FETE Model Solutions
Spring 2012

1. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

Sources:

Hull, John *Options, Futures, and Other Derivatives,* Chapter 17 – The Greek Letters

Hull, John *Options, Futures, and Other Derivatives,* Chapter 13 – The Black-Sholes-Merton Method

Commentary on Question:

This question explores delta and vega hedging in practice. It also tests the candidates' knowledge of futures contracts including their delta and gain/loss.

Solution:

(a) Identify the features of Wonka Life's Equity-Linked GIC product that present risk exposures that may require hedging.

Commentary on Question:

Many candidates provided a shopping list of typical risks, without identifying any features that present risk exposures. A list of typical risks does not adequately demonstrate a candidate's ability to recognize and communicate exposures to risks inherent in financial products. Full credit could only be given where these features were explicitly identified.

Features requiring hedging:

- Return of principal after 5 years as floor
- 75% of percentage increase in S&P 500 total return index as S&P participation

(b) Recommend which of the following approaches is more appropriate for hedging the Wonka Life's Equity-Linked GIC block. Justify your recommendation.

1. Continued

Commentary on Question:
A common problem for candidates was to confuse put and call options. Candidates are expected to be comfortable with a variety of derivative products and know when and how they can be used. Many candidates correctly noted that options for the specified period (4.5 years) may not be available.

Candidates overall tended to ignore the fact that futures provide only delta hedging. Some answers noted that gamma and vega exposure require frequent rebalancing of the futures position.

Overall, justification of recommendations was not well thought out. A recommendation on its own would not receive full credit as this does not demonstrate analysis and synthesis of course material. Also, many justifications were simply a list of the characteristics of the chosen approach. Many of the recommendations for an approach relied on cons of the opposite approach. The most complete responses recognized that pros and cons exist for either approach, identified the most important factors, and stated a recommendation. Knowledge and understanding of the case study material was essential to answering this part adequately and receiving full credit.

(i) Static hedging strategy using options

- Set up the hedge initially and never adjust it ("hedge and forget")
- Can buy S&P 500 call options to hedge the 75% participation
- May not have options with required term in the market

(ii) Dynamic hedging strategy using futures

- Little transaction costs for futures
- Requires resources to maintain it
- Only delta hedging with futures
- Set up a position so that the net delta of the asset and liability is zero (delta neutral position)
- The hedge has to be adjusted frequently (rebalancing)

Recommend static hedging with justification (example):
Given the size of the 55M business, it is not worth setting up the resources for dynamic hedging since it requires daily modeling and trading of the hedge portfolio, or

1. Continued

Alternatively, recommend dynamic hedging with justification (example): S&P index changes daily → delta of liability changes as index moves. Therefore should adjust hedge assets frequently (daily?), interest rate changes in market also affect delta of portfolio.

(c) Explain why a static hedge may not work perfectly.

Commentary on Question:
Most candidates were able to adequately explain why static hedges may not work perfectly.

- The delta hedge only works for a short period of time.
- The hedge becomes less effective as liability changes.
- The delta hedge does not work well during market gap
- Need to consider other Greeks.

(d) Explain why a dynamic hedge may not work perfectly.

Commentary on Question:
Most candidates identified that dynamic hedging needs to rebalance frequently (continuously) which incurs transaction costs. Candidates should also realize that with both approaches there is a need to consider the "other Greeks." Very few candidates completely explained that the BSM model is based on several limiting assumptions, none of which apply readily in the real world. Most if not all financial models are developed in theory based on definite assumptions which are ultimately relaxed when applied to the real world. Ignoring this reality can have perilous consequences to a hedging strategy, as many companies discovered in 2008.

- Need to continuously rebalance
- Transaction costs are involved
- Need to consider other Greeks.
- The BSM formula based on unrealistic assumptions:
  - Return and vol are constant
  - Short selling of securities with full use of proceeds
  - No transaction costs or taxes
  - All securities are perfectly divisible
  - No dividends during life of the derivatives
  - No riskless arbitrage opportunities
  - Security trading is continuous
  - Risk free rate is constant & the same for all maturities

1. Continued

(e) Recommend enhancements to the dynamic hedging strategy to improve hedge effectiveness.

Commentary on Question:
Many candidates suggested daily rebalancing as the sole enhancement. The candidate should recognize that in dynamic hedging with futures only delta is hedged, so the hedge strategy should be extended to consider Greeks beyond the delta (in line with earlier comments above). Also, candidates should recognize that more frequent rebalancing is only useful if necessary (why daily if markets are steady?), and that to use set limits to trigger rebalancing could make the process more cost efficient.

Hedge second order Greeks: Gamma or Vega

Rebalancing tolerances (set limits to trigger rebalancing)

2. Learning Objectives:

Learning Outcomes:

Sources:
Hull, John, *Options, Futures, and Other Derivatives,* Chapter 17

Solution:
(a) Define the following risks for this VA benefit:

Commentary on Question:
Candidates generally did well on this part. It is important to remember that GMDB is a put option that Wonka sells to its policyholder.

(i) Delta

Delta is the rate of price change of the instrument with respect to the price change of the underlying asset.

Δ = Change in option price / Change in underlying asset price.

The return of Principal feature is a put option that Wonka sold to its policyholder.

A put option has a negative delta.

(ii) Gamma

Gamma is the rate of the change of the instrument's delta with respect to the price change of the underlying asset.

The 2nd partial derivative of the instrument with respect to asset price.

$\Gamma = (\partial^2 \Pi)/ (\partial S^2)$

If Gamma is small, then delta changes slowly.

(iii) Vega

Vega is the rate of change of the instrument's value with respect to the volatility of the underlying asset.

$\Upsilon = \partial\Pi/\partial\sigma$

2. Continued

(b) Confirm that the Greek values of the liability that Wonka gave you are reasonably correct. Show your work.

Commentary on Question:
Candidates generally did well on this part, however many candidates did not calculate Gamma correctly.

The Delta may be confirmed in one of the following two ways:

(i) "Increase 1% - Decrease 1%"
(-59 - (-66)) / (1100 * 2%) = 0.318

(ii) Consider
"Increase 1%" - "Current" = (-62 - (-66)) / (1100 - 1100 * 0.99) = 0.3636
"Current" - Decrease 1%" = (-59 - (-62)) / (1100 * 1.01 - 1100) = 0.2727

The average of these two 1-sided changes are (0.3636 … + 0.2727 …) / 2 = 0.318

The Gamma may be confirmed in one of the following two ways:

(i) [-62 * 2 - (-59)-(-66)] / [(1100 * 1%)^2] = 0.008

(ii) Using the above results in the confirmation of Delta, we get:

(0.3636 - 0.2726) / (0.5 * (1100 *1.01 - 1100 * 0.99)) = 0.0083

The Vega may be confirmed in one of the following two ways:

(i) Vega: (-68-(-56))/2%= -600

(ii) "Increase 1%" - "Current"
= (-68 - (-62))/ (0.26 - 0.25) = -600

2) "Current" - Decrease 1%" = (-62 - (-56)) / (0.25 - 0.24)
= -600

(c) Determine the notional amount of the forward contract and whether it is to be short or long.

2. Continued

Commentary on Question:
Candidates should remember that the GMDB liability becomes more negative as the S&P 500 drops, and we have a positive delta. This means we should short the forward contract so that it can offset the liability.

We need 0.318 / 1 = 0.318 of S&P 500 forward contract, because the forward contract's delta = 1

We should short sell forward contract, to offset the positive delta from the liability.

(d) Determine the number of units of the options and the notional amount of the forward contract and whether it is to be short or long.

Commentary on Question:
Some candidates were confused about the sign of the Gamma and Vega in the 2 equations below. Also, a lot of candidates were unable to calculate the updated Delta correctly because they omit the given liability of 0.318 in the calculation of -8.6 (see below).

X as number of units for option A
Y as number of units for option B
0.05X + 0.08Y = -0.008
2X + 1.2Y = 600
X = 480 (buy long) for option A
Y = -300 (sell short) for option B
Delta becomes 480*(-0.3)-300*(-.45) + 0.318 = -8.6
Buy long 8.6M of forward contract to offset the negative delta.

(e) Demonstrate that your full hedging strategy works when the S&P 500 decreases 1% and its volatility increases 1%.

Commentary on Question:
Some candidates omitted the amounts of hedge assets calculated in Part (d) and the current level of the S&P 500 (1100) in the calculations. Furthermore, some candidates incorrectly assumed that in a full hedge strategy some or all of the three given Greeks will automatically become 0 and so there will be no change in the value of net liability.

From the asset side:
Forward Contract Change = 8.6 * 1 * 1100 * -1% = -95

Option A Price Change = 480(-0.3*1100*-1%+ 0.5*0.05*(1100*-1%)^2 + 2*1%) = 3,046

2. Continued

Option B Price Change = -300(-0.45*1100*-1% + 0.5*0.08*(1100*-1%)^2 + 1.2*1%)
= -2,942

Total change from assets = -95 + 3046 - 2942 = 9

From the (pre-hedge) liability side:
The change from GMDB liability = -72-(-62) = - 10

Net change in position (net liability) = -1, which shows the offset expected from the hedging program.

3. Learning Objectives:

1. Modern Corporate Financial Theory

Learning Outcomes:

(1d) Define and compare risk metrics used to quantify economic capital and describe their limitations.

Sources:

Application of Coherent Risk Measures to Capital Requirements in Insurance," Artzner, NAAJ, Vol. 3, No. 2, http://www.soa.org/library/journals/north-american-actuarial-journal/1999/april/naaj9904_1.pdf

Commentary on Question:

This question was derived from material in a new study note so there was an expectation that candidates would put some effort towards learning this material. The question was designed to test the primary calculation in the study note which could be derived by well prepared candidates. In retrospect, this question may have been too difficult to do under exam conditions. While exam performance did not meet our initial expectations, the performance was consistent in that very few candidates scored points. This was taken into account in setting the pass mark.

Solution:

(a) Calculate margin requirements for the option portfolio under each of the following approaches:

(i) The SEC method

Commentary on Question:

Candidates got the pairings correctly, however many did not realize that zero margin is required to be held if strike of long call is less than the strike of the short call.

Need to pair call spreads with strike prices nearest to each other. We get the following pairings:

(long, short)

(10, 20)
(45, 35)
(65,60)
(10,10)
(100,70)

If strike of long call is less than or equal to the strike of short call, no margin is required to be held. This is because the spread position in this case will never have a negative value.

3. Continued

If strike of long call is higher than the strike of the short call, margin required is the difference between the strike prices. This is the maximum possible loss of the spread position.

Margin for (10,20) = 0 (since strike of long call is less than strike of short call)

Margin for (45,35) = 10
Margin for (65,60) = 5
Margin for (10,10) = 0
Margin for (100,70) = 30

Total margin required is: 0 + 10 + 5 + 0 + 30 = 45

(ii) The SPAN method

Commentary on Question:
Candidates did very poorly on this part of the question; many understood that the SPAN method required calculations at different scenarios, but few remembered the details of the calculations. For example, some took the average or sum of the scenarios' margin values instead of the maximum. Some also got distracted with the probabilities given within the stem which were not needed in order to calculate the margin requirement.

The SPAN method requires calculations under 16 scenarios.

