ERM-INV Model Solutions
Fall 2012

1. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

Learning Outcomes:

(2b) Evaluate how risks are correlated, and give examples of risks that are positively correlated and risks that are negatively correlated.

(2h) Construct approaches to managing various risks and evaluate how an entity makes decisions about techniques to model, measure and aggregate risks including but not limited to stochastic processes.

(3e) Define and evaluate credit risk. Explain how to incorporate best practices in credit risk measurement, modeling and management.

Sources:

Financial Enterprise Risk Management, Sweeting, 2011, chapter 14

Value-at-Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion, 2007, chapter 18

Commentary on Question:

The focus of this question was credit risk and the variety of credit risk models presented in the syllabus. Successful candidates were expected to demonstrate their understanding of these models in the specific context of the question.

Solution:

(a) Explain how Hamsik's perspective on credit risk would differ if it reported assets at market value rather than at book value.

Commentary on Question:

Most candidates performed poorly on this part of the question as they confused the financial reporting of assets by Hamsik with the book and market value of collateral supporting the Hamsik loan portfolio. Very few candidates commented that accounting convention did not impact the economic reality of risk.

1. Continued

To the extent that Hamsik reports at Book Value, its financial reporting is unaffected by temporary changes in the market value of its assets. Nevertheless, it is concerned with the ultimate ability to collect payments (both coupons and principal).

If Hamsik instead reported at Market Value, its financial reporting would be affected by both temporary changes in the market value (as a result of credit spread movements or downgrades) and by actual defaults.

Accounting convention does not impact the economic reality of credit risk.

(b) Explain how credit risk and market risk can interact in the context of the Hamsik loans to Cayuga.

Commentary on Question:
Many candidates received full credit for this section. Almost all were able to get one of the two key points.

A downturn in the market may reduce investment portfolio income for Cayuga. Since Cayuga's income is highly dependent on his investment portfolio, a market downturn could lead to Cayuga's defaulting on his obligations to Hamsik.

The market value of the collateral backing the loans may fluctuate (regardless of the likelihood of default), which will increase/decrease the loss given default for Hamsik.

(c)

(i) Identify and describe the three broad types of quantitative credit risk models.

Commentary on Question:
Most candidates performed well on this part of the question.

Credit Scoring Models- use features of the entity (e.g. accounting ratios) to assign a score that represents the likelihood of insolvency. These are general linear models taking on values between 0 and 1.

Structural Models- model the value of the underlying entity (asset value, debt levels) to arrive at the probability of default.

Reduced Form Models- use the exogenously derived credit rating to arrive at the probability of default.

1. Continued

(ii) Categorize each of the following credit risk models into one of the three model types identified in part (i) above.

(A) Probit/Logit Models
(B) Merton Model
(C) Credit Risk +
(D) K-nearest neighbor (kNN)
(E) Discriminant Analysis
(F) KMV
(G) Altman's Z Score
(H) Support Vector Machines

Credit Scoring Models: A, D, E, G, H
Structural Models: B, F
Reduced Form Models: C

(d) Explain why each of the following specified approaches to modeling credit risk may or may not be appropriate for the GreatKibble.com loan.

(i) Merton Model
(ii) Discriminant Analysis
(iii) Credit Migration Model

Commentary on Question:
Most candidates performed well on this section. Candidates lost points when they did not relate each approach specifically to Great Kibble.

Merton Model – Not good for Great Kibble. It's better for large borrowers with liquid and frequently traded equity, which Kibble is not. There are no tangible assets held by Kibble, which is required by the Merton Model. The volatility of newer companies is also not stationary, which is a requirement in the Merton Model.

Discriminant Analysis- Requires an entity to have a history of financial ratios. Great Kibble does not have a reliable history. It has a short history and the financials are being restated (i.e. current financials may not be reliable). However, this approach works well for smaller companies, so once the numbers have been restated this approach may be suitable.

Credit Migration Model- Requires an entity to be publicly rated. It is very unlikely that Kibble is publicly rated, so this approach is not appropriate.

2. Learning Objectives:

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

5. The candidate will understand the concept of economic capital, risk measures in economic capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:

(4c) Demonstrate means for transferring risk to a third party, and estimate the costs and benefits of doing so.

(5a) Describe the concept of economic measures of value and demonstrate their uses in the risk management and corporate decision-making processes.

(5d) Apply risk measures and demonstrate how to use them in economic capital assessment. Contrast and understand regulatory, accounting, statutory and economic capital.

(5e) Propose techniques for allocating/appropriating the cost of risks/capital/hedge strategy to business units in order to gauge performance (risk adjusted performance measures).

Sources:

Value-at-Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion, 2007

- Chapter 5 Computing VaR

Article: Variance of the CTE Estimator, Risk Management, August 2008

ERM-106-12: Economic Capital-Practical Considerations-Milliman

Tiller, Life, Health and Annuity Reinsurance, 3rd Edition, 2005

- Chapter 5 Advanced Methods of Reinsurance

Financial Enterprise Risk Management, Sweeting, 2001

- Chapter 18 Economic Capital

Commentary on Question:

The question asked candidates to compute VaR and CTE risk metrics for the specified loss distribution and to use this information to make a decision regarding the cost / benefit of entering into a reinsurance arrangement. The question required the use of calculus, which proved to be a stumbling block for some candidates.

2. Continued

Solution:

(a) Erie holds economic capital based on the tail risk of claims paid.

Compute the amount of required economic capital Erie should hold, prior to purchasing any reinsurance, for the following tail risk metrics:

(i) VaR(95%)

(ii) CTE(95%)

Commentary on Question:
Most candidates could compute VaR and CTE. Some candidates did not subtract the expected claim amount from the VAR and CTE to arrive at the capital amount held by Erie. This oversight did not cost the candidate significant points in part (a) but failure to use the correct capital amount in subsequent parts led to incorrect results and conclusions.

Weaker candidates incorrectly computed a VAR amount which was greater than the computed CTE amount and failed to comment on this or otherwise indicate that this was not a reasonable outcome.

