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**SOCIETY OF ACTUARIES**  
**Exam FETE**  
**Financial Economic Theory and Engineering Exam (Finance/ERM/Investment)**

# Exam FETE

## MORNING SESSION

**Date:** Thursday, November 1, 2012

**Time:** 8:30 a.m. – 11:45 a.m.

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### INSTRUCTIONS TO CANDIDATES

#### General Instructions

1. This examination has a total of 120 points. It consists of a morning session (worth 60 points) and an afternoon session (worth 60 points).
  - a) The morning session consists of 8 questions numbered 1 through 8.
  - b) The afternoon session consists of 9 questions numbered 9 through 17.The points for each question are indicated at the beginning of the question.
2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.
3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

#### Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.
2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.
3. The answer should be confined to the question as set.
4. When you are asked to calculate, show all your work including any applicable formulas.
5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate morning or afternoon session for Exam FETE.
6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

Tournez le cahier d'examen pour la version française.



**\*\*BEGINNING OF EXAMINATION\*\***  
**MORNING SESSION**

- 1.** (*6 points*) Insurers Colonial Creek (CC) and Johannesburg (JH) have net operating cash flows which are proportional to each other over time. A project, which will cost \$10,000,000, is assumed to generate an IRR of 9.56% and the following estimated cash flows:

Year End	1	2	3	4+
Cash Flow	\$2,500,000	\$4,700,000	\$5,000,000	\$0

Assume that:

- Both companies can borrow at the risk free rate of 4%.
- The expected market return next year is 10%.
- The beta of the project without leverage is estimated to be 1.2.

Company	Debt-to-Asset Ratio	Tax Rate	WACC
Colonial Creek	20%	35%	9.96%
Johannesburg	40%	35%	To be determined

- (a) (*3 points*)
- Explain why Colonial Creek and Johannesburg have the same systematic risk  $\beta_U$  of net operating cash flows;
  - Determine the value of  $\beta_U$ ;
  - Calculate the WACC for Johannesburg.
- (b) (*3 points*) Determine whether the project should be accepted by each of Colonial Creek and Johannesburg:
- (*1 point*) By the NPV criterion;
  - (*2 points*) By a cost of capital analysis.

- 2.** (9 points) Tugela Life's equity is currently priced at 300 million and plans to issue a 150 million long term corporate bond to fund one of two projects.

As the new CFO you are concerned that two classic agency problems, underinvestment and asset substitution, could arise because of the planned bond issuance.

- (a) (2 points) Explain reasons for your concern.

You are considering three funding methods:

1. A bond with call option under which the issuer can buy back the bond at the face value (Callable bond)
  2. A bond under which bondholders have option to convert bond to certain number of company shares (Convertible bond)
  3. A bond under which company has option to convert bond to certain number of company shares (Reverse convertible bond)
- (b) (3 points) Assess each funding method as it relates to underinvestment and asset substitution.

## 2. Continued

Two projects have been proposed in the Board meeting. You have collected the following information regarding the projects:

Projects	Capital Cost	PV of Earnings	Probabilities	NPV	Expected NPV
<b>Project A</b>	150	225	100%	75	75
<b>Project B</b>	150	45	50%	-105	10
		275	50%	125	

Note 1: Values are in millions

Note 2: The outcomes for Projects A and B are independent of funding method

Note 3: Projects A and B are independent

You have also collected the following information on the Ex Ante values:

Probability	Equity	Debt	Firm Value
50%	250	50	300
50%	350	50	400

Number of Existing Company Shares	10 million shares
New Bond Issue	150 million
Coupon Rate	Market Rate
Call Options Price (Ex Ante)	7.5 million
Convertible Option Price (Ex Ante)	5 million
Conversion Ratio of Convertible Bond	5 million shares
Bankruptcy Cost	75 million
Other Revenues/Costs	Not Considered
Tax	Not Considered

- (c) (4 points) Recommend a Project and the appropriate funding method which maximizes the Ex Post value of the firm based on the above information.

- 3.** (12 points) Three months ago Tres Hermanas Life (THL) issued 100,000 variable deferred annuity contracts with a new Guaranteed Minimum Maturity Benefit feature (GMMB), which guarantees a return of a portion of the principal 5 years after inception. This closed block of business will not have any further inflows.