Scenarios 1 – 7: UP move in volatility along with 3 up moves in CEI price, 3 down moves in CEI prices, and no move in CEI price.

Scenarios 8 – 14: DOWN move in volatility along with 3 up moves in CEI price, 3 down moves in CEI prices, and no move in CEI price.

Scenario 15 & 16: 2 "extreme" moves in CEI price

In order to calculate the margin requirement, we take max loss of:
[scenario 1 – 14; 35% of loss in extreme scenarios]

The specified range for the down move is 50 →20, and the specified range of the up move is 50 → 80. So, the specified range for scenarios 1 – 14 is 20 through 80.

3. Continued

Up & Down moves are calculated at 1/3, 2/3, and 3/3 of the specified range, therefore we need values of the portfolio at these 7 CEI prices: 20, 30, 40, 50, 60, 70, 80. Note that the portfolio value is same in UP and DOWN volatility scenarios.

V(70) = V(65) + 5 - 5 = -5 since now both Short(60) and Long(65) have moved "in the money" by 5 which nets to 0

V(80) = V(75) - 5 = -15 since Short(70) is now "in the money" by another $5

V(100) = V(95) - 5 = -35 since Short(70) is now "in the money" by another $5. The Long(100) is now "at the money" but contributes nothing

Scenario 1 = V(20) = 0 = down 3/3 of range; margin = 0
Scenario 2 = V(30) = 10 = down 2/3 of range; margin = 0
Scenario 3 = V(40) = 5 = down 1/3 of range; margin = 0
Scenario 4 = V(50) = 0 = no move; margin = 0
Scenario 5 = V(60) = 0 = up 1/3 of range; margin = 0
Scenario 6 = V(70) = -5 = up 2/3 of range; margin = 5
Scenario 7 = V(80) = -15 = up 3/3 of range; margin = 15

Values of scenarios 8 through 14 are the same as the values for scenarios 1 through 7 because the volatility path doesn't affect the option value at expiration.

Scenario 15: 35% * V(0) = 0
Scenario 16: 35% * V(100) = 35% * -35 = -12.25; margin = 12.25

Final Margin requirement under SPAN: max(0,0,0,0,0,5,**15**, 0,0,0,0,0,5,15,0,12.25) = **15**

(b) Outline the disadvantages of:

(i) The SEC method

- May not be coherent if it cannot be used to effectively regulate or manage risks
- Considers too many scenarios
- Considers too few risks
- Scenarios may not fulfill the model's independent, arbitrage-free conditions
- Can consider impossible risks or scenarios

3. Continued

- Margin required exceeds empirically observed worst case loss of V(100)=35
- Takes no account of current market conditions
- Too strict for some portfolios

(ii) Using VaR

Commentary on Question:
Candidates who scored points on this question generally did so in this section.

- Fails subadditivity property of coherence
- Fails subadditivity because prices are not jointly normally distributed
- Prone to computational and statistical problems
- Not used by any organized exchanges so a regulator may not have much familiarity
- Unable to recognize undue concentrations of risks
- Diversification of risks may lead to an increasing risk measure
- Fails to encourage a reasonable allocation of risk
- May create severe aggregation problems
- May prohibit diversification because it does not take into account the economic consequences of events
- VaR quite different from TailVaR for heavy-tailed distributions

4. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2a) Describe the steps necessary to obtain funds for a given project or firm from any specified source, and be able to recommend a specific approach to raising capital in a given situation.

(2g) Recommend an optimal capital structure and how to implement it for a given business or strategy and be able to justify the recommendation.

Sources:

FET-180-07

Commentary on Question:

The question tests candidates understanding of how to value a firm using different forms of debt for an acquisition. There is a similar numerical example in the study note.

Solution:

(a) Analyze possible reasons for using convertible debt and reverse convertible debt instead of nonconvertible debt.

Commentary on Question:

In general, candidates were able to provide some reasons for using convertible debt and reverse convertible debt.

However, not many candidates had included the points that

- *shareholders keep more of the downside risk under convertible debt*
- *equity is diluted by exercise of the conversion option*
- *crowding out costs are reduced under reverse convertible debt.*

In addition, some candidates provided similar listings not relevant to the question.

<u>Convertible Debt</u>

Probability of bankruptcy declines
Shareholders keep more of the downside risk
Bondholders participate in the potential upside
Risky projects less attractive
Interests of the shareholders and bondholders more aligned

Reverse Convertible Debt (RCD)

Probability of bankruptcy declines

- When debt becomes a problem, issuer can convert it to equity

4. Continued

Changes incentives to undertake new investments
- Equity is diluted by exercise of option
- Closer alignment of interests

Crowding out costs are reduced
- Will unlever firm when it has a loss – countercyclical

(b) Calculate the value of equity prior to any acquisitions.

Commentary on Question:
In general, candidates did well in calculating the value of equity prior to any acquisitions. However, some candidates incorrectly calculated the value of equity to be 0 instead of -200 under Scenario 1. This required a more nuanced response than the example in the reading.

At time zero the scenario is unknown. Candidates using -200 showed (c) balancing out, for instance if a parent company were able to work out the wholly owned Manchego subsidiary. Alternatively, the equity holders hold a put option worth – (-200)/3 = 67 at the expense of the debtholders, in addition to a direct equity value of (-200+300+800)/3 = 300 value calculated below. Candidates didn't explain this put option value or why they floored the equity value at 0 in scenario 1. There would be a number of options for recapitalizing the company if scenario 1 emerges.

Scenario	1	2	3	Expected Value
PV Earnings	0	500	1000	500 = (0+500+ 1000)/3
Debt	200-200	200	200	133 = (0+200+200)/3
Equity	-200+200	300	800	367 = (0+ 300+800)/3 Or = 500 – 133

Value of firm is 500.
Equity is 500-133=367. OR (-200+300+800)/3 + 67 = 367
Equity holders also effectively have a put option worth (200+0+0)/3 = 67
Debt holders are short this put option.

(c) Calculate the value of the firm and the value of the equity after the acquisition, under each form of financing. Assume no bankruptcy costs.

Commentary on Question:
In general, the majority of the candidates did very well in calculating the value of the firm and the value of equity under the non-convertible debt.

4. Continued

Some candidates correctly calculated the value of the firm and the value of equity under the convertible debt and reverse convertible debt. In order for candidates to receive maximum credit they needed to demonstrate and calculate under what value of the firm it would convert under convertible and reverse convertible debt structures.

Acquisition	Capital Cost	PV Earnings	E(NPV)
Chevre Company	300	500	200 = 500 – 300

(i) Non convertible debt

Acquisition of Chevre Company

Scenario	1	2	3	Expected Value
Old Debt	200	200	200	200
New Debt	300	300	300	300
Equity	-200 + 200 = 0	300 + 200 = 500	800 + 200 = 1000	500
Value of Firm (Total)	500	1000	1500	1000

(ii) Convertible debt

Acquisition of Chevre Company
Convertible to 300 * 0.25 = 75 shares
Convertible debt becomes 75/(75+100) = 3/7 of total.
Convert if 3/7 * value of equity > 300
3/7 * (value of firm – senior debt) > 300
3/7 * (value of firm – 200) > 300
Value of firm > 900

Scenario	1	2	3	Expected Value
Old Debt	200	200	200	200
New Debt	300	3/7 * (1000-200) = 342.86	3/7 * (1500-200) = 557.14	400
Equity	0	4/7 * (1000-200) = 457.14	4/7 * (1500-200) = 742.86	400
Value of Firm (Total)	500	1000	1500	1000

4. Continued

(iii) Reverse convertible debt (RCD)

Acquisition of Chevre Company
Convertible to 300 * 1.0 = 300 shares
RCD becomes 300/(300+100) = 3/4 of total.
Convert if 3/4 * value of equity < 300
3/4 * (value of firm – senior debt) < 300
3/4 * (value of firm – 200) < 300
Value of firm < 600

Scenario	1	2	3	Expected Value
Old Debt	200	200	200	200
New Debt	¾ * (500 – 200) = 225	300	300	275
Equity	1/4 * (500 – 200) = 75	500	1000	525
Value of Firm (Total)	500	1000	1500	1000

(d) Explain in words whether Manchego Corp. should pursue the Chevre acquisition.

Commentary on Question:
In general, the majority of the candidates did not do very well on this part.

Many candidates stated that Manchego Corp. should pursue the Chevre acquisition due to an increase in equity or firm value after the acquisition or a reduction of the probability of default or bankruptcy.

However, in order to receive maximum credit an answer should include items such as concerns of bond holders, dilution of shareholder value through the use of convertible debt, and increasing shareholder value.

Considerations surrounding which option increases the value of equity:
Discussion surrounding options relating to firm value.
Considerations surrounding concerns of bond holders.
Considerations surrounding increasing shareholder value.
Concerns surrounding risk/probability of default.
Concerns surrounding dilution of shareholder value through the use of convertible debt.
Incentives of firm to undertake further investments under different options.
Discussion surrounding the potential risks of each of the options asked to indicate.

4. Continued

Proposal in favor or against pursuing the acquisition (valid based on above or related criteria or not)

5. Learning Objectives:

2. Corporate Financial Applications

3. Derivatives and Pricing

Learning Outcomes:

(2a) Describe the steps necessary to obtain funds for a given project or firm from any specified source, and be able to recommend a specific approach to raising capital in a given situation.

(2e) Apply real options analysis to recommend and evaluate firm decisions on capital utilization.

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

Sources:

Copeland, Weston, and Shastri, *Financial Theory and Corporate Policy*, Multi-Period Capital Budgeting Under Uncertainty: Real Option Analysis, Chapter 9

Greenblatt and Titman, *Financial Markets and Corporate Strategy*, 2nd Edition, The Information Conveyed by Financial Decisions, Chapter 19

Megginson, W.L., *Corporate Finance Theory*, Understanding and Accessing Financial Markets, Chapter 9

Doherty, *Integrated Risk Management*, Contingent Leverage strategies and Hybrid Debt, Chapter 13

Hull, *Options Futures & Other Derivatives*, Basic Numerical Procedures, Chapter 19

Solution:

(a) Outline the advantages and disadvantages of financing the new product line through each of the following methods:

(i) Equity

(ii) Hybrid debt

(iii) Private debt

5. Continued

Commentary on Question:
Candidates did well on this section which was generally recall.

(i) Equity

Advantages:
- Do not have to repay principal
- Agency viewed favorably

Disadvantages
- Negative signal to the market
- Dilution to earnings to existing shareholders

(ii) Hybrid debt

Advantages:
- Use embedded option to provide offset to lower wholesale milk costs
- Increase the firm value by reducing the chance the firm will encounter bankruptcy

Disadvantages:
- Complex
- Basis risk

(iii) Private debt

Advantages:
- Locked in fixed cost
- Tax deductibility of interest
- Keep proprietary all confidential information

Disadvantages:
- Increase chance of default
- Contains covenants that limit the borrower's operating and financial discretion
- Restrict future financing

5. Continued

(b) Calculate the value to Peyniri of the new product line under each of the following expansion scenarios using the risk-neutral probability approach. Show your work.

(i) Fund using 100% common stock

(ii) Fund using 60% common stock and 40% private debt financing

(iii) Fund using 100% private debt financing

Commentary on Question:
The question asked the candidates to apply real option analysis to analyze different financing options.