(i) VaR (p) = x_p, so that Pr(X>x_p) =1 - p

F (x_p) = Pr (X≤x_p) = $\int_0^{x_p} f(x)dx = \int_0^{x_p} e^{-x}dx = 1-e^{-x_p} = p = 0.95$

x_p = -ln(0.05) = 2.996

So VaR (95%) = 2.996 billion

Expected Liability = E[x] = $\int_0^{\infty} xf(x)dx = \int_0^{\infty} xe^{-x}dx$ = 1 (using the formula given in the stem of the question with φ = 0)

Capital Held Based on VaR Measure = VaR(95%) - E[X] = 2.996 - 1 = 1.996 billion

(ii) CTE (95%) = E [X|X>x_p] =

$$\frac{\int_{x_p}^{\infty} xf(x)dx}{\int_{x_p}^{\infty} f(x)dx} = \frac{\int_{x_p}^{\infty} xe^{-x}dx}{\int_{x_p}^{\infty} e^{-x}dx} = \frac{(1+x_p)\times e^{-x_p}}{0.05} = \frac{(1+2.996)\times e^{-2.996}}{0.05} = 3.996 \text{ billion}$$

Capital Held Based on CTE Measure = CTE (95%) - E[X] = 3.996 - 1 = 2.996 billion

2. Continued

(b) Compute the pre-tax return on economic capital, prior to the purchase of reinsurance, for both (i) and (ii) from part (a) assuming an investment earned rate of 0%.

Commentary on Question:
The most common mistake candidates made was to use VaR and CTE values as capital in the calculation without subtracting the expected claim amount.

	VaR(95%)	CTE(95%)
Premium	1.25	1.25
Investment Income	0	0
Claims	-1	-1
Pre-tax Return	0.25	0.25
Capital	1.996	2.996

*Table in billions

Return on capital = Pre-tax Return/Capital

VaR(95%)	CTE(95%)
12.53%	8.34%

(c) Assume that Erie's only objective is to maximize return on capital.

Recommend whether Erie should purchase the reinsurance, assuming the following capital levels:

(i) VaR(95%)

(ii) CTE(95%)

Commentary on Question:
Most candidates could not compute the expected reinsurance claims correctly. Some candidates did not realize that VaR and CTE would change after the reinsurance was purchased.

Candidates are encouraged to signal to the grader that they understand when computed amounts seem unreasonable. For example, candidates who computed a VAR value which was greater than CTE or computed a reinsurance premium which exceeded the direct premium should signal to the grader that they understand or at least suspect that the computed value is incorrect before proceeding to use it further.

2. Continued

E [Reinsurance Claims]

$$= \int_0^{\infty} \max[0, x-2.5] f(x)dx = \int_{2.5}^{\infty} xe^{-x}dx - \int_{2.5}^{\infty} 2.5e^{-x}dx = 3.5\,e^{-2.5} - 2.5e^{-2.5} = 0.082 \text{ billion}$$

Reinsurance Premium = 200% of Expected Reinsurance Claims
= 200% × 0.082 = 0.164 billion
Liability after reinsurance = E [X−Reinsurance Claims] = E[X] –
E [Reinsurance Claims] =1 − 0.082 = 0.918

(i) VaR(95%) without reinsurance is 2.996 billion (from part a). Reinsurer will cover claims above 2.5 billion; therefore, for any claim above 2.5 billion, the liability to the insurance company is 2.5 billion. Therefore, at the 95th percentile, its exposure will no longer be 2.996 billion, it will be 2.5 billion.

VaR Capital = VaR(95%) − E[X] = 2.5 − 0.918 = 1.582

	VaR(95%)
Premium	1.25
Reinsurance Premium	-0.164
Investment Income	0
Claims	-0.918
Pre-tax Return	0.168
Capital	1.582

*Table in billions

RoC = Pre-tax Return/Capital = 10.62%

VaR: RoC post reinsurance is reduced from 12.53% to 10.62%, so NO, Erie should NOT purchase the reinsurance if it holds capital based on VAR(95%) and Erie's only objective is to maximize return on capital.

(ii) VaR (95%) with reinsurance is 2.5 billion (from part c (i)). Each of the worst 5% of claims is greater than 2.5 billon, and Erie is only responsible for 2.5 billion for each claim above VaR (95%) due to reinsurance. As a result, the average of the claims above VaR(95%) is 2.5 billion, which is CTE (95%).
CTE (95%) = 2.5 billion
CTE Capital = CTE (95%) − E[X] = 2.5 − .918 = 1.582

2. Continued

	VaR(95%)
Premium	1.25
Reinsurance Premium	-0.164
Investment Income	0
Claims	-0.918
Pre-tax Return	0.168
Capital	1.582

*Table in billions

RoC = Pre-tax Return/Capital = 10.62%

CTE: RoC post reinsurance is increased from 8.34% to 10.62%, so YES, Erie should purchase the reinsurance if it holds capital based on CTE(95%) and Erie's sole objective is to maximize return on capital.

(d) Explain why Erie may want to purchase reinsurance even though it may reduce expected net income.

Commentary on Question:
Most candidates did this part of the question well. However, while there are a range of reasonable responses, the most pertinent response (given the analysis performed in parts (a) through (c)) is that Erie may be willing to reduce expected profits in exchange for limiting losses in the worst case scenarios. Weaker candidates failed to mention this but rather provided a generic list of benefits associated with reinsurance.

- Reinsurance helps Erie to reduce expected claims especially in the worst case scenarios (or tail of the claims distribution). While this reduces the expected profit for Erie, it also reduces the volatility of profits for Erie.
- Erie may relieve capital strain.
- Erie can get expertise of reinsurer regarding liability.
- Erie may be looked upon positively from rating agency, regulator, etc.
- Reinsurance may allow Erie to take on more business.
- Reinsurance can help with tax planning.
- Reinsurance may enhance balance sheet.
- Reinsurance can help with strategic business planning.

3. **Learning Objectives:**

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

(1a) Explain risk concepts and be able to apply risk definitions to different entities.

(1b) Explain risk taxonomy and its application to different frameworks.