You are given the following:

Value of Separate Account at Inception (Principal)	10 billion
Guarantee Level as % of the Principal	90%
Implied Volatility	25%
Risk-Free Rate (Continuous)	2%
Dividend Rate (Continuous)	0%
The policy count declines by equal percent each quarter at the rate of 7% per annum.	
Policy Count Today (End of 3 months)	98,202
Expected Policy Count 1 Year from Inception	93,000
Expected Policy Count 5 Years from Inception (Maturity)	69,568
Quarterly Charge (charged at end of quarter)	0.50%
Since inception, the rate of return on assets underlying the Separate Account was negative 3% (excluding Quarterly Charge).	

Assume:

- A flat yield curve
- Flat volatility surface
- All policies sold are of the same size
- Policy count changes are not correlated with the market
- No hedging has been done on the block until today

- (a) (1 point) Calculate the total value of the Separate Account as of today.
- (b) (3 points) Calculate the option value of the GMMB block as of today.
- The Separate Account will now be hedged with a US equity index currently trading at 1300, which pays a 2.5% continuous dividend. Three-month futures contracts on the index have a multiplier of 50.
- (c) (3 points) Calculate the number of three-month futures contracts required to delta hedge the GMMB.
- (d) (2 points) Calculate what the margin offset would be as of today if the guarantee were to be repriced.
- (e) (1 point) Discuss the sufficiency of the Quarterly Charges to finance the GMMB.

### **3. Continued**

- (f) (*2 points*) In the context of a delta-hedging strategy:
- (i) Define the concept of hedging error.
  - (ii) Explain the impact on the hedging error of frequency of rebalancing.

- 4.** (8 points) The Multnomah Options Company has just written 250 European call option contracts on a non-dividend paying stock.

You are given:

Exercise Price = \$150

Current Stock Price = \$145

Risk free rate = 2%

Time to maturity = 18 months

Options per contract = 100

Call option price = \$25

The CFO is inquiring about a stop-loss strategy.

- (a) (2 points) Describe the strategy and explain its pros and cons.

You are given the following information:

Volatility	$N(d_1)$	$N(d_2)$
32%	0.574	0.418
34%	0.579	0.414
36%	0.584	0.409
38%	0.589	0.405

- (b) (2 points) Estimate the implied volatility given the above option price.

Assume that the stock price is now \$146 and the implied volatility is now 32%.

- (c) (2 points) Calculate

- (i) The new delta of the option written by the company.
- (ii) The number of shares of the stock that need to be bought or sold to make the combined position delta neutral.

- (d) (1 point) Calculate the rho of the option.

Assume the risk free rate has dropped by 50 basis points.

- (e) (1 point) Estimate the profit or loss to be realized if the Company closes the open position.

- 5.** (5 points) You are given the GARCH(1,1) model as follows:

$$\sigma_n^2 = \omega + \alpha \mu_{n-1}^2 + \beta \sigma_{n-1}^2$$

Where  $\mu_n$  is the percentage change in the asset price between the end of day  $n-1$  and the end of day  $n$ ,  $\sigma_n^2$  is the variance rate per day, and  $\omega$ ,  $\alpha$ , and  $\beta$  are constants.

You are modeling the daily volatility,  $\sigma_n$ , of the S&P 500 index using the above model, with the following information.

- S&P 500 index closed at 1400 yesterday.
  - The daily volatility of the index was estimated at 1.00% per day yesterday.
  - $\omega=0.00002$ ,  $\alpha=0.05$  and  $\beta=0.93$ .
- (a) (2 points)
- (i) Estimate the daily volatility, if S&P 500 index closed at 1440 today.
  - (ii) Calculate the expected value of  $\sigma_n^2$  at the end of five days from today.
  - (iii) Interpret parameter  $\beta$  in the GARCH(1,1) model.
  - (iv) Determine whether the given GARCH(1,1) model is a stable model and justify your answer.
- (b) (3 points) Compare autoregressive models (ARCH and GARCH) to models from the stable distribution family and recommend one for modeling stock returns.

- 6.** (*7 points*) As the chief investment officer of Tahquamenon Insurance Company (TIC) you are evaluating two projects using a decision tree analysis (DTA) and real option analysis (ROA).