Candidates did poorly on this section. Common mistakes:

- *Not recognizing that this is a real option analysis problem*
- *Forgetting to use the risk-free discount rate and risk-neutral q to determine the value*
- *Unable to identify the items that are changed under the 3 scenarios:*
  - *I(0)*
  - *Option strike prices*

1. Use risk neutral valuation, use risk-free rate discount and risk-neutral q
   $V(0) = (uV * q + dV * (1-q))/(1+rf) – I(0)$
   $q = (1+rf-d)/(u-d)=14.35\%$
   $I(0) = 10$ since using 100% common stock to fund
   $V(0) = (30 * 0.1435 + 7 * (1-.1435))/ (1.03) − 10$
   $V(0) = 0$

2. 6 m stock 4 m debt => pay debt in 1 year, can default on debt, this option has value
   Debt cost of capital is 5%,
   $= 4 * 1.05 = 4.2$ m
   $V(0) = (Max(uV – D, 0)* q + max(dV – D, 0) * (1 − q))/(1+rf) – I(0)$
   $I(0) = 6$ since using 60% common stock to fund
   $V(0) = ((30 − 4.2) * 0.1435 + (7 − 4.2) *(1 − 0.1435))/ (1.03) – 6$
   $V(0) = −0.0772$ m

3. 10 m debt => pay debt in 1year $= 10 * 1.05 = 10.5$ m
   $V(0) = (max(uV – D, 0)* q + max(dV – D, 0) * (1 − q))/(1+rf) – I(0)$
   $I(0) = 0$ since using 100% private debt financing
   $V(0) = ((30 − 10.5) * 0.1435 + (0) * (1 − 0.1435))/ (1.03) − 0$
   $V(0) = 2.717$ m

5. Continued

(c) Calculate the price of the 1-year vanilla European call option.

Commentary on Question:
The question asked the candidates to demonstrate understanding of option pricing techniques to determine the price of a 1-year vanilla European option.

In general, the candidates did well on this part, but common mistakes were to:

- *Mistake the convenience yield as the risk-free rate*
- *Not using the semi-annual time-step*
- *Using a discrete interest rather than the continuous discounting specified in the question*

$\sigma = 0.15$, $t = 0.5$, $u = \exp(\sigma t^{1/2}) = 1.1119$
$d = 1/u = 0.8994$

t	0	0.5	1
			Node 4
		Node 2	
	Node 1		Node 5
		Node 3	
			Node 6

Price at Node 2, Up = u * 2 = 2.2238
Price at Node 3 = Down = d * 2 = 1.7987
Price at Node 4 = Up Up = u * u * 2 = 2.4726
Price at Node 5 = Up Down = Down Up = u * d * 2 = 2
Price at Node 6 = Down Down = d * d * 2 = 1.6177
Node 4 payoff = 2.4726 - 2 = 0.4726
Node 5 payoff = 2 - 2 = 0
Node 6 payoff = MAX(1.6177 - 2,0) = 0

$p = (\exp(rt) - d) / (u - d) = 0.5446$

Option value at Node 2 = exp(- rf * t) * (p * Node 4 payoff + (1 − p) * Node 5 payoff) = 0.2536

Option value at Node 3 = exp(- rf * t) * (p * Node 5 payoff + (1 − p) * Node 6 payoff) = 0

5. Continued

Option value at Node 1 = exp(- rf * t) * (p * Node 2 option value + (1 – p) * Node 3 option value) = 0.136

Total Option value for 1 million lbs of milk = 136,041

(d) Recommend a capital structure for the new product line based on your analysis in (a), (b) and (c) above. Support your position.

Commentary on Question:
This question asked the candidates to put everything together in analyzing the capital financing options and recommend the most appropriate method.

In general, this part was answered poorly.

Common mistakes:

- *Most candidates failed to do the calculations in (b) correctly which affects their choice of financing method.*
- *Most candidates did not relate part (c) to the rest of the question.*

- From part (b), the 100% Private Debt financing structure has the highest value
- Private Debt has more advantages than equity, from reasons in part (a)
- Hedging against an increase in raw materials costs will increase the firm value by reducing the likelihood of bankruptcy
- From part (c), the cost of hedging is relatively small compared to all these advantages

6. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3h) Describe and evaluate equity and interest rate models.

(3i) Contrast commonly used equity and interest rate models.

Sources:

Hull, *Options, Futures, and Other Derivatives*, Chapter 30

FET-158-09: Babbel and Fabozzi, Investment Management for Insurers, Chapter 11, The Four Faces of an Interest Rate Model

Commentary on Question:

This question was trying to test the candidate's understanding of interest rate models and under which applications they can be appropriately used. In particular, candidates had to demonstrate a solid understanding of the differences between an equilibrium model (i.e. Vasicek) vs. a no-arbitrage model (i.e. Hull-White).

To receive maximum points, candidates were required to not only identify the correct interest rate model, but also explain why it should be used for a particular application.

Solution:

(a) Identify the interest rate model $dr = a(b-r)dt + \sigma dz$ and describe the following components of the model:

(i) dr

(ii) $a(b-r)$

(iii) σdz

Commentary on Question:

Most candidates answered this question well.

This model is the Vasicek Model, which is an Equilibrium Model.

(i) dr is an Ito process to describe the short rate, r.

(ii) *a(b-r):* The short rate is pulled to a level b at rate a. b is the mean reversion target, a constant. a is the mean reversion strength, a constant.

(iii) σdz is a normally distributed stochastic shock term. σ is a constant scaling of the shock, i.e. the volatility.

6. Continued

(b) Compare the model $dr = a(b - r)dt + \sigma dz$ to the following model:

$$dr = [\theta(t) - ar]dt + \sigma dz$$

for the following applications:

(i) Valuing and hedging interest rate derivatives

(ii) Modeling financial projections

Commentary on Question:
Most candidates properly identified the second interest rate model and which model was appropriate to use under each of the two applications. However, many candidates did not clearly explain why a particular model was suitable for the application.

This model is the Hull-White Model. It is a no-arbitrage model:

The Hull-White Model is similar to the Vasicek Model as follows:
- Both incorporate mean reversion
- Both can have negative values for r
- Hull-White is an extension of the Vasicek model by letting mean reversion vary over time, i.e. changing *ab* into time-varying $\theta(t)$

(i) When valuing and hedging interest rate derivatives, it is important that the model used be consistent with the initial term structure observed in the market. Furthermore, risk neutral interest rate scenarios are preferred for pricing interest rate derivatives as the term premium embedded in each spot rate does not have to be identified.

A no-arbitrage, risk-neutral model (e.g. Hull-White) is more appropriate for valuing and hedging interest rate derivatives *when market prices are complete and reliable*. Hull-White lets the mean reversion target vary by time. This lets the modeler calibrate the model to match currently observed prices.

An equilibrium, risk-neutral model (e.g. Vasicek) may be used for valuing and hedging interest rate derivatives when market prices are unavailable or unreliable. It may also be used for horizon pricing, where the derivative is priced on some assumed future state of the market.

6. Continued

(ii) When modeling financial projections, a realistic simulation is required for the test results to be useful. A no-arbitrage, realistic model is not of practical use as it is impossible to discriminate between model misspecification error and the term premia (i.e. confounding). Since the no-arbitrage form of a realistic model is not available, the equilibrium form (e.g. Vasicek) *must be used* for modeling financial projections. An equilibrium model attempts to capture the behaviors of the term structure over time.

(c) Explain why each of the models in (b) might be appropriate for stress testing.

Commentary on Question:
Only a minority of candidates were able to properly explain why an equilibrium model under the realistic measure was the appropriate model for stress testing.

For stress testing, a realistic term structure process is required for the test results to be useful. A risk-neutral term structure process would have zero term premia, which is not realistic. A no-arbitrage, realistic model is not of practical use as it is impossible to discriminate between model misspecification error and the term premia. Since the no-arbitrage form of a realistic model is not appropriate, the equilibrium form (e.g. Vasicek) must be used for stress testing.

The equilibrium form does not require that the predictions be used instead of the current market prices as the first point in a scenario. The equilibrium model is a statistical model of term structure behavior; by taking this approach it explicitly recognizes that this prediction will deviate from observed values, which is what one is doing in stress testing.

7. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3h) Describe and evaluate equity and interest rate models.

(3i) Contrast commonly used equity and interest rate models.

(3l) Recommend an equity or interest rate model for a given situation.

Sources:

Hull, *Options, Futures, and Other Derivatives*, Chapters 13 and 21

Hardy, *Investment Guarantees*, Chapter 2 - Modeling Long-Term Stock Returns

FET-115-08: Specialty Guide on Economic Capital

FET-178-11: Economic Capital Modeling Practical Considerations

Commentary on Question:

This question tests candidates' knowledge of the properties of a lognormal world of a stock price by calculating key elements (confidence interval, future expected price, and standard deviation) of a specific lognormal model.

This question also looks at the appropriateness of different stock price models (lognormal vs. regime switching lognormal in particular, though other models such as the generalized autoregressive conditional heteroskedastic model will also be given credit) and apply such knowledge to form an opinion.

For part (a) and part (b), candidates were divided. Many candidates were able to recall the right formula from Hardy's textbook and precisely implement the formula. The rest of candidates generally understood the concepts of confidence of intervals and standard deviation but either did not use right formula or did not do the calculation correctly.

For parts (c) and (d), candidates generally were able to provide some discussion of the appropriateness of the lognormal model vs. the regime switching lognormal model. Candidates need to illustrate all major points and form opinion based on them to get full credit.

Solution:

(a) Calculate the 95% confidence interval for the stock price in one year.

7. Continued

Use formula
$\ln S_T \sim N(\ln S_0 + (\text{mu} - \text{sigma}^2/2)*T, \text{sigma}^2*T)$
$\ln(25)+(8\%-(20\%)^2/2)*1=3.279$
$(20\%)^2*1=0.04$
$\ln S_T \sim N(3.279, 0.04)$
Standard deviation $= (0.04)^{0.5} = 0.2$
Hence, with 95% confidence,
$3.279-1.96*0.2 < \ln S_T < 3.279+1.96*0.2$
i.e. $2.89 < \ln S_T < 3.67$
This can be written as $\exp(2.89) < S_T < \exp(3.67)$
or $17.94 < S_T < 39.29$

(b) Calculate the expected stock price and the standard deviation of the stock price in one year.

$E(S_T) = S_0 * \exp(\text{mu}*T)$
$=25*\exp(8\%*1)$
$=27.08$
$\text{Var}(S_T) = S_0^2*\exp(2*\text{mu}*T)*(\exp(\text{sigma}^2*T)-1)$
$= 25^2*\exp(2*.08*1)*(\exp((20\%)^2*1)-1)$
$= 29.93$
StDev = sqrt(29.93) = 5.47

(c) Critique the assumption of lognormally distributed returns for modeling the stock for each of the two applications above.

Empirical studies indicate that LN fails to capture more extreme price movements, i.e. it fails to capture fatter tail risk. Not appropriate for EC as EC is a tail metric (use metrics such as CTE, VAR, Tail VAR, etc.). So LN is not appropriate for hedging as the crucial left tail risk is under-represented.

LN also fails to capture volatility bunching (i.e. volatility clustering), which is not appropriate for hedging.