(1c) Identify and analyze risks faced by an entity, including but not limited to market risk, currency risk, credit risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk.

(2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.

(2e) Evaluate the theory and applications of extreme value theory in the measuring and modeling of risk.

(2h) Construct approaches to managing various risks and evaluate how entity makes decisions about techniques to model, measure and aggregate risks including but not limited to stochastic processes.

(4d) Demonstrate means for reducing risk without transferring it.

(4k) Analyze methods of managing other risks (operational, strategic, legal and insurance) both pre-event and post-event.

Sources:

Financial Enterprise Risk Management, Sweeting, 2011

- Chapter 7 Definitions of Risk
- Chapter 8 Risk Identification
- Chapter 10.4 Copulas
- Chapter 12 Extreme Value Theory
- Chapter 15.5 Unquantifiable Risks
- Chapter 16 Responses to Risk

3. Continued

Value-at-Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion, 2007

- Chapter 19 Operational Risk Management (excluding 19.5)

Commentary on Question:
The focus of this question was operational risk. The question aimed to test the ability of the candidate to understand how operational risk could be managed in a specific context.

Solution:

(a) Define operational risk.

Commentary on Question:
This was a simple retrieval question.

The risk of loss resulting from inadequate or failed processes (projects, data, products, models), people (crime, errors), and systems, or from external events.

(b) The interviewer has asked for your opinion regarding the viability of Niagara offering this product for the U.S. banking market. Provide arguments for and against Niagara offering the product.

Commentary on Question:
The purpose of this part of the question was to test a candidate's perspective on operational risk, i.e., based on the candidate's reading of the syllabus, what would be the challenges and opportunities of offering such a product.

For:

- Operational risk is a material risk for banks with a history of leading to failure (Barings, AIB, etc.) or sizable losses (JPM). Thus an opportunity for Niagara.
- Could provide capital relief to banks' capital charge for op risk (third largest contributor to economic capital for banks).
- Could provide a natural hedge to Niagara compared to its exposures to credit and market risks.

3. Continued

Against:

- Moral hazard issues.
- Op risk is difficult to model as it involves the estimation of extreme events.
- Operational risk is highly correlated with Niagara's current exposure to market risk hence may compound capital requirements for Niagara
  - Op risk is also related to systemic risk; the traditional pooling effect of insurance may not apply.
- Definition of op risk is not standard in the industry
  - Legal issues likely to be significant given difficulty in defining indemnification provision

(c) Outline for the interviewer what you believe to be the most important considerations in structuring and underwriting such a product.

Commentary on Question:
This knowledge utilization question builds upon the previous high level strategic analysis and aims at having the candidate explain how the different risks embedded in such a product might be managed.

Stronger candidates provided specific considerations (such as those listed below) and demonstrated their ability to apply the source readings in this specific context. Weaker candidates tended to answer in more general terms using a "list" approach and did not provide any structuring or client specific factors they considered important in underwriting this product.

- Product Structuring Considerations:
  - Have a meaningful Deductible as covering first dollar losses would engender moral hazard
  - Consider coverage exclusions

- Consider Specifics of Client
  - Lines of Business: Operational loss experience differs depending on the types of business policy owners are involved in (e.g. higher exposure for retail banking / retail brokerage and commercial banking than corporate finance and agency services).
  - Geographical dispersion of activities and related concentration of risk issues.
  - Consider corporate governance structure, organizational culture, independence of functions, compensation structure as these will provide insight into incentives provided to management with respect to op risk.

3. Continued

- Consider organization's risk measurement / management programs and systems, and management competency at managing risk. Assess how effective client is going to be at limiting future losses.
- Consider historical loss experience.

- Well Defined Indemnification Provision:
  - Exposure to legal risk when entering into such agreements -- ensure that the conditions under which indemnification payments are due are well defined

(d)

(i) Explain the significance of ξ and whether using $\xi = 0$ would be appropriate in this context.

Commentary on Question:
This analysis question is testing the comprehension of Extreme Value Theory and what the EVT model parameters represent.

The ξ represents the shape parameter for the generalized Pareto distribution. It is used to model the tail of the distribution when using EVT. The choice of ξ will determine how heavy tailed the resulting distribution is. Selecting $\xi = 0$ corresponds to a relatively light tailed assumption (consistent with normal distribution assumption or exponential reduction in tail). It is <u>not</u> appropriate for operational type risks which tend to have small probabilities for potentially huge losses, particularly for risk related to human and external events.

(ii) Using $\xi = 0.95$ and given a scale parameter of 20, determine the probability that Niagara will need to make an indemnification payment on this policy in excess of \$0.75 billion.

3. Continued

Commentary on Question:
This analysis question is testing the application of one model of EVT to a specific context. A common error on this part was the failure to adjust by the 1% probability of making a payment (see last step). Another common error was failure to account for the deductible (see second line below).

Prob[Indemnification payment > 0.75B]
= Prob [Loss – Deductible > 0.75B]
= Prob [Loss > 1.25B]
= Prob [(Loss – u)/ β > (1.25 – 0.5)/20]
= Prob [y > .0375]
= 1 – F(y = .0375)
= $(1 + .0375\xi)^{-1/\xi}$
= $(1 + .0375 * .95)^{-1/.95}$
= 0.96382

This last number is the conditional probability of loss exceeding threshold given that we are above threshold. We are looking for the unconditional probability of having a loss in excess of .75B; hence, this needs to be adjusted by the probability of making payment, which equals 1%, resulting in a final answer of 0.009638.

(iii) Determine the probability that Niagara will experience a loss in excess of $0.75 billion for **at least one** of market risk, credit risk or Pirlo product risk, assuming these risks are independent.

Commentary on Question:
This analysis question is to determine the probability that Niagara will incur a total loss in excess of .75B level from at least one risk.

1 – (Probability that each of Market / Credit / Pirlo losses are < 0.75B)

Probability of market loss below .75 billion = .95 from the given table.
Probability of credit loss below .75 billion = .957 from the give table.
Probability of Pirlo loss below .75 billion = 1 – .009638 as calculated in part (d)(ii).

