing

- Mental accounting

13. (*8 points*) You are an investment actuary and have been given an assignment to provide the management team an introduction to principal-agency theory and behavioral finance. A crucial component of any model of financial markets is a specification of how agents form expectations.

     (a)   (*2 points*) Identify and explain four ways psychologists have learned about how people appear to form beliefs.

You will also give examples to help the management team better understand the concept.

Behavioral finance has some success in explaining how certain groups of investors behave. Typical behaviors include:
- Regret
- Self-Control
- Overconfidence
- Attention Effect
- Mental Accounting
- Ambiguity and Familiarity
- Irrational Belief in Mean-Reverting
- Prospective Theory and Narrow Framing

The behaviors above can be associated with the following actions:

1. Investors diversify their portfolio holdings much less than is recommended by normative models of portfolio choice.

2. One of the clearest predictions of rational models of investing is that there should be very little trading. In contrast to this prediction, the volume of trading is very high.

3. Investors are reluctant to sell assets trading at a loss relative to the price at which they were purchased.

4. Firms choose to pay dividends, and in so doing, apparently make some shareholders worse off.

     (b)

          (i)   (*1 point*) Map each of the behaviors to the action to which it is associated.

          (ii)   (*2 points*) Explain, for each of the four actions above, why one of the mapped behaviors is associated with it.

13. Continued

Rational models have been developed and tested against behavioral alternatives. You want the management team to understand the challenges.

(c)   (*1 point*) Explain the general obstacles faced by rational approaches to cross-sectional evidence.

You have been given the task of pricing a brand new privately-issued debt instrument. Management has suggested pricing the debt instrument using the speculative equilibrium hypothesis or a replicating portfolio approach based on Emanuel Derman's interpretation of the Law of One Price. You are told to select one method.

(d)   (*2 points*)   Describe the steps you would take to obtain a price for this asset under your chosen method.

14. (*6 points*) A consulting firm has developed a new valuation system to calculate the value of insurance liabilities with embedded options. Their system uses a Monte Carlo Simulation model with three independent variables (*a*, *b*, and *c*). The liability value *V* for each trial is calculated as $V = a + 2b + 0.75c$. No variance reduction techniques are employed. A simulation with 10,000 trials has been performed on a block of insurance liabilities. The following values have been produced:

	Average	Standard Deviation
Variable *a*	6.50	2.00
Variable *b*	−2.10	1.50
Variable *c*	1.00	0.50
Estimated value of insurance liability	3.05	
Estimated value of plain-vanilla European Put Option	3.00	1.00

The value of the plain-vanilla European put option using Black-Scholes is 2.95.

(a)   (*2 points*) Estimate a 95% percent confidence interval for the value of:

(i)   the insurance liability, and

(ii)   the European put option.

Assume that the option embedded in the insurance liability being modeled is similar to a short position in a European put option.

(b)   (*1 point*) Estimate the value of the insurance liability using the control variate technique.

(c)   (*2 points*) Describe briefly three other variance reduction techniques that may be applied to Monte Carlo models.

(d)   (*1 point*) Recommend whether or not to use the proposed valuation system and justify your recommendation.

15. (*9 points*) Kakabeka Life Insurance Company sells Single Premium Deferred Annuities (SPDAs). The company is in very weak financial condition and is seeking immediate statutory surplus relief. Angel Re has offered to reinsure 100% of this block of business. Angel Re is not licensed in Kakabeka Life's state of domicile.

(a)   (*1 point*) Outline the uses of reinsurance in financial planning.

(b)   (*1 point*) Outline the features of a simple co-insurance transaction.

(c)   (*1.5 points*) Explain why Kakabeka Life will prefer to retain the assets in this situation.

Kakabeka Life and Angel Re are considering modified coinsurance (Mod-Co) and funds withheld modified coinsurance.

Kakabeka Life
- Issues $1,000 of SPDAs on Dec. 31st.
- $100 of commissions is paid to its agents.
- Reserves on Dec. 31st are $1,000.
- Benefits of $5 and experience refund of $1 are assumed to be incurred in the second calendar year.
- The second year reserve increase on the remaining contract is assumed to be $70.

Angel Re
- Angel Re is assumed to have $100 of free surplus available for this reinsurance transaction.
- Angel Re grants Kakabeka Life an allowance equal to 10% of the premium.

Both Kakabeka Life and Angel Re earn 10% on invested assets held at the beginning of each year.

Other expenses, assets or interests on cash flows are not considered in this transaction.

(d)   (*3 points*) Calculate the first-year and second-year Mod-Co adjustment for both reinsurance options.

(e)   (*2.5 points*) Prepare Kakabeka Life's first-year and second-year Mod-Co Reinsurance Report.

16. (*5 points*) Two securities, S and U, are both positively dependent on the same underlying source of uncertainty.

Security S has price *S* that follows the process:

$$dS = \mu S dt + \sigma S dW$$, where *W* is a standard Brownian motion.

Security U has price *U* that follows the process:

$$dU = v U dt + \theta U dW$$, with the same underlying Brownian motion.

U's volatility is a function of S's volatility and is defined as:

$$\theta = \sigma \exp(-kt) \text{ for some constant } k.$$

The risk-free rate *r* is 4% per annum.

(a)   (*1 point*) Explain the market price of risk.

(b)   (*2 points*) Calculate the expected return of U at time $t = 10$, if S has a constant expected return of 6% per annum and if $k = 0.04$.

You are given another security G which follows the process

$$dG = b(G_0 - G)dt + kG^{\frac{3}{2}}dW$$

Where b, G_0 and k are positive constants; and dW is a Wiener process.

λ is the market price of risk for W.

(c)   (*2 points*) Determine the process for *G* in the traditional risk-neutral world.

17. (*8 points*) Kootenai Inc. is considering two potential projects whose payoffs are independent of its existing operations. Both projects have the same initial cost of $100. The following table gives the payoffs and associated probabilities. The Project payoffs exclude the initial cost.

	Probability	Payoff
Existing Operations	50%	300
	50%	500
Project A	100%	150
Project B	50%	260
	50%	50

Kootenai currently has $100 of senior debt, and 300 shares of equity. In order to finance the two projects, it is considering issuing junior debt of $100.

(a)   (*3 points*) Calculate the expected value of the Firm, Senior Debt, Junior Debt, and Equity value for both projects.

(b)   (*1 point*) Explain why the company may have difficulty securing debt financing for either project.

Kootenai issues junior convertible debt that can be converted to 200 shares of equity.

(c)   (*4 points*) Calculate the expected value of the Firm, Senior Debt, Junior Debt, and Equity value for both projects.

END OF EXAMINATION
Afternoon Session

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