LN does not fit for long time intervals, which is not appropriate to model long term stock price characteristics such as long-term mean-reversion.

LN does not allow for stock price autocorrelations.

(d) Propose alternative assumption(s) to that in (c) which would be more appropriate for modeling stock returns for each of the two projects above. Justify your proposals.

7. Continued

Regime Switching Lognormal Model (RSLN) would be more appropriate for both economic capital and hedging.

RSLN more accurately captures extreme observed behavior and fatter tails. This is more appropriate for EC since EC is a tail risk metric.

RSLN introduces stochastic volatility (by allowing the volatility to switch randomly between the volatility values corresponding to each regime) to capture volatility bunching. This is more appropriate for hedging. Reasons include, but are not limited to:
1. volatility is a key factor for option price;
2. volatility skew can be modeled (high volatility partnered with lower mean).

RSLN maintains the tractability of the LN model, which is good for both EC and hedging.

RSLN has been shown to provide a good fit to model stock return characteristics such as:

long-term mean-reversion
autocorrelation through the serial dependence on regimes.

8. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3m) Describe issues and best practices in the estimation or calibration of financial models.

Sources:

Hardy, *Investment Guarantees*, Chapters 2

Commentary on Question:

This question tests the application and understanding of regime-switching lognormal models. The candidates need to apply basic statistical knowledge such as mean, standard deviation, and correlation to answer this question.

Solution:

(a) Calculate the unconditional regime probabilities for Model A.

Commentary on Question:

Candidates did well in this part. This is a straightforward question if one recalls the unconditional regime probabilities formulas correctly. Some candidates illustrated how π_1 and π_2 are developed from p_{12} and p_{21} but no extra credit was given for this.

$\pi_1 = \Pr(p_t = 1) = p_{21}/(p_{21} + p_{12}) = 0.4/(0.4+0.1) = 0.8$
$\pi_2 = \Pr(p_t = 2) = 1 - \pi_1 = p_{12}/(p_{21} + p_{12}) = 0.1/(0.4+0.1) = 0.2$

(b) Calculate the following using Model A:

(i) The mean and standard deviation of Y_t

(ii) The correlation coefficient between Y_t and Y_{t+1}, $t > 0$

Commentary on Question:

Candidates were able to calculate the mean. A common mistake in the standard deviation calculation was missing the term $V[E[Y_t|\rho_t]]$. Most of the candidates did not attempt the correlation coefficient calculation at all.

$E[Y_t] = \pi_1\mu_1 + \pi_2\mu_2 = 0.8\text{x}0.01+0.2\text{x}-0.02 = 0.004$
$V[Y_t] = E[V[Y_t|\rho_t]] + V[E[Y_t|\rho_t]]$
$= \pi_1 \sigma^2_1 + \pi_2 \sigma^2_2 + (\pi_1 \mu_1^2 + \pi_2 \mu_2^2 - E[Y_t]^2)$
$= 0.8\text{x}0.04^2 + 0.2\text{x}0.1^2 + (0.8\text{x}0.01^2 + 0.2\text{x}-0.02^2 - 0.004^2)$
$= 0.003424$
SD of $Y_t = 0.003424^{0.5} = 0.0585$

8. Continued

$\text{Corr}(Y_t, Y_{t+1}) = \text{Cov}(Y_t, Y_{t+1})/(\text{V}(Y_t)\text{V}(Y_{t+1}))^{0.5}$
$\text{Cov}(Y_t, Y_{t+1}) = \text{E}[Y_t\, Y_{t+1}] - \text{E}[Y_t]\text{E}[Y_{t+1}]$

$p_{11} = 1 - p_{12} = 0.9$; $p_{22} = 1 - p_{21} = 0.6$
$\text{E}[Y_t\, Y_{t+1} | \rho_t = 1] = \mu_1(p_{11}\mu_1 + p_{12}\mu_2) = 0.01\text{x}(0.9\ \text{x}0.01+0.1\ \text{x}-0.02)=0.00007$
$\text{E}[Y_t\, Y_{t+1} | \rho_t = 2] = \mu_2(p_{21}\mu_1 + p_{22}\mu_2) = -0.02\text{x}(0.4\ \text{x}0.01+0.6\ \text{x}-0.02)=0.00016$

$\text{E}[Y_t\, Y_{t+1}] = \pi_1\, \text{E}[Y_t\, Y_{t+1} | \rho_t = 1] + \pi_2\, \text{E}[Y_t\, Y_{t+1} | \rho_t = 2]$
$= 0.8\text{x}0.00007+0.2\text{x}0.00016 = 0.000088$

$\text{Cov}(Y_t, Y_{t+1}) = 0.000088 - 0.004^2 = 0.000072$
$\text{Corr}(Y_t, Y_{t+1}) = 0.000072 / 0.0585^2 = 0.021$

(c) Calculate the following using Model B:

(i) The mean and standard deviation of Y_t

(ii) The correlation coefficient between Y_t and Y_{t+1}, $t > 0$

Commentary on Question:
Similar to (b), candidates did well on the calculation of mean but missed the term $V[E[Y_t|\rho_t]]$ in standard deviation formula. For the correlation coefficient, full credit was given only for an explanation of zero correlation.

Since the probabilities in Model B are the same as in Model A, the mean and standard deviation are the same as in question (b)(i).

$\text{E}[Y_t] = p_1\mu_1 + p_2\mu_2 = 0.8\text{x}0.01+0.2\text{x}-0.02 = 0.004$
$\text{V}[Y_t] = \text{E}[\text{V}[Y_t|\rho_t]] + \text{V}[\text{E}[Y_t|\rho_t]]$
$= p_1\sigma^2_1 + p_2\sigma^2_2 + (p_1\mu_1^2 + p_2\mu_2^2 - \text{E}[Y_t]^2)$
$= 0.8\text{x}0.04^2 + 0.2\text{x}0.1^2 + (0.8\text{x}0.01^2 + 0.2\text{x}-0.02^2 - 0.004^2)$
$= 0.003424$
SD of $Y_t = 0.003424^{0.5} = 0.0585$

The correlation coefficient is 0 in Model B. In the model structure $Y_t|(\rho_t = k) = \mu_k + \sigma_k\, \varepsilon_t$, Y_{t+1} depends on Y_t through the regime-switching process $\{\rho_t\}$ only. For Model B, $\{\rho_t\}$ is a sequence of independent random variables, which means Y_{t+1} does not depend on Y_t. Therefore, the correlation between Y_t and Y_{t+1} is 0.

8. Continued

(d) Outline the advantages and disadvantages of using Model A, compared with Model B, for hedging VA guarantees.

Commentary on Question:
Candidates did moderately well on this question. Some restated the advantages of Model A as disadvantages of Model B. No extra credit was given for a repeated answer.

Advantages of Model A:
Fatter tailed as model is aggregated over many periods. It is also caused by correlation and second order dependence.
There is more volatility clustering than Model B. The probability of a high volatility followed by a high volatility regime is 60% (p_{22}) in Model A, while it is 20% in Model B.
Volatility Clustering and fatter tailed are common characteristics of the stock price modeling process. This will affect the hedge effectiveness.
Model A displays more complex path dependent behavior. It is better for modeling policyholder behavior.

Disadvantages of Model A:
The model is more complex. There are 6 parameters in Model A compared to 5 parameters in Model B.

9. Learning Objectives:

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

Learning Outcomes:

(4a) Define capital market efficiency and the value of information.

(4c) Describe empirical evidence and results regarding market efficiency.

Sources:

Copeland, Weston, Shastri, *Financial Theory and Corporate Policy*, 4th Edition, Chapters 10 and 11

Commentary on Question:

The purpose of this question was to test understanding of the Efficient Markets Hypothesis and its application to market anomalies. In addition, the question tested the candidate's understanding and application of the common errors of empirical studies. In general, candidates did well on part (a) of this question but struggled when applying the concepts.

Solution:

(a) Identify and describe the efficiency form of markets to which you subscribe.

Commentary on Question:

Candidates did well on this part of the question. Most noted which form of the EMH they thought applied and described it correctly. Some candidates listed all 3 forms of efficiency without clearly identifying the form to which they subscribe. A few candidates clearly stated that they assumed market experts have private information and thus they subscribed to the "Strong" form. From their written answer, it was clear they understood the difference between the three forms and they were able to apply to 3 reasons from the problem. These responses earned partial credit even though the conclusion may have been incorrect.

I subscribe to the semi-strong form of efficiency, which states that no investor can earn excess returns from trading rules based on publicly available information. Since the historical performance (reasons (i) and (iii)) is publicly known and the experts' future predictions (reason (ii)) are public, you don't believe that will earn you excess returns.

(b) Identify and describe 4 market anomalies which don't support your efficiency form.

9. Continued

Commentary on Question:
Candidates did not do well on this question. Quite a few candidates did not understand what the question was asking and answered using behavioral finance examples. It was important to note that the question asked for anomalies that applied to the EMH in part (a) semi-strong form. Several other candidates included other anomalies that would have described the strong form. Some candidates were able to list a few key points from the list, but only a few candidates provided some general description, even fewer candidates were able to provide the examples from the reading.

4 market anomalies which don't support the semi-strong form of efficiency are:

1. **Initial Public Offerings (IPOs):** New issues generally show short-run positive and long-run negative abnormal returns. Studies of short-term return (by Ibbotson) showed that it was possible for new issues to earn excess returns of 11.4% in month of issue. Loughran and Ritter reported IPO'd equities underperformed benchmark portfolios by 30% over 5 year period following listing.
2. **Stock splits:** Positive abnormal returns are earned before the split but not afterward.
3. **Mutual Funds:** Minimize unsystematic risk and use professional expertise to earn abnormal returns. However, studies show above average gross returns can be earned but net of costs they cannot.
4. **Self Tenders and Share Repurchases:** Abnormal returns from a trading rule where one purchases stock the day following the announcement of a planned self-tender or open market repurchase.

Additional Commentary:
In addition to the 4 market anomalies listed in the model solution above, any of the examples of market anomalies below received full credit:

5. **Value Line Investor Survey:** Securities ranked 1 to 5 according to 4 criteria 1) E/P rank relative to others 2) price momentum factor 3) year to year relative changes in quarterly earnings 4) earnings "surprise" factor. Predictions use historical data and filter rule to predict future performance. 18 year price performance record assuming all value line ratings had been followed. Outperformance in securities ranked 1 and underperformance in securities ranked 5, so could go long 1 and short 5 to capture excess returns.
6. **Dual purpose funds:** A closed end fund which has two pieces: capital shares and income shares. Funds often sell at a discount. Closed end discounted funds tended to outperform the market. Closed end funds at a premium tended to underperform the market.

9. Continued

7. **Timing Strategies:** If VIX rises unexpectedly, it can earn statistically significant returns by buying futures on large-cap stocks and selling futures on small-cap stocks.
8. **Stocks that over/under shoot:** Extreme losers outperform extreme winners over next 5 years.

(c) Identify and describe 5 common errors of empirical studies.

Commentary on Question:
This part was a list question with a little application. Most candidates were able to list 5 key points; short descriptions were needed to earn full credit.