Since risks are assumed independent, the total probability is = .95 * .957 * (1 – .009638) = 0.9003876
Thus, the probability of at least one loss exceeding 0.75B = 10%.
.

3. Continued

(iv) Determine the probability that Niagara will experience a loss in excess of \$0.75 billion for **at least one** of market risk, credit risk or Pirlo product risk using a Gumbel copula with alpha parameter equal to 5.

Commentary on Question:
This analysis question is the same as the previous subpart, but instead of assuming independence, we assume that there is a relationship between the risks that we model using the given Copula.

$C(\pi 1, \pi 2, \pi 3) = \exp - [(-\ln(0.95))^5 + (-\ln(0.957))^5 + (-\ln(0.99036))^5]^{0.2}$

Probability $= 1 - C(\pi 1, \pi 2, \pi 3) = 1 - .94574 = 0.053839 = 5.38\%$

4. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

(1c) Identify and analyze risks faced by an entity, including but not limited to market risk, currency risk, credit risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk.

(4i) Analyze funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Explain the concepts of immunization including modern refinements and practical limitations. Contrast the various risk measures and be able to apply these risk measures to various entities.

(4k) Analyze methods of managing other risks (operational, strategic, legal and insurance) both pre-event and post-event.

Sources:

Financial Enterprise Risk Management, Sweeting, 2011

- Chapter 16 Responses to Risk

ERM-110-12: Derivatives: Practice and Principles, Recommendations 9- 24 and Section III

Commentary on Question:

The overall performance on this question was below expectations. The question is focused on understanding and managing interest rate risk. Candidates had an awareness of these topics but struggled to apply them in the context of the question. In general, they were able to provide a "list" of responses but could not demonstrate a deeper understanding and failed to recognize which items from this "list" were applicable in the context of the question. Most candidates did well on the calculation in part (c).

Solution:

(a) Compare and contrast the exposure to interest rate risk that the USFIS portfolio has relative to a typical pension plan.

Commentary on Question:

Most candidates described the differences in the durations of the USFIS portfolio and a typical pension plan. However, very few recognized that the USFIS portfolio did not have any liabilities to consider and most were unable to give a good description of the underfunding risk of a pension plan.

4. Continued

Partial credit was given for the duration comparison but to receive full credit the candidate needed to recognize that there are no liabilities in the USFIS portfolio and describe the risk of underfunding in a pension plan, a risk that does not exist with the USFIS portfolio.

Exposure in the USFIS portfolio is to changes in market value resulting from changes in interest rates. There are no liabilities to consider in this portfolio since fund redemptions are at market value and the fund is merely investing the bank's own money. There is no risk of underfunding nor an inability to meet obligations.

Exposure in a pension fund arises from having assets and liabilities with different sensitivities to interest rates (i.e., duration mismatch between assets and liabilities). The risk is that changes in interest rates result in underfunding or the inability to meet obligations because of the duration mismatch.

(b) Describe Redington's immunization, and explain whether it can or cannot be used to manage the interest rate exposure of the USFIS portfolio. Justify your response.

Commentary on Question:
Most candidates failed to fully describe Reddington's immunization technique. They defined this technique as a modified duration matching technique and did not consider the convexity requirement. In addition, candidates did not recognize that since there are no liabilities in the USFIS portfolio, Reddington's criteria cannot be used.

Reddington's Criteria:

- Present value match – the present value of bonds/swaps' fixed legs is equal to the present value of the liabilities.
- Modified duration match – modified duration of bonds/swaps' fixed legs is equal to the modified duration of the liabilities.
- If present values and modified durations are matched, then a small change in interest rates will result in both the assets and the liabilities changing by the same amount.
- Asset convexity > liability convexity – convexity of bonds/swaps' fixed legs is greater than the convexity of the liabilities.
- The advantage of the asset convexity being greater than the liability convexity is that small changes in interest rates would result in asset values increasing/decreasing more/less than the liability value.

Reddington's criteria cannot be used in this context because there are no liabilities supported by the assets in the USFIS portfolio.

4. Continued

(c) Compute the estimated change in the USFIS portfolio value for the alternate interest rate scenario in the table below:

Scenario	i_{short}	i_{medium}	i_{long}
Current / Baseline	5.0%	5.0%	5.0%
Alternate Scenario	4.5%	5.5%	6.0%

Commentary on Question:
Most candidates did well on this part. Some candidates performed the calculation but failed to explain how this could be used to estimate the magnitude and direction of the change in portfolio value as a result of the specified interest rate shift.

1.3 * (5.0% − 4.5%) + 2.7 * (5.0% − 5.5%) + 0.8 * (5.0% − 6.0%) = 0.65% − 1.35% − 0.80% = -1.50%
This shift will produce a 1.50% x P decrease in portfolio value.

(d) Explain how the concept of key rate durations might be used to manage the volatility risk associated with an options portfolio.

Commentary on Question:
This was a challenging part of this question which required candidates to have a good conceptual understanding of KRD's as well as option volatility risk. Candidates did not do well on this part. Most candidates provided a list of uses for key rate durations which did not address the specific context of this question. To get credit for this part, candidates needed to describe that option volatility has a term structure and explain how KRDs could be used to understand this volatility.

The concept of key rate durations can be applied to volatility since volatility has a term structure just like interest rates. The price of options is contingent on the volatility of the underlying and variations in implied volatility/volatility assumptions will differ for short dated versus long dated options. Much like using key interest rates to understand one's exposure to changes in interest rates along the entirety of the yield curve, one could use key volatility rates to understand one's exposure to changes in volatility along the maturity spectrum for a portfolio of options.

(e) In response to your information, the compliance department has drafted the following disclosure statement:

4. Continued

"Given that this portfolio aims to enter into offsetting positions with various counterparties, the risk of experiencing a loss on this portfolio has been virtually eliminated."

Explain why you disagree with this statement by identifying and explaining two key risks that you believe this fund is exposed to.