Both projects have a contraction option to scale down operations by 40% at any time during the next two years. Each project can be sold at fair value at the end of the second year. You are given the following:

Project	Initial Investment	PV without Flexibility	Annual Volatility	Cash Received from Contracting Operation
1	100 million	125 million	35%	50 million
2	100 million	125 million	25%	54 million

Assume that:

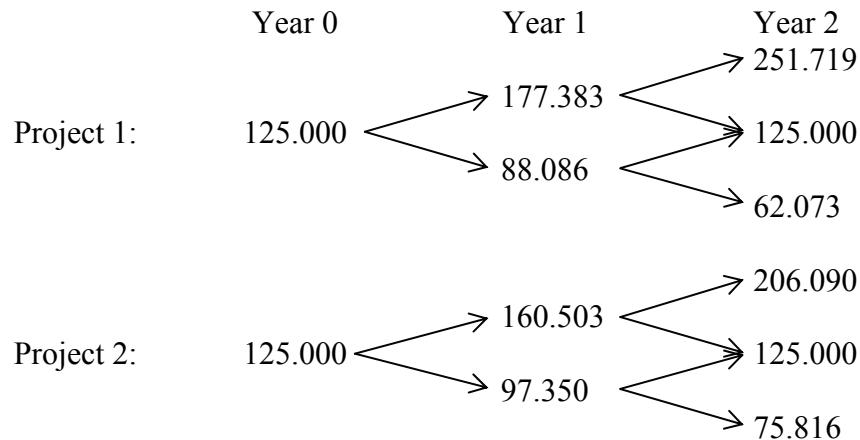
- The WACC for Tahquamenon Insurance Company is 12%.
- The risk-free rate is 2%.

(a) (*1 point*)

- (i) Describe how NPV and ROA respond to the resolution of uncertainty in different ways;
- (ii) Describe how DTA and ROA assume discount rates in different ways.

## 6. Continued

The binomial trees of present values (PV) are given below for the two projects:



(b) Calculate the value of

- (i) (2 points) Project 1 using DTA
- (ii) (2 points) Project 2 using ROA

The value of Project 2 using DTA is 30.035

The value of Project 1 using ROA is 33.039

(c) (2 points) Recommend and justify which project the company should invest in.

- 7.** (*7 points*) Olo'upena Life Insurance Co. ran 1000 stochastic scenarios on its variable deferred annuity business. Below are the total balance sheet requirement results  $X_i$  for these 1000 scenarios in descending order.

2000, 1500, 1200, 1100, 1050, 950, 880, 700, 650, 580, ..... 50, 40,  
35, 30, 30, 20, 15, 10, 5, 0

The company sets the total balance sheet provision at CTE(99.5%) level.

- (a) (*1 point*) Calculate the CTE(99.5%) of the given results.
- (b) (*2 points*) Estimate the standard error of CTE(99.5%).

You ran another set of 1000 scenarios and the CTE(99.5%) is changed to 850. Your manager is concerned that the sampling error is “too large,” but she doesn’t want to increase the sampling size. You are considering two approaches to reduce sampling errors.

- 1. Importance Sampling Technique
- 2. Control Portfolio approach
- (c) (*1 point*) Describe the Importance Sampling Technique and show how the CTE formula needs to be adjusted to account for this method.
- (d) (*1 point*) Assess the appropriateness of Importance Sampling Technique for this VA block modeling based on results in (a) and (b) above.
- (e) (*2 points*) Describe the steps to implement the Control Portfolio approach.

- 8.** (6 points) The owners of Yosemite Insurance Company are considering different management compensation plans:
- Plan 1 bases compensation solely on the company's growth in earnings during the last 12 months.
  - Plan 2 bases compensation on the company's stock price performance relative to other companies in the industry.
- (a) (3 points) Describe the strengths and weaknesses of each Plan above.
- (b) (1 point) Explain how a value based management plan could provide better incentives to the managers.
- Lloyd and Harry are the head managers of Yosemite's Life and P&C segments, respectively. Under Plan 1, both managers would be paid \$10,000 for every 1% of growth in total company earnings. Plan 3, a value based management compensation plan, would pay Lloyd and Harry 20% of the value each creates in his business segment.
- You are given the following:
- Total company earnings grow by 7% in 2012.
  - The Life segment economic cash flow for 2012 equals 1,000,000.
  - The P&C segment economic cash flow is 2,000,000.
  - The Life segment Cost Of Capital charge is 1,000,000.
  - The P&C Cost Of Capital charge is 1,000,000.
- (c) (1 point) Calculate each of Lloyd's and Harry's compensation for 2012 under both Plan 1 and Plan 3.
- (d) (1 point) Recommend and justify which compensation plan should be implemented.

**\*\*END OF EXAMINATION\*\***  
**Morning Session**

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