5 common errors of empirical studies are:

1. **Biased model of equilibrium returns:** Abnormal returns are not evidence of market inefficiency, but a result of a bad model.
2. **Specification searches:** It is always possible, if one tries long enough, to find a model that beat the market over a test period, but often can't when that model is extended to a different time period.
3. **Sample Selection bias:** Stocks that show abnormal returns before splitting do so because only stocks that have risen will split. Expecting post split returns because of pre-split data doesn't work.
4. **Survivorship Bias:** Historical data of only funds in existence today (survivors) introduces a bias to exclude funds that did not perform well and were shut down. Survivorship bias implies upward-biased results.
5. **Biased measures of return:** Geometric returns over time can overstate performance. The false conclusion can be avoided by measuring abnormal return as ratio of geometric returns on test portfolio to the geometric returns on the benchmark portfolio.

Additional Commentary:
In addition to the 5 common errors of empirical studies in the model solution above, any of the examples of common errors of empirical studies received full credit:

6. **Inappropriate portfolio weightings:** Should use value weightings of outcomes instead of equal weightings. Often equally weighted abnormal returns are reported with average driven by small firms.
7. **Failure to distinguish between statistical and economic significance:** To be economically inefficient, must show abnormal returns after transaction costs.

9. Continued

8. **Overestimating the frequency of opportunities for arbitrage:** In practice, arbitrage opportunities do not appear that often due to transaction costs. For the market to be inefficient, the frequency of opportunities must be high enough to allow economically significant arbitrage returns.

10. Learning Objectives:

1. Modern Corporate Financial Theory

Learning Outcomes:

(1a) Explain the various definitions of capital, including regulatory, rating agency and other risk-based capital requirements, the context in which they are appropriate, and how they affect decisions.

(1e) Apply the concept of economic capital and describe methodologies for allocating capital within a financial organization.

Sources:

FET-115-08: Specialty Guide on Economic Capital

Solution:

(a) Define and calculate the following for the Stilton Company:

(i) Tier 1 capital

(ii) Tier 2 capital

(iii) Tier 3 capital

Commentary on Question:

Most candidates were able to calculate each type of capital correctly. Some had trouble linking Tier 1 to Economic Capital, Tier 2 to Face Capital, and Tier 3 to Free Capital. Though the question asked for it, very few candidates gave the definition of economic capital.

(i) Economic capital

Sufficient surplus to maintain solvency at a given level of risk tolerance, over a specified time horizon.

EC = 78 + 38 + 29 = $145 million

(ii) Face capital

Difference between required capital and economic capital.

Face capital = 150 – 145 = $5 million

(iii) Free capital

Any capital held over and above the larger of the economic and required capital.

Free capital = 185 – 150 = $35 million

10. Continued

(b) List the advantages and disadvantages of 4 methods to allocate capital within an organization.

Commentary on Question:
Most candidates were able to answer this question well. Candidates lost points if they didn't mention "face" capital for 2 and 3. The concept of allocation of "face" capital is crucial, especially for part (c).

1. Hold in Corporate Line
   Adv – Simple
   Adv – Insulates product lines from vagaries of required capital formulas
   Disadv – Could lead to over-investment in line of business that tie up excessive required capital

2. Marginal
   Adv – Attempts to allocate true cost of face capital for adding given line of business
   Disadv – Complicated

3. Pro-rata
   Adv – Simple
   Adv – Allocates face capital to business units
   Disadv – May allocate a large amount of face capital to LOB that generated none and vice-versa

4. Treat each line of business as if monoline
   Adv – Somewhat easier to understand than marginal approach
   Adv – Neither helps nor hurts a given LOB due to presence of other LOB
   Disadv – May allocate a large amount of capital to LOB that adds little required capital due to diversification

(c) Allocate Stilton's required capital under each method.

Commentary on Question:
Most candidates did not answer this question well. Some did not list the required capital for each line of business, and many did not calculate an allocation to a Corporate line. Some didn't calculate the required capital correctly.

Few answered correctly in parts 2 and 3. Most didn't do the pro-rata calculation on face capital. Most answered parts 1 and 4 well.

10. Continued

1. Hold in Corporate Line
   Annuities = $78mm
   Life = $38mm
   Health = $29mm
   Corporate = 150 –78 – 38 - 29 = $5mm

2. Marginal
   Annuities = 78 + 5 – 0 = $83mm
   Life = 38 + 5 – 5 = $38mm
   Health = 29 + 5 -7 = $27mm
   Corporate = 150 – 83 - 38 – 27 = $2mm

3. Pro-rata
   Annuities = 78 + 5 * (900/1800) = $80.5mm
   Life = 38 + 5 * (600/1800) = $39.67mm
   Health = 29 + 5 * (300/1800) = $29.83mm
   Corporate = 150 – 80.5 – 39.67 – 29.83 = $0mm

4. Treat each line of business as if monoline
   Annuities = $80mm
   Life = $40mm
   Health = $30mm
   Corporate = 150 – 80 – 40 – 30 = $0mm

11. Learning Objectives:

1. Modern Corporate Financial Theory

Learning Outcomes:

(1g) Recommend a specific legal form of organization and justify the choice.

(1h) Recommend specific firm governance measures and justify the recommendation.

Sources:

FET-166-09

Financial Theory and Corporate Policy, Chapter 2

Commentary on Question:

The question tested candidates' knowledge of ownership structure with focus on non-US ownership models. Part (c) covers anti-takeover measures.

Most candidates were able to correctly identify Markets 1 and 2. Generally they were able to provide some characteristics of the two Markets, though candidates were less able to provide weaknesses for each of the Markets. Candidates did well identifying anti-takeover measures but many had trouble explaining the likely impact to shareholder wealth.

Solution:

(a)

(i) Identify the ownership structure models of Market 1 and Market 2

Commentary on Question:

Most candidates correctly identified Market 2 with somewhat fewer candidates correctly identifying Market 1. The most common error was "open corporate model" while less mentioned errors were "partnership form" or "S-Corporation".

Market 1 is financial intermediary or closed, entrepreneurial model.
Market 2 is industrial group model.

(ii) Describe additional characteristics for Market 1 and Market 2 that are likely to be present.

11. Continued

Commentary on Question:
The answer for this was basically the listings provided on pages 60 and 65 of chapter 2 of Corporate Finance Theory. *Many candidates correctly listed some characteristics, though sometimes there was confusion between Market 1 and Market 2. Understanding the role of governments and banks was generally accurate as was understanding that management has basically no direct financial interest in an organization (i.e., little or no stock compensation). While candidates may have identified the role of banks in a given market, to receive points for this a candidate had to provide more detail other than that already provided in the question itself. For example for Market 1, a candidate would have to remark that the commercial bank usually plays a governance role or that the commercial bank has an investment banking role.*

If a candidate confused the identifications of Market 1 and Market 2 in (a)(i), the candidate could still receive points in (a)(ii) depending on how the answers were written. When a candidate was describing something other than the closed model or industrial group model because of misidentification in (a)(i) (e.g., the incorrect response, "S-corporation"), a candidate would receive no points in a(ii).

Market 1:

- Private or infrequently traded
- Mid-sized, closely held; often family owned
- Capital markets play small role; small investors have limited participation
- Bond markets small and illiquid
- Strong reliance on personal relationships; there is little regulation and little transparency
- Few professional managers, and little equity compensation
- Inactive market for corporate control, and few hostile takeovers
- Few but strong commercial banks dominating all aspects of corporate financing
- commercial banks have strong ties to clients; usually serve on boards and play strong governance role

Market 2:

- Groups compete with each other in large number of industries
- Groups include several industries usually with major commercial bank at center of group
- Group companies own each other's shares; economy is intertwined
- Commercial bank exhibits control thru majority shareholder position or thru managerial authority

11. Continued

- Industrial groups dominate home market and are lead exporters
- Groups have close relationships with governments and foreign ownership is prohibited
- Groups controlled by founding families but run by professional managers who receive little or no stock compensation
- Capital markets play small role and shareholders have little power
- Takeovers are rare and hostile takeovers are non-existent
- Small investors have more participation in Market 2 than in Market 1

(b) Describe the weaknesses of both Market 1 and Market 2 ownership structures

Commentary on Question:
Most candidates received some points here. The weaknesses for the two markets are not the same, and the expectation was there would be separate descriptions for Market 1 and Market 2. Some candidates answered without specifying which market is intended (i.e., the answer implied that the weaknesses were for both markets), and in this case partial credit was given if the answer applied to one of the markets.

The most common mistake was to suggest that technology is lacking in Market 1. This is a misunderstanding of the material. What is happening is that technology is diminishing the competitive advantage that commercial banks have had in Market 1.

Many candidates did not supply sufficient detail regarding a particular weakness. For example, simply stating "conflict of interest" did not receive as many marking points as a description of how the conflict of interest arose.

Market 1:
- Conflict of interest for bankers who are both creditors and shareholders
- Little transparency so potential for abuse
- Higher costs of large scale financing
- Information processing is weakening value of commercial bank franchises
- Deregulation is reducing access to low-cost financing

Market 2:
- All members of group must grow similarly or else subsidies occur
- Strongest industrial groups will likely seek external, international financing
- Groups must fight natural tendency to purchase inputs from other group members, which leads to inefficiencies

11. Continued

- Domestic market essentially closed to imported goods, so consumers pay higher costs for goods than otherwise
- No evidence industrial group system works outside Japan or Korea

(c)

(i) Describe four additional ways to defend against a hostile takeover.

Commentary on Question:
Generally candidates did well on this part as there were many possible answers, though only 4 were requested. If more than four were described then credit was given for only the first four. To receive full credit for one of the 4 ways the candidate had to identify the method (e.g., "defensive restructuring" or "anti-takeover amendments"), then accurately provide detail on the method. Points were frequently lost because a candidate named a method but provided no detail, and so failed to "describe" as the question required.

Most candidates mentioned poison pills and poison puts, and most correctly described them as involving stock shares and bonds, respectively. However some candidates confused pills for puts or suggested both pills and puts involve stock purchases.

- "Scorched earth" defensive restructuring by selling of attractive units, incurring large debt, and using proceeds to declare large dividend
- Defensive restructuring by selling off "crown jewels", the unit(s) the buyer is most interested in
- Defensive restructuring by consolidating a voting block that is aligned with current management, though may need to dilute bidder's equity by diluting shares
- Defensive restructuring by share repurchase without management sale, and so increases leverage and equity position of current management, possibly enabling management to have enough shares to defeat a takeover bid
- Defensive restructuring by issuing new securities to parties partial to current management
- Defensive restructuring by creating anti-trust barriers specific to the bidder (e.g., buy a business that can accomplish this)
- Poison pills – issue warrants to current shareholders providing the rights to purchase surviving firm's stock at low price in event of merger

11. Continued

- Poison puts – give bondholders rights to put bonds to issuer or successor at par or premium in event of merger or takeover
- Anti-takeover amendments such as fair price provisions defending against two-tier offers; or super-majority amendment requiring 67%-80% shareholder approval for change in control; or staggered or classified board of directors making change of control difficult or delayed; or charter amendment to reincorporate in state with laws more protective against takeovers
- Golden parachutes – separation provisions of employee contract providing payments to managers in event of change of control; this reduces conflicts between shareholders and management and so aligns their interests, encourages managers to take long-term view and make firm specific investments of human capital

(ii) Explain the likely impact to shareholder wealth if you implement these defenses.