Commentary on Question:
Most candidates were able to list and define one or two risks that the fund is exposed to but tended to focus on less important risks at the expense of the key risks. The stronger candidates identified the key risks and described how these risks applied in the context of this portfolio. Weaker candidates only described the risks in general.

Several candidates included basis risk as a key risk for the portfolio. The stem of the question states "…the portfolio aims to enter into offsetting positions…" Therefore, while basis risk may be a key risk for some portfolios, it is not a key risk for this portfolio.

This statement is incorrect because offsetting positions only serve to reduce the exposure to market risk (and not to the other risks this portfolio is exposed to). Even if the portfolio has perfectly offsetting positions (i.e. no basis risk), the portfolio is exposed to credit risk, operational risk, legal risk, and liquidity risk.

Candidates only needed to describe two of the following:

1. Legal risk – exposure relates to the risk that the derivatives contract (in whole or in part) with the counterparty owing you monies is unenforceable.
    a. The USFIS portfolio is specifically targeting "less sophisticated parties" who may not legally be eligible to enter into such contracts.
2. Liquidity risk – exposure relates to the risk of having to make a payment or post collateral at unexpected or inopportune times.
    a. While the portfolio may have entered into offsetting positions, there is no assurance that the settlement provisions or credit enhancement provisions for offsetting positions are equivalent.
    b. While the fund may have a perfectly offsetting position it may still have exposure to liquidity risk if it needs to post collateral on a losing contract while its counterpart on the winning contract only needs to settle at maturity.
3. Operational risk – exposure relates to the risk that the company is operationally inefficient or unable to function, including business continuity risk, people risk, and technology risk.
    a. Specific examples include company technology failing, company not hiring good employees, natural disasters (earthquake, fire, weather, etc.) making it impossible for employees to work.

4. Continued

4. Credit risk – exposure relates to the risk that a counterparty will not be able to fulfill its obligations.
    a. While the fund enters into offsetting positions with major investment banks, the specialized derivative contracts are with less sophisticated counterparties which might impose some additional risk.

5. Learning Objectives:

1. The candidate will understand the types of risks faced by an entity and be able to identify and analyze these risks.

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

Learning Outcomes:

(1a) Explain risk concepts and be able to apply risk definitions to different entities.

(1c) Identify and analyze risks faced by an entity, including but not limited to market risk, currency risk, credit risk, spread risk, liquidity risk, interest rate risk, equity risk, hazard/insurance risk, inflationary risk, environmental risk, pricing risk, product risk, operational risk.

(2d) Apply and analyze scenario and stress testing in the risk measurement process.

Sources:

Value-at-Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion, 2007

- Chapter 13 Liquidity Risk

Commentary on Question:

The focus of this question was on assessing the impact to liquidity risk exposure under a set of specific scenarios.

Solution:

(a) Describe the different kinds of liquidity risk that a firm, in general, may be exposed to.

Commentary on Question:

Most candidates were able to list and explain the two types of liquidity risks.

Liquidity risk can be categorized as asset (market) or funding related.

- Asset liquidity risk is the risk that liquidation value of assets may differ significantly from their current mark-to-market values.
- Funding liquidity risk is the risk of being unable to meet the payment obligations to creditors of investors
  - Risk increases with mismatches in the timing of payments

(b) Identify and explain a stress scenario which could create a liquidity crunch specifically for LWD.

5. Continued

Commentary on Question:
This question requires the candidate to create a scenario based on the context of the question. While there are many possible responses, some are more relevant to the context and hence would result in higher grades. Generally candidates did well in this part; however weaker candidates provided scenarios which did not relate well to LWD or were not very plausible. Below is a sample answer.

- The LWD business model requires LWD to acquire a lot of inventory (pay out cash) and then get repaid from customers.
- To the extent that LWD customers extend their repayment periods / opt to forgo the Pay-As-You-Go Program, this could create a liquidity crunch for LWD.

(c) Describe the exposure to liquidity risk that each of the six financing alternatives creates for **Horseshoe**.

Commentary on Question:
This part of the question asked about the liquidity risk exposure for Horseshoe. Some candidates mistakenly addressed this question from the perspective of LWD. Instead of focusing on liquidity risk exposure, some candidates addressed the other types of risks that these six financing alternatives may create.

- All of the options will increase the asset related liquidity exposure because Horseshoe will hold a sizable position in LWD ($10M stake in a company with $76M in market value) and this position will tend to have high liquidity risk.
- Option I will also create funding liquidity risk as LWD may elect to draw on the LOC at some future date creating a cash demand on Horseshoe.
- Option II will be relatively more liquid compared to the other options as it has an acceleration of principal provision in the event that LWD fails to maintain certain financial ratios.
- Option III is less liquid as Horseshoe would hold LWD stock and the stock is traded (presumably thinly traded) on the OTC market.
- Option IV is less liquid as private placements usually involve less transparent offerings with non-standard provisions, which are less readily transferrable. Further, since these are zero coupon bonds, they will result in an increasing exposure over time.
- Option V is relatively more liquid than IV as the securities are publically traded (hence more standard / transparent than private placements). Also, the periodic coupon rate will either keep the exposure constant or accelerate payment to Horseshoe in the event that the quality of the LWD notes decline.

5. Continued

- Option VI is less liquid as structured notes usually involve less transparent offerings with non-standard provisions.

(d) Assess each of the alternatives with respect to its impact on the liquidity risk exposure it creates for **LWD**, and rank them from least to greatest liquidity risk exposure. Justify your assessment.

Commentary on Question:
Most candidates were able to correctly rank these options (with some exception on option VI)

Best to Worst: III – VI – I – IV – V – II

- Option I does not create liquidity risk exposure for LWD; it helps to solve it by providing a source of additional funding to LWD if / when required.
- Option II's acceleration provision creates funding liquidity risk as it will require unanticipated payment of principal in times of distress.
- Option III does not create liquidity risk as there is no obligation to repay equity investment or even to pay the dividend.
- Option IV does not create liquidity risk as it only requires LWD to make the expected repayment as scheduled.
- Option V creates liquidity risk exposure as it may increase LWD's interest burden in times of distress.
- Option VI does not create liquidity risk; rather it helps solve it by eliminating cash flow mismatch between LWD cash inflows and debt payment outflows.

6. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

Learning Outcomes:

(2b) Evaluate how risks are correlated, and give examples of risks that are positively correlated and risks that are negatively correlated.

(2c) Analyze and evaluate risk aggregation techniques, including use of correlation, integrated risk distributions and copulas.

(3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.

Sources:

Value-at- Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion Ch. 7

Value-at- Risk, Third Edition, The New Benchmark for Managing Financial Risk, Jorion Ch. 9

Commentary on Question:

This question tests candidates' ability to calculate Value-at-Risk for a portfolio of assets. Candidates' knowledge of time-varying risk models, such as the exponentially weighted moving average (EWMA) process, is also assessed. The last part of the question requires the candidates to analyze the impact of a proposed trade on the risk of the portfolio.

Solution:

(a) Calculate the 99% VaR for the portfolio as of today.

Commentary on Question:

Full points were also awarded for candidates who expressed the portfolio variance formula in matrix form (assuming done correctly), instead of writing out the equations, as shown in the response that follows.

First, we need the portfolio variance:

$$\sigma_p^2 = w_{wolf}^2\sigma_{wolf}^2 + w_{lion}^2\sigma_{lion}^2 + 2w_{wolf}w_{lion}\sigma_{wolf,lion}^2$$
$$= 0.5\text{^}2 * 0.0225 + 0.5\text{^}2 * 0.01 + 2 * 0.5 * 0.5 * 0.005$$
$$= 0.010625$$

6. Continued

The 99% VaR for the portfolio is:

$$VaR_p = \alpha \sigma_p W$$
$$= 2.33 * \text{sqrt}(0.010625) * (500 + 500)$$
$$= 240.17$$

(b) Calculate the expected conditional covariance of the portfolio to use in tomorrow's calculation.

Under the EWMA process, we have the conditional covariance tomorrow as:

$$\sigma_{wolf,lion;\, t+1} = \lambda \sigma_{wolf,lion;\, t} + (1-\lambda) r_{wolf;\, t} r_{lion;\, t}$$
$$= 0.95 * 0.005 + 0.05 * 5\% * 5\%$$
$$= 0.004875$$

(c) Tyrion wants to increase one of his commodity positions by 1%. His goal is to increase his expected future profits while minimizing the change in VaR. Determine whether Tyrion should increase the investment in wolf fur or lion mane. Justify your response.

Commentary on Question:
Partial points were given to candidates who used a marginal VaR approach. This is because the marginal VaR measures the change in the portfolio VaR resulting from an additional dollar of exposure to a given component, and could inadequately capture the change in portfolio VaR when a component changes by a larger amount (in which case VaR changes nonlinearly). To receive full points under the marginal VaR approach, the calculated marginal VaR of a dollar change in either commodity needs to be scaled to the level of change specified by the question (1% or $5).

To assess the exact change in VaR, the incremental VaR approach is used:

First, test the incremental VaR of increasing the wolf fur position:

Increase wolf fur position by 1% → 505 in wolf fur and 500 in lion mane

$$\sigma'^2_p = w'^2_{wolf}\sigma^2_{wolf} + w^2_{lion}\sigma^2_{lion} + 2w'_{wolf}w_{lion}\sigma^2_{wolf,lion}$$
$$= (505/1005)^2 * 0.0225 + (500/1005)^2 * 0.01 + 2 * (505/1005) * (500/1005) * 0.005$$
$$= 0.01066$$

6. Continued

$VaR'_p = \alpha \sigma'_p W'$
$= 2.33 * \text{sqrt}(0.01066) * (500+505)$
$= 241.73$

Incremental VaR = $VaR'_p - VaR_p$ (from part a)
$= 241.73 – 240.17$
$= 1.56$

Repeating the above steps for increasing the lion mane position by 1%, we get an incremental VaR of 0.85, which is less than the incremental VaR from increasing the wolf fur position.

As such, assuming both commodities have the same expected rate of return, Tyrion should increase his position in lion mane to meet his goal.

7. Learning Objectives:

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

5. The candidate will understand the concept of economic capital, risk measures in economic capital assessment and techniques to allocate the cost of risks within business units.

Learning Outcomes:

(2b) Evaluate how risks are correlated, and give examples of risks that are positively correlated and risks that are negatively correlated.

(3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.

(3d) Describe and evaluate risk management techniques in terms of best practice that can be used to deal with financial and non-financial risks.

(5e) Propose techniques for allocating/appropriating the cost of risks/capital/hedge strategy to business units in order to gauge performance (risk adjusted performance measures).

Sources:

Value at Risk, Jorion, Chapter 17, VAR and Risk Budgeting in Investment Management

Commentary on Question:

The question focuses on risk budgeting and how VaR can be used to manage investment risk in the context of a defined benefit pension plan. Candidates generally started out well on the first question part, showing understanding of the concepts, but then had problems with some of the later calculations and with applying the intermediate results to reach a final solution.

Solution:

(a)

(i) Determine whether or not the proposed portfolio allocation results in a portfolio that satisfies SLIC's risk appetite objective.

Commentary on Question:

Candidates generally did well on this part. It is important to recognize the covariance portion while calculating the portfolio volatility.

7. Continued

Targeted Risk Appetite = portfolio volatility of 10%

Variance(portfolio) = $\sigma_1^2 w_1^2 + \sigma_2^2 w_2^2 + 2\rho\sigma_1 w_1 \sigma_2 w_2$
= $(0.15)^2(0.65)^2 + (0.037)^2(0.35)^2 + 2(.06)(.15)(.65)(.037)(.35)$
= 0.982547%

Volatility(portfolio) = sqrt(0.982547%) = 9.91235%

Thus, the 35% bond, 65% equity allocation produces a portfolio volatility slightly below the target risk appetite.

(ii) Using the proposed allocation, determine the portfolio VaR (in $000s) at a 95% confidence level.