Commentary on Question:
Few candidates answered this part well. Many papers argued that shareholder wealth increased when defenses were implemented, and this is not the case. The reading is clear that research has shown that shareholder wealth is minimally impacted and usually negatively.

Candidates could have answered by either listing the defenses mentioned in (c)(i) and explaining the impact to shareholder wealth by item (which most candidates did) or by discussing defense tactics in general (e.g., defense measures requiring shareholder approval) and explaining the impact to shareholder wealth.

- Methods requiring shareholder approval (e.g., anti-takeover amendments) generally have little impact to shareholder wealth
- Methods requiring no shareholder approval (e.g., poison pills) generally have a more negative shareholder impact though still small

12. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2f) Describe the process, methods and effects of a potential acquisition or reinsurance of a business including its effect on capital structure, return on equity, price/earnings multiples, and share price.

Sources:

Financial Theory and Corporate Policy, 4th Edition, Chapter 18

Insurance Industry Mergers & Acquisitions, Chapter 4

Commentary on Question:

Many candidates had trouble with parts (a) and (d). Part (a) was a list from the source. Instead of listing merger types as requested, most candidates described alternative ownership strategies for value creation as mentioned in table 18.3 of page 755 of the reading, or they listed means of acquisitions (all stock, etc) rather than the purpose. Most candidates were not able to list key points; see table 18.7 on page 759.

Part (d) answers lacked detailed calculations for the Actuarial Appraisal Value. Most candidates were able to provide a general formula for the Actuarial Appraisal Value but were unable to develop detailed calculations for components other the Value of Inforce and Future Business.

Candidates earned most of the points for parts (b) and (c).

Solution:

(a) List and describe 5 Merger Types and Critical Managerial Issues.

Merger Types:

To expand business with product extension and market extension through actual and future products, attracting new markets and increasing market share.

To develop synergy by increasing size mass and benefit of economics of size.

Adjust to changes from new technology.

With a merger of different locations and cultures, can benefit from cross-border combinations.

Merger can help the company to maintain industry leadership.

Critical Managerial issues:

12. Continued

Act quickly for a rapid integration of companies.

Improve operating margins.

Respect cultural and organizational differences.

Selection of managers and impact of layoffs if any.

Shut down less efficient operations.

Identifying sources of economies and retain the most efficient operations.

Reduce cost of production to increase profitability.

Share knowledge and promote best practices.

Gain knowledge of different laws and structures.

Learn new technologies.

(b) Calculate the maximum price that Chhena would be willing to pay to acquire Paneer Corp.

Commentary on Question:
In addition of identifying the value added from acquiring Paneer, candidates had to recognize the opportunity cost of not acquiring it.

The value added if Chhena acquired Paneer Corp. is the actual value of Paneer = 85
Plus the value of Paneer Corp. after restructuring = 42
Plus the reduction in value of Chhena if it fails to acquire = 99 – 78 = 21

Thus the total amount Chhena is willing to pay is = 85 + 42 + 21 = 148.

(c) Calculate the minimum price Chhena could pay and expect to win.

Commentary on Question:
Here, a successful answer consisted in providing the same answer than in (b) for the 2 other companies and recognize that Chhena had to offer just slightly more than the maximum price from the 2 other companies.

As for Chhena we must value for Akkawi and Bergkäse the amount that each is willing to bid. Therefore, to win the bid, the minimum price would be the amount that exceeds the maximum of the other two companies' maximum bids.

12. Continued

The value added if Akkawi acquired Paneer Corp. is the actual value of Paneer = 85
Plus the value of Paneer Corp. after restructure = 42
Plus the reduction in value of Akkawi if failed to acquired =141 − 127 = 14

Maximum bid for Akkawi = 85 + 42 + (141 − 127) = 141

The value added if Bergkäse acquired Paneer Corp. is the actual value of Paneer = 85
Plus the value of Paneer Corp. after restructure = 42
Plus the reduction in value of Bergkäse if failed to acquired =113 − 99 = 14

Maximum bid for Bergkäse = 85 + 42 + (113 − 99) = 141

Therefore, to win the bid, the minimum price would be greater than 141 but must not exceed 148. So 141 + 0,01.

(d) Calculate the Actuarial Appraisal value if acquiring Paneer Corp. and entering into the treaty and advise whether the company should proceed with the treaty.

Commentary on Question:
Candidates were able to describe the general formula for AAV and the calculation of reduction in value of inforce and new business. For the value of WACC all candidates wrongly factored in income taxes. Many of the remaining values were not calculated.

For many candidates the reduction of the value of inforce and new business was too large for the remaining value, so they concluded that the company should not enter into the treaty.

Step 1: Identify Formula
Actuarial Appraisal Value (AAV) is equal to the Value of Inforce and Future Business + Adjusted Book Value - Cost of Required Capital

This formula is translated into:

= NPV(Distributable Earnings)
= Excess Capital$_0$ + NPV(After-tax Earnings on the Business $_t$ − Increase in RC $_t$)
= NPV(After-tax Earnings on the Business $_t$) + Excess Capital$_0$ + NPV(RC $_{t-1}$*i$_t$)-(NPV(RC $_t$) − NPV(RC$_{t-1}$))
= NPV(After-tax Earnings on the Business $_t$) + Excess Capital$_0$ + NPV(RC $_{t-1}$*i$_t$) − ((1 + d)NPV(RC $_{t-1}$) − RC0 − NPV(RC $_{t-1}$))

12. Continued

= NPV(After-tax Earnings on the Business $_t$) + Excess Capital$_0$ + i_t*NPV(RC $_{t-1}$) − ((1 + d)NPV(RC $_{t-1}$) − RC0 − NPV(RC $_{t-1}$))

Step 2: Get Components of the formula
NPV(After-tax Earnings on the Business)

Pre Treaty
= NPV(After-tax Earnings on the Business) = 171
Post Treaty
= NPV(After-tax Earnings on the Business)=171*(1 − 0.05)
= 162.45

Excess Capital$_0$

The appraisal value is equivalent to Value of Inforce and Future Business + Adjusted Book Value − Cost of Required Capital and solve for the Adjusted Book Value
Pre Treaty
226 = 171 + Adjusted Book Value − 15
Adjusted Book Value = 70

Adjusted Book Value = Excess Capital$_0$ + RC$_0$
Excess Capital$_0$ = Adjusted Book Value − RC$_0$
Excess Capital$_0$ = 70 − 20
Excess Capital$_0$ = 50
Post Treaty
Excess Capital$_0$ = 50 + 25
Excess Capital$_0$ = 75

Derive Weighted Average Cost of Capital with CAPM Formula
d = r D*D/(D + E) + E/(D + E)*(r_f + B(r_m − r_f))
d = 0.04*0.2/(0.2 + 0.8) + 0.8/(0.2 + 0.8)(0.03 + 1.4(0.09 − 0.03)
d = 0,0992

Given:
RC$_0$ = 25
NPV$_0$(returnt * RC $_{t-1}$) = 50
Capital Interest should be after tax
i(after tax) = 0.03(1 − 0.2)
i(after tax) = 0.024

12. Continued

Step 3: Plug into Formula from Step 1

= NPV(After-tax Earnings on the Business $_t$) + Excess Capital$_0$ + NPV(RC $_{t-1}$*i_t) − ((1 + d)NPV(RC $_{t-1}$) − RC0 − NPV(RC $_{t-1}$))

= NPV(After-tax Earnings on the Businesst) + Excess Capital$_0$ + i_t*NPV(RC $_{t-1}$) −((1 + d)NPV(RC $_{t-1}$) − RC$_0$ − NPV(RC $_{t-1}$))

= 162.45 + 75 + 0.024(50) − ((1 + 0.0992)(50) – 25 −50)
= 258.69

Step 4: Advice
Value before treaty = 226
Value after treaty = 258.69

Since the Actuarial Appraisal Value increases then the company should proceed with the treaty

Other method for calculation:

Step 1: Identify Formula
= NPV(After-tax Earnings on the Business $_t$) + Excess Capital$_0$ + i_t*NPV(RC $_{t-1}$) − ((1 + d)NPV(RC $_{t-1}$) − RC$_0$ − NPV(RC $_{t-1}$)))

Step 2: Get Components of the formula
Pre treaty

Actuarial Appraisal = Value of Inforce and Future Business + Adjusted Book Value − Cost of Required Capital

Actuarial Appraisal (Pre Treaty) with Paneer (see matrix) = AAV (Pre Treaty) = 226
Adjusted Book Value = Actuarial Appraisal value − Value of Inforce and New Business + Cost of Required Capital = 226 – 171 + 15 =70

Adjusted Book Value= Excess Capital + RC$_0$

Excess Capital = Adjusted Book Value − RC$_0$ = 70 – 20 = 50

Post treaty

Given RC$_0$ (Post Treaty) 25

12. Continued

Given NPV(RC_{t-1}) = 50 50

Given Reduction of value of inforce and new business with treaty 5%

= NPV(After-tax Earnings on the Business $_t$) Post treaty 171 minus 5% = 162,45

= NPV(After-tax Earnings on the Business $_t$) + Excess Capital$_0$ + i_t*NPV(RC_{t-1}) − ((1+d)NPV(RC_{t-1}) − RC_0 − NPV(RC_{t-1})))

IF Adjusted Book Value (Post Treaty) = Excess Capital (Post Treaty) + RC_0

And Excess Capital (Post treaty) = Excess Capital (Pre treaty) + 25 = 75

Then **Adjusted Book Value (Post Treaty)** = 75 + 25 = **100**

If **Cost of Required Capital = (d − i (after tax)) x NPV(RC_{t-1})**

From formula of Capital Asset Pricing Model the Weighted Average Cost of Capital <d> =
d = (1-tax rate)*r D*D/(D + E) + E/(D + E)*(rf + B(r m − r f))
Then d = 9,9200%

Also i(after tax) = (1 − tax rate) x r_f = 3%x(1 − tax rate) = 2,4000%
Then **Cost of Required Capital (Post Treaty) = (0,0992 − 0,024) x (50) = 3,76**

<u>Step 3: Plug into Formula from Step 1</u>
Actuarial Appraisal (Post Treaty) = Value of Inforce and Future Business + Adjusted Book Value − Cost of Required Capital
Actuarial Appraisal (Post Treaty) = 171x(1-0,05) + 100 - 3,76 = 258,69

<u>Step 4: Advice</u>
Since the Actuarial Appraisal Value increases, the company should proceed with the treaty.

13. Learning Objectives:

2. Corporate Financial Applications

Learning Outcomes:

(2e) Apply real options analysis to recommend and evaluate firm decisions on capital utilization.

Sources:

Financial Theory and Corporate Policy, 4th Edition, Chapter 9

Commentary on Question:

This question was trying to test the candidate's ability to recognize and evaluate real options. To receive maximum points, candidates had to recognize that there were two options (the option to defer and the option to abandon) and had to be able to quantify the value of the project with those options. The candidates were also expected to make a recommendation based on their work. Candidates generally did well on part (a). Parts (b) and (c) were varied.

Solution:

(a) Show that the risk-neutral probability of an increase in revenue is 56%.

$$q = (1 + r - d) / (u - d)$$

$$q = (1 + 0.05 - 0.8) / (1.25 - 0.8)$$

$$= \quad 56\%$$

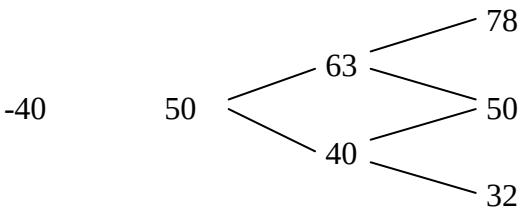
(b) Calculate the value of this project using real options analysis and recommend whether the company should pursue the project.

Commentary on Question:

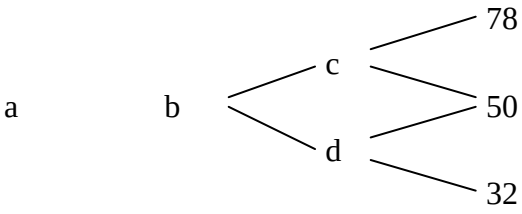
Ideally candidates would have evaluated the option both if the system build was undertaken right away and if the system build was undertaken in year 2. Also, they should have recognized that in year 2, the build should not be taken if the expected PV of future revenue is less than the cost of building the system. Many students failed to do this. A lot of candidates compared the cost of the project in year two to the revenue in year two instead of the PV of future income. At the end, they should have compared the two possibilities and recommended that the system build is deferred as that would result in a higher project value. Many candidates did not make a final recommendation or state the final value of the project, as the question asked.

13. Continued

First Value the project assuming the systems build is undertaken immediately.
Revenue in various states



Value of the project at various nodes

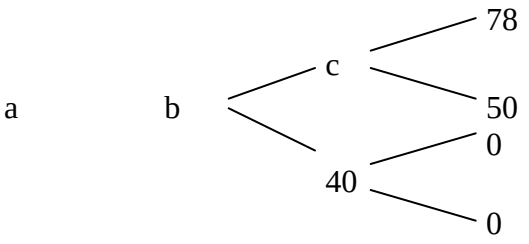


c = 62.5 + (0.56 * 78 + 0.44 * 50)/1.05
c = 124.93
d = 40 + (0.56 * 50 + 0.44 * 32)/1.05
d = 80.00
b = 50 + (0.56 * 125.05 + 0.44 * 80)/1.05
b = 149.96
a = -40 +150/1.05
a = 102.82

Now value the project assuming the systems build is postponed until year 2
Expected present value of future CFs at node c = (0.56 * 78 + 0.44 * 50)/1.05
Expected present value of future CFs at node c = 62.55
At node c, the build would be undertaken
Expected present value of future CFs at node d = (0.56 * 50 + 0.44 * 32)/1.05
Expected present value of future CFs at node d = 40.08
At node d, the build would not be undertaken

13. Continued

New value of the project at various nodes



c = 63 + (0.56 * 78 + 0.44 * 50)/1.05 – 47
c = 78.12
b = 50 + (0.56 * 78.12 + 0.44 * 40)/1.05
b = 108.43
a = 108.43/1.05
a = 103.27
Postpone the systems build until the end of year 2 because the resulting value of the project is higher.
The value of the project is 103.27.

(c) Explain why using the NPV approach undervalues all projects.

NPV implicitly assumes that no decision can be made in the future.
Therefore, discounts all CFs at a constant discount rate.
NPV doesn't capture the value of real options - the value of deferring an investment, abandoning or expanding a project, etc.
For example, in this case, you get a higher value if you recognize the deferral option and the option to abandon.

14. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3o) Use numerical methods to effectively model complex assets or liabilities.

Sources:

Hull, Chapter 24, 7th Edition

Commentary on Question:

Commentary is listed beneath question component.

Solution:

(a) Describe variance swaps and their uses.

Commentary on Question:

Many candidates did not comment on the uses of variance swaps, especially speculation.

A variance swap is an agreement to exchange the realized variance of an asset for a prescribed fixed variance in a given time period.

The payoff from the variance swap at time T to the payer of the fixed volatility is the notional amount times the difference between the realized variance and the fixed variance.

Uses of variance swaps are hedging or speculating on market volatility.

(b) Estimate the variance of Golka stock using the option quotes in the table above.

Commentary on Question:

Some candidates knew the formula but did not apply it correctly. Many candidates used the Black-Scholes formula to derive the implied volatility from the call and put prices which was not the right approach.

$E(V) = 2/T [\ln F_0/S^*] - 2/T [F_0/S^* -1] + 2/T * \sum [\Delta K_i/K_i^2 * e^{rT} * Q(K_i)]$

$F_0 = Se^{rT} = 354.77$ where S is the current stock price of 353.

S^* is 350, the first strike price below F_0.

$Q(K_i)$ = put price if $K_i < S^*$
$Q(K_i)$ = call price if $K_i > S^*$
When $K_i = S^*$ use the average of the call and put prices.

14. Continued

Strike	Call	Put	Q(Ki)	ΔK	$\Delta K_i/K_i^2 * e^{rT} * Q(K_i)$
345	20.92	11.20	11.20	5	0.000473
350	18.00	13.25	= (18+13.25)/2 = 15.63	5	0.000641
355	15.35	15.58	15.35	5	0.000612
360	12.90	18.10	12.90	5	0.000500

E(V) = 0.01707.

(c) Calculate the value of the variance swap.

Commentary on Question:
Some candidates calculated the payoff but forgot to discount it to get today's value of the swap.

$L_{var} [E(V) - V_k]e^{-rT}$

500 mm* $[0.0171 - 0.012] e^{-.02*0.25}$

= $ 2.488 million

15. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3f) Demonstrate understanding of option pricing techniques and theory for equity and interest rate derivatives.

(3g) Identify limitations of each option pricing technique.

Sources:

Hardy, Ch. 8, pages 143-150

Hull, Ch. 17

Commentary on Question:

As presented, this question is defective since the stock part of the hedge should have had negative signs in front of all values, and the bond part of the hedge should have had positive signs in front of all values. This question on the exam was the opposite.

Several candidates wrote on their papers that they knew the signs were wrong but proceeded to answer the question as they normally would have. Most candidates didn't make comment on the sign issue and proceeded to use the figures as is or assumed they were wrong and did what they thought made sense.

We gave the candidates the benefit of the doubt and disregarded the signs in their solutions. We felt this was the only way to give credit for those who ignored the defective nature and did it correctly and also for those who took what was presented in the question and used the information as is. We believe this is fair.

There was also another issue with respect to defining terms in the question. "T" was defined as 24 months but it needs to be 2 years to be inputed into the formula as stated. Some candidates understood this difference and used 2 years while others blindly plugged in the information as is. We question why there is a need to define terms and assign values in the first place for an exam at this level. This seemed rather generous given that the formula is provided as well.

Even with the defective nature of the question, most candidates performed fairly well. All answers were acceptable as long as they were "close" as a result of the interpolation performed from (a)(i) and (a)(ii).

Overall, 9 points for this question is very high given that the question was essentially a substitution into formulas. Very little commentary was asked for and candidates complied.

15. Continued

Solution:

(a)

(i) Verify that the stock part of the hedge at $t = 1$ is 28.7280 dollars.

Commentary on Question:
The negative sign indicates a short position in the stock as part of the hedge. This is opposite to how the question was written on the exam.

Candidates generally performed well on this but had some challenge in applying the math correctly. Given that the terms were primarily defined, it was a simple matter of substitution. Candidates applied the Normal interpolation liberally and had many different answers. All were acceptable as long as they were "close" and were a result of the interpolation approach.

Using the Black-Scholes formula, the stock part of the hedge is $-S_0N(-d_1)$

$d_1 = [\ln(105/100) + (0.06 + 0.3^2/2)*(2 - 1/12)] / (0.3*(2 - 1/12)^{1/2}) = -0.60202597$

$N(-d_1) = N(-0.60202597) = 0.27361117$

Therefore, the value of the stock part of the hedge is -105*0.27361117 = -$28.73

(ii) Verify that the bond part of the hedge at $t = 1$ is -37.9633 dollars.

Commentary on Question:
The positive sign indicates a long position in the bond as part of the hedge. This is opposite to how the question was written on the exam.

Candidates generally performed well on this but had some challengse in applying the math correctly. Given that the terms were primarily defined, it was a simple matter of substitution. Candidates applied the Normal interpolation liberally and had many different answers. All were acceptable as long as they were "close" and were a result of the interpolation approach.

Using the Black-Scholes formula, the bond part of the hedge is $K \exp(-r(T-t/12))\ N(-d_2)$

$d_2 = d_1 - 0.3*(21/12)^{1/2} = 0.18669478$

$N(-d_2) = N(-0.18669478) = 0.42598904$

15. Continued

This implies the bond part of the hedge is worth
100*0.42598904*0.891366 = $37.97 (or $37.9633, both are acceptable outcomes).

(iii) Calculate the hedging error (dollars of price drift) prior to rebalancing at the end of month 1.

Commentary on Question:
The negative sign indicates a source of profit. This means that the replicating portfolio brought forward is worth more than we need to set up the rebalanced portfolio. Disregarding the negative sign in the solution, most candidates were able to answer this question. While very few candidates successfully answered this question with the correct sign on the hedging error, this was ignored in grading due to the sign problem in the question.

Almost no candidates commented on what the hedging error meant – is it a gain or a loss. The question asked for the error to be calculated but again at this level, we're not sure it's completely valuable to just do math and not be asked to make comments on what it means and what you would then do in the next period. A refinement in the future could be to ask the candidate what the hedging error means to really test their understanding.

At the end of the first month, immediately before rebalancing, the stock part of the hedge has a value of -31.03 x 105/100 = -32.5815

At the end of the first month, immediately before rebalancing, the bond part of the hedge has a value of 41.8449*exp(0.06 x 1/12) = 41.8449*1.0050125 = 42.0546

So, at the end of the first month, immediately before rebalancing, the total value of the hedge is $H(1^-)$ = -32.5815 + 42.0546 = 9.4731

According to the Black-Scholes formula, at the end of the first month, the value of the hedge after rebalancing should be
$H(1) = -S_1N(-d_1) + K \exp(-r(2-1/12))N(-d_2)$
= -105 x 0.27361117 + 100 * 0.891366144 x 0.4260
= -28.728 + 37.9633
= 9.2353

Therefore, the hedging error at the end of the first month is 9.2353 – 9.4731 = -0.2378

15. Continued

(iv) Calculate the transaction costs from rebalancing the hedge at $t = 1$.

Commentary on Question:
The majority of candidates were able to calculate the transaction cost correctly. Those who didn't simply took the hedge error from the prior part and applied the 0.2% transaction fee to it and said that was the transaction cost – suggesting the candidate didn't really understand the nature of the hedging error.

At time 1, immediately before rebalancing, the hedge consists of a short position of 0.3103 units of the underlying stock

At time 1, however, the hedge requires $-N(-d_1) = -0.2736$, or 0.2736 units of the underlying stock (short)

Therefore, $|0.2736 - 0.3103| = 0.0367$ units of the stock must be traded at time 1

This means that the transaction cost at time 1 is $\$105 \times 0.2\% \times 0.0367 = \$0.0077$

(b) Describe the expected change in the aggregate hedging error and the aggregate transaction cost. Explain your answer.

Commentary on Question:
Most candidates were only able to indicate that hedging error would decrease and transaction costs would increase, and that there would be more transactions. This is likely because only 1 point was assigned so no one spent any time on this part of the question.