Portfolio VAR = ασW = 1.65 x 9.91235% x $450,957 = $73,756

(b) Determine the risk budget (in $000s) for each of U.S. Bonds and U.S. Equity, consistent with your results in (a)

Commentary on Question:
A common problem for candidates was confusing the individual VaR, which was asked for here, with component VaR or marginal VaR. A few candidates failed to recognize the risk budget as a function of the underlying volatility and simply multiplied the total risk budget by the respective allocation percentage.

VaR(US Equity) = ασW = 1.65 x 15.0% x $293,122 = $72,548
VaR(US Bonds) = ασW = 1.65 x 3.7% x $157,835 = $9,636

(c) Determine the optimal allocation to each of the U.S. Equity managers and to the S&P 500 Index if a total U.S. Equity portfolio TEV of 4% is targeted.

Commentary on Question:
Many candidates were able to correctly produce the information ratio, but failed to apply the ratio to the risk budgeting calculation. The TEV equation with relative risk budgets allocation was essential to answering this part adequately and receiving full credit.

Manager C: $\omega_C = 6\%, \mu_C = 3.6\%$
Manager C: $IR_C = \mu_C / \omega_C = 3.6\% / 6\% = 0.60$
Manager D: $\omega_D = 6\%, \mu_D = 2.4\%$
Manager D: $IR_D = \mu_D / \omega_D = 2.4\% / 6\% = 0.40$
Also note Equity Index I: $\omega_I = 0\%, \mu_I = 0\%$

7. Continued

Maximizing the Portfolio Information Ratio subject to a fixed tracking error volatility requires the following formula:

$$x_i\omega_i = IR_i\left[\omega_p / IR_p\right]$$

That is, the relative risk budgets should be proportional to the information ratio.

Thus, we have: $x_C(6\%) = 0.60 \times \left[\omega_p / IR_p\right]$

and, $x_D(6\%) = 0.40 \times \left[\omega_p / IR_p\right]$

therefore: $x_D = (2/3)x_C$

With a target of total US Equity portfolio TEV at 4%, we have:

$$\omega_p = \sqrt{x_C{}^2\omega_C{}^2 + x_D{}^2\omega_D{}^2 + x_I{}^2\omega_I{}^2} = \sqrt{.0036x_C{}^2 + (4/9)(.0036)x_C{}^2 + 0}$$

$$= \sqrt{.0052x_C{}^2} = 4\%$$

Then, $x_C{}^2 = (0.04^2/0.0052)$ or $x_C = 55.5\%$

$x_D = (2/3)(55.5\%) = 37\%$

$x_I = 1 - 55.5\% - 37\% = 7.5\%$

Thus an allocation of 55.5% / 37% / 7.5% to each of Manager C / Manager D / S&P500 Index respectively is the optimal allocation within the risk budget.

(d) Determine the allocated principal and relative risk budget (in $000s) for each U.S. Equity manager under the optimal manager allocation, at a confidence level of 95%.

Manager C: $x_C W$ = 55.5% x $293,122 = $162,683 allocated principal

Manager C: ασW = 1.65 x 6% x $162,683 = $16,106 relative risk budget

Manager D: $x_D W$ = 37% x $293,122 = $108,455 allocated principal

Manager D: ασW = 1.65 x 6% x $108,455 = $10,730 relative risk budget

8. **Learning Objectives:**

2. The candidate will understand the concepts of risk modeling and be able to evaluate and understand the importance of risk models.

3. The candidate will understand how the risks faced by an entity can be quantified and the use of metrics to measure risk.

4. The candidate will understand the approaches for managing risks and how an entity makes decisions about appropriate techniques.

Learning Outcomes:

(2a) Demonstrate how each of the financial risks faced by an entity can be amendable to quantitative analysis including an explanation of the advantages and disadvantages of various techniques such as Value at Risk (VaR), stochastic analysis, scenario analysis.

(2f) Analyze the importance of tails of distributions, tail correlations, and low frequency/high severity events.

(3a) Apply and construct risk metrics to quantify major types of risk exposure such as market risk, credit risk, liquidity risk, regulatory risk, etc., and tolerances in the context of an integrated risk management process.

(3b) Analyze and evaluate the properties of risk measures (e.g. Delta, volatility, duration, VaR, TVaR, etc.) and their limitations.

(4i) Analyze funding and portfolio management strategies to control equity and interest rate risk, including key rate risks. Explain the concepts of immunization including modern refinements and practical limitations. Contrast the various risk measures and be able to apply these risk measures to various entities.

(4j) Analyze the application of Asset Liability Management and Liability Driven Investment principles to Investment Policy and Asset Allocation/

Sources:

ERM 604-12: Impact of Skewness and Fat Tails on Asset Allocation Decisions

ERM 605-12: *Modern Investment Management*, Litterman, Chapter 10, Strategic Asset Allocation in the Presence of Uncertain Liabilities

8. Continued

Commentary on Question:
The objective of this question is to measure candidates' knowledge and understanding of different risk metrics, including their advantages and limitations, in the asset allocation decision process for a DB pension plan.

*In particular, this question aims to evaluate candidates' understanding of the impact of liabilities, skewness and fat tails on asset allocation decisions for a DB pension pla*n.

Candidates generally found the question challenging. Many were able to complete the calculations in part (a) and provide some appropriate comments in part (b). Most could not give complete answers to the other parts of the question.

Solution:
(a) Using the index mapping in Table B above, calculate the Sharpe ratio for each of:

(i) Bonds

(ii) Equity

(iii) Total Assets

Sharpe Ratio$_i$ = (μ_i - r_f) / σ_i

Bonds: Sharpe Ratio$_B$ = (2.3% - 1%) / 3.7% = 0.351

Equity: Sharpe Ratio$_E$ = (6.5% - 1%) / 15% = 0.367

Total Assets: Sharpe Ratio$_P$ = (μ_P - r_f) / σ_P , where:

μ_P = $w_B\mu_B + w_E\mu_E$ = (0.32*2.3%) + (0.68*6.5%) = 5.156%
σ_P = sqrt($w^2_B\sigma^2_B + w^2_E\sigma^2_E + 2w_Bw_E\rho_{BE}\sigma_B\sigma_E$)
= sqrt(0.32^2*$3.7\%^2$ + 0.68^2*$15\%^2$ + 2*0.32*0.68*0.06*3.7%*15%) = 10.34%

Sharpe Ratio$_P$ = (5.156% - 1%) / 10.34% = 0.402

(b) Provide your insights regarding the DB Plan's Current bond/equity split based on the Sharpe Ratio calculations above.

The objective when making asset allocation decisions is to maximize the portfolio Sharpe ratio.

The equity Sharpe ratio is higher than the bond Sharpe ratio, so the higher allocation to equity in the portfolio makes sense.

8. Continued

The portfolio Sharpe ratio is higher than both the equity and bond Sharpe ratios. This shows that an allocation to both asset classes in the portfolio is beneficial and better than each asset class in isolation. This is due to the diversification effect provided by the low correlation between bonds and equity.

(c) Identify and describe two shortcomings of using the Sharpe ratio approach for asset allocation decisions with regard to the DB Plan.

Commentary on Question:
In this question, it is important for candidates to both identify and include a description of the two shortcomings chosen. Candidates generally earned some points on this part, but not full credit.

First shortcoming: Sharpe ratio only considers the risk and return of assets and ignores the characteristics of the liabilities of a DB plan

- Sharpe ratio is only theoretically well-founded in asset-only framework.
- Some investment strategies are better for liability matching; this characteristic should be taken into account when evaluating the inclusion of a particular investment strategy in a portfolio, but it is not captured by the Sharpe ratio.

Second shortcoming: Sharpe ratio is only a theoretically well-founded measure in a one-period model

- Assuming that a DB plan only focuses on asset and surplus distributions at one future point in time is inappropriate.
- DB plans generally have a long or indefinite horizon and therefore need to meet funding requirements at any point of time.

(d)

Commentary on Question:
In this part, RACS formulas from the static version of the model were given to candidates, so they needed to demonstrate knowledge of how to use the formula. The main challenge for candidates was to know how to calculate β from the liability duration and the given liability proxy duration (Barclays Capital Aggregate 10+ Year Maturity Index).

In addition, candidates needed to recognize that μ_B, σ_B and ρ to be used in the given formulas should be inputs from the invested bonds index (Barclays Capital Aggregate) and not from the liability proxy index (Barclays Capital Aggregate 10+ Year Maturity Index).

Most candidates were able to do portions of this question, but did not provide a complete answer.

8. Continued

(i) Determine the static RACS framework's bond/equity split that minimizes surplus risk.

β = Liability Duration / Liability Proxy Duration = 18.4/14.2 = 1.296

Using the given formula and known inputs:

$$\alpha = \frac{\left(1-\frac{\beta L_t}{A_t}\right)\left(\sigma_B^2 - \rho\sigma_B\sigma_E\right)}{\sigma_E^2 + \sigma_B^2 - 2\rho\sigma_B\sigma_E}$$

α = [(1-1.296*192,949/169,289) * (3.7%^{2} – 0.06*3.7%*15%)] /
[15%^{2} + 3.7%^{2} – 2*0.06*3.7%*15%]
= -2.13%

(ii) Determine the static RACS framework's bond/equity split that prevents a surplus decline.

Using the given formula, known inputs and β found in (i):

$$\alpha = \frac{\mu_B\left(\frac{\beta L_t}{A_t}-1\right)+\frac{L_t}{A_t}R_f\left(1-\beta\right)}{\mu_E-\mu_B}$$

α = [2.3%*(1.296*192,949/169,289-1) + 192,949/169,289*1%*(1-1.296)] /
[6.5%-2.3%]
= 18.1%

(e)

(i) Interpret each of your static RACS analyses from (d).

A negative 2% allocation to equity would minimize surplus risk. This means we would have to do a short sale of equities equal to 2% of total assets and invest the proceeds in bonds. Alternatively, this would lead to no allocation to equities if short sales are not permitted.

On the other hand, the DB plan would have to invest about 18% in equity if the objective is to avoid a surplus decline and therefore preserve the current surplus level.

8. Continued

The RACS formula for that particular objective takes into account the current underfunded position of the DB plan and higher one-year expected return from equity. It however fails to take into account the higher expected volatility of equity.

(Other valid observations would also receive credit.)

(ii) Assess the appropriateness of the DB plan's current bond/equity split.

From a simple Sharpe ratio perspective, equity appears to have a higher excess return by unit of risk than bonds and the liability proxy. However, in an asset-liability framework, which is used in RACS analyses, equity appears less optimal than bonds and the liability proxy.

Equity is indeed not a natural hedge for DB plans' liabilities and this is captured in both RACS formulas.

It therefore shows that the DB plan should invest more in bonds, particularly in the liability proxy (Barclays Capital Aggregate 10+ Year Maturity Index), which is a better hedge than the current assets invested in a Barclays Capital Aggregate mandate.

Even if the liability proxy duration is less than the liability duration, it is significantly closer to it than the current bonds duration. Moreover, this shortcoming can be addressed using a bond overlay (derivatives) in order to lengthen the portfolio duration.

(f) Define the M-CVaR Optimization framework, and identify two major advantages it would provide over the MVO framework in making asset allocation decisions for the DB Plan.

Commentary on Question:
In this question, candidates have to first define the M-CVaR Optimization framework and then, identify two major advantages over the MVO framework. Candidates who were familiar with the material were able to do well on this straightforward question.

Definition of the M-CVaR Optimization framework:

M-CVaR is "Mean Conditional Value at Risk" and it takes non-Normal return characteristics (skewness and kurtosis) into account. Conditional VaR is calculated by taking a probability-weighted average of the possible losses, conditional on the loss being equal to or exceeding the specified VaR.

8. Continued

M-CVaR maximizes return for a given level of CVaR, or equivalently, minimizes CVaR for a given level of return. In general, M-CVaR optimization prefers assets with higher positive skewness, lower kurtosis and lower variance.

Two major advantages over the MVO framework:

- Better modeling of asset returns
- Focus on tail risk