It is expected that the aggregate hedging error will decrease.

The magnitude of a hedging error is positively related to the time gap between which the hedge is rebalanced.

In the extreme case when the portfolio is rebalanced continuously, the hedging error would become zero.

The aggregate transaction cost, however, will increase.

A higher frequency of rebalancing would mean more trading.

16. Learning Objectives:

3. Derivatives and Pricing

Learning Outcomes:

(3h) Describe and evaluate equity and interest rate models.

(3p) Understand the differences and implications of P Measures and Q Measures.

Sources:

Hull, Chapter 30, 7th Edition

Babbel and Fabozzi, Chapter 11

Commentary on Question:

Commentary is listed beneath question component.

Solution:

(a) Calculate the price of this option.

Commentary on Question:

This question is designed to gauge candidate's understanding of the Vasicek model, with an emphasis on the application of the model in bond option valuation. In order to get the full credit, the candidate needed to display that the candidate understood the particulars that are involved in using the model for bond option pricing.

Candidates generally did poorly on (a), frequently trying Black-Scholes-Merton equity option valuation techniques rather than the Vasicek bond option formulae. See Hull 7th edition pages 684 and 690.

$L = 200$
$K = 180$
$s = 10$
$T = 5$
$r_0 = 3\%$
$\sigma = 0.04$
$P(t,T) = A(t,T)\, e^{-B(t,T)r(t)}$
$P(0,5) = A(0,5)\, e^{-B(0,5)r(0)} = 0.8372$

(To calculate P(0,10) we need:)
$A(t,T) = \exp\left[(B(t,T) - T + t)(a^2b - \sigma^2/2)/a^2 - \sigma^2 B(t,T)^2/4a\right]$
$A(0,10) = \exp\left[(B(0,10) - 10)(a^2b - \sigma^2/2)/a^2 - \sigma^2 B(0,10)^2/4a\right] = 0.9061$
$B(t,T) = [1 - e^{-a(T-t)}]/a$
$B(0,10) = [1 - e^{-a(10)}]/a = 7.5198$

16. Continued

Then we can calculate: $P(0,10) = A(0,10)\, e^{-B(0,10)r(0)} = 0.7231$
$\sigma_P = \sigma/a\,[1 - e^{-a(s-T)}]*\sqrt{[(1 - e^{-2aT})/2a]} = 0.3350$
$h = 1/\sigma_P * \ln (L*P(0,s)/P(0,T)*K) + \sigma_P/2$
$h = 1/\sigma_P * \ln (200*P(0,10)/P(0,5)*180) + \sigma_P/2 = 0.0444$

Then the price of a T=5 year call option on a bond maturing at s=10
$call(T = 5,s = 10) = L*P(0,s)*N(h) - K*P(0,T)*N(h - \sigma_P)$
$= 200*P(0,10)*N(0.0444) - 180*P(0,5)*N(0.0444-0.3350)$
$= 16.75$

(b) Explain why this could be an appropriate proposal.

Commentary on Question:
The candidate needed to show that the candidate understood the differences between the current model and the newly proposed model for this part. First, candidate needed to distinguish that one of the models is equilibrium and the other no-arbitrage. Just describing the details of the Hull-White model wasn't enough to get full credit for this part. The candidate needed to compare and contrast the two models and explain how the Hull-White could be a more appropriate choice than the Vasicek.

Vasicek model is an equilibrium model and it does not fit the term structure perfectly. Choosing parameters judiciously, one can find an approximate fit for the term structure. However, sometimes no reasonable fit can be found. If the model does not fit the term structure perfectly, then the model will not price the underlying bond correctly. As a consequence, we cannot be confident that the bond option price can be calculated correctly by using the same model.

Hull-White is a no-arbitrage model and is designed to be exactly consistent with today's term structure of interest rates. This two-factor model provides a richer pattern of term structure movements and a richer pattern of volatilities than one-factor model of r and could be more appropriate choice in calculating the European call option.

(c) Explain how the Vasicek and the Hull-White Two Factor models can be used when:

(i) *Q* measures are used.

(ii) *P* measures are used.

16. Continued

Commentary on Question:
Many candidates confused the Q and the P measures and the model types. The candidate needed to be able to distinguish between the Q and the P measures. Because Vasicek and Hull White are two different types of models, the Q and the P measures have different uses depending on the model type.

(i) When risk neutral probability is used, the Vasicek model can be used for current pricing, where inputs (market prices) are unreliable or unavailable. The Vasicek model can also be used for horizon pricing.

When risk neutral probability is used, the Hull-White Two Factor model can be used for current pricing, where input data (market prices) are reliable.

(ii) When realistic probability is used, the Vasicek model can be used for stress testing, and reserve and asset adequacy testing.

When realistic probability is used, the Hull-White Two Factor model is unusable, since term premium cannot be reliably estimated.

17. Learning Objectives:

1. Modern Corporate Financial Theory

3. Derivatives and Pricing

Learning Outcomes:

(1d) Define and compare risk metrics used to quantify economic capital and describe their limitations.

(3m) Describe issues and best practices in the estimation or calibration of financial models.

Sources:

Variance of the CTE Estimator by Manistre and Hancock

Commentary on Question:

In part (a), many students stated that SDE is not a valid estimator. However, the rationale to support that statement, the majority of the time, was not correct. Half of the students wrote down that ordered statistics are not independent of each other.

*In part (b), most students got the correct formula and calculated CTE correctly. But a lot of them did not get the VAR calculation correct {i.e. Var(CTE) = [**VAR** + alpha * (CTE − x(k))^2] / k); **VAR** = k * SDE^2}. The most common mistake was not multiplying by the correct k which is 5 for VAR.*

Solution:

(a) Determine whether the quantity SDE defined above is a valid estimate for the true variance of the CTE.

VAR(CTE) = E[VAR(CTE|X(k)] + VAR(E(CTE|X(k))]
Not a valid estimate because

1. Understate the true variance
2. Uncertainty in estimating the 99.95% quantile
3. Uncertainty in estimating the distribution to the right of the 99.95% quantile

(b) Estimate the variance of CTE 99.95 for Lingallin's capital requirement based on the approach outlined in the Manistre and Hancock paper.

Use the formula to estimate Var(CTE) = [VAR + alpha * (CTE − x(k))^2] / k
where alpha = 99.95%, k = 5, x(k) = 80
CTE = average(80, 84, 90, 110, 120) = 96.8
VAR = k * SDE^2 = 5 * 7.76^2 = 301.1
Var(CTE) = (301.1 + 99.95% * (96.8 − 80)^2) / 5 = 116.6

18. Learning Objectives:

4. Efficient and Inefficient Markets, Complete and Incomplete Markets, Information Theory & Market Misbehavior

Learning Outcomes:

(4d) Explain information asymmetry and how it can affect financial markets, especially insurance markets.

Sources:

Financial Theory and Corporate Policy, 4th Edition, Chapters 12 and 16

Commentary on Question:

This question is about the effect of a tender offer as a way of delivering shareholder value in corporate finance. The question then asks the candidates to demonstrate their understanding by calculating the tender offer effect. This question addresses learning outcome 4e.

Solution:

(a)

(i) Identify and describe the hypotheses related to the effect of the tender offer on the wealth of shareholders and bondholders.

Commentary on Question:

Candidates did relatively well in part (a)(i)

The information or signaling hypothesis, leverage tax shield hypothesis, dividend tax avoidance hypothesis, bondholder expropriation hypothesis, and the wealth transfers among shareholders hypothesis.

(ii) Identify the hypothesis that received the strongest support from the empirical evidence.

Commentary on Question:

Many candidates did not correctly identify the strongest hypothesis

Empirical evidence provides strong support to the information or signaling hypotheses that the tender offer for repurchase is interpreted by the marketplace as favorable information regarding future prospects of the firm.

(b) Construct the company's balance sheet as of 7/31/2012 similar to Table 2. Show your work.

18. Continued

Commentary on Question:
Candidates did not do well in part (b).

The question did not give the following information on 6/30/2012: risk-free interest rate, Zero-Coupon Bond (ZCB) spread and dividend yield on "Other Assets". In addition, ZCB spread is subject to change from time to time due to changes in the issuer's risk profile. It is incorrect to apply the ZCB discount factor (= risk free rate + ZCB spread) derived from 6/30/2012 data in computing the ZCB market value on 7/31/2012. One must use the Option Pricing Method to derive the new market value of the ZCB or equity after the tender offer under the given information of this question.

Even if the candidate failed to compute correctly the ZCB market value on 7/31/2012, partial credit is given if the candidate completed a valid balance sheet.

The Tender Offer reduces the company's total assets, which may affect the market value of the zero coupon bond (ZCB). Need to derive the market value of the ZCB as of 7/31/2012 using the Option Pricing Method

V = Total Asset = Cash + Other Assets = (150- 10*11.5) + 352 = 387
s = Implied volatility of V = 16.5% *352/387 = 15.01% since cash has 0% volatility
D = Bond maturity value = 300, r = 0%, T = 4 years
d1= [ln(V/D) + (r + 0.5*s^2)*T]/(s*T^0.5) = 1.00
d2 = d1 – s*T ^0.5 = 0.70
Put = D*exp(–r*T) *N(-d2) – V*N(-d1) = 11.20
B = 300 – Put = 288.80
Equity = Total Assets – Bond =387 – 288.80 = 98.20

Balance Sheet as of 7/31/2012 *(Market Value basis)*

Assets ($millions)		Liability & Equity ($millions)	
Cash:	35	Zero-coupon Bond:	288.80
Other Assets:	352	Equity	98.20
Total Assets:	387	Total Liability & Equity:	387

(c)

(i) Demonstrate the wealth effect created or lost due to the tender offer as of 7/31/2012.

18. Continued

Commentary on Question:
This is follow-up of part (b). Credit is given as long as the candidate demonstrated correctly the wealth effect based on the balance sheet derived in part (b). No credit is given if the answer is inconsistent with the balance sheet in part (b).

After the Tender Offer:
Bondholders gained = 288.80 – 285 = 3.80
Shareholders receive 10*11.5 = 115 in cash
Shareholders gained = 98.20 + 115 – 215 = -1.80
Total net gain = bondholder gain + shareholder gain = 3.8 – 1.8 = 2

(ii) Explain where this wealth came from.

Commentary on Question:
This is a follow-up to part (c)(i). Credit is given if the candidate's answer is consistent with his/her analysis in part (c)(i)

The $2 million net wealth came from the increase of the company's "Other Assets" = 352 – 350.

(iii) Describe in words (not numbers) the circumstances under which wealth would not be created.

Commentary on Question:
This is a follow-up to part (c)(i) and (c)(ii). Credit is given if the candidate's answer is reasonable even though it does not match the Model Solution.

Above a certain tender offer price, enough cash has been stripped out of the company so that the put option increases in value enough that the bondholders lose value (this price is $12.97). In this case, shareholder gains at the expense of the bondholder.

(iv) Determine whether the tender offer really created wealth.

Commentary on Question:
This is a follow-up to part (c)(i), (c)(ii), and (c)(iii). Credit is given if the candidate's answer is reasonable.

No, the tender offer did not create wealth other than the wealth transfer between bondholders and shareholders.