SOCIETY OF ACTUARIES
Enterprise Risk Management – Investment Extension

Exam ERM-INV

Date: Wednesday, October 31, 2012
Time: 8:30 a.m. – 12:45 p.m.

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has a total of 80 points.

   This exam consists of 8 questions, numbered 1 through 8.

   The points for each question are indicated at the beginning of the question. Questions 7 and 8 pertain to the Case Study, which is enclosed inside the front cover of this exam booklet.

2. Failure to stop writing after time is called will result in the disqualification of your answers or further disciplinary action.

3. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the exam booklet.

Written-Answer Instructions

1. Write your candidate number at the top of each sheet. Your name must not appear.

2. Write on only one side of a sheet. Start each question on a fresh sheet. On each sheet, write the number of the question that you are answering. Do not answer more than one question on a single sheet.

3. The answer should be confined to the question as set.

4. When you are asked to calculate, show all your work including any applicable formulas.

5. When you finish, insert all your written-answer sheets into the Essay Answer Envelope. Be sure to hand in all your answer sheets since they cannot be accepted later. Seal the envelope and write your candidate number in the space provided on the outside of the envelope. Check the appropriate box to indicate Exam ERM-INV.

6. Be sure your written-answer envelope is signed because if it is not, your examination will not be graded.

CASE STUDY INSTRUCTIONS

The case study will be used as a basis for some examination questions. Be sure to answer the question asked by referring to the case study. For example, when asked for advantages of a particular plan design to a company referenced in the case study, your response should be limited to that company. Other advantages should not be listed, as they are extraneous to the question and will result in no additional credit. Further, if they conflict with the applicable advantages, no credit will be given.

BEGINNING OF EXAMINATION

1. (*10 points*) Banca di Hamsik operates in a jurisdiction which requires financial reporting on a book value basis. You are given the following information regarding positions in the Banca di Hamsik loan portfolio:

    I. As part of its wealth management services, Hamsik has extended loans of approximately $125 million to M. Cayuga, a wealthy individual whose primary source of income comes from earnings on a substantial investment portfolio. Cayuga has provided collateral in the form of various private homes, yachts and aircraft he owns.

    II. Hamsik has issued a $2 million private placement loan to GreatKibble.com, an internet startup company. While this company shows great promise, it is in the process of restating its earnings for the entire 2-year period it has been in existence.

(a)   (*2 points*) Explain how Hamsik's perspective on credit risk would differ if it reported assets at market value rather than at book value.

(b)   (*2 points*) Explain how credit risk and market risk can interact in the context of the Hamsik loans to Cayuga.

(c)   (*3 points*)

    (i)   Identify and describe the three broad types of quantitative credit risk models.

    (ii)   Categorize each of the following credit risk models into one of the three model types identified in part (i) above.
        (A)   Probit/Logit Models
        (B)   Merton Model
        (C)   Credit Risk +
        (D)   K-nearest neighbor (kNN)
        (E)   Discriminant Analysis
        (F)   KMV
        (G)   Altman's Z Score
        (H)   Support Vector Machines

(d)   (*3 points*) Explain why each of the following specified approaches to modeling credit risk may or may not be appropriate for the GreatKibble.com loan.

    (i)   Merton Model

    (ii)   Discriminant Analysis

    (iii)   Credit Migration Model

2.   (*12 points*) The annual claims distribution for Erie Property and Casualty follows the exponential distribution $f(x)=e^{-x}$, where x represents claims in billions of dollars. Expected annual premium for Erie is $1.25 billion and is collected in full at the beginning of the year, while claims are paid at the end of the year.

Erie is considering the purchase of reinsurance for the current year. Under the proposed reinsurance contract, Erie would pay the reinsurance premium at the beginning of the year, and receive payment from the reinsurer at the end of the year for claims in excess of a $2.5 billion deductible. The premium for this coverage is 200% of expected reinsurance claims.

You are given:

$$\int udv = uv - \int vdu$$

$$\int_{\varphi}^{\infty} xe^{-x}dx = (1+\varphi)\,e^{-\varphi}$$

(a)   (*3 points*) Erie holds economic capital based on the tail risk of claims paid.

Compute the amount of required economic capital Erie should hold, prior to purchasing any reinsurance, for the following tail risk metrics:

(i)   VaR(95%)

(ii)   CTE(95%)

Show your work.

(b)   (*2 points*) Compute the pre-tax return on economic capital, prior to the purchase of reinsurance, for both (i) and (ii) from part (a) assuming an investment earned rate of 0%.

Show your work.

2. **Continued**

(c)   (*6 points*) Assume that Erie's only objective is to maximize return on capital.

Recommend whether Erie should purchase the reinsurance, assuming the following capital levels:

(i)   VaR(95%)

(ii) CTE(95%)

Justify your responses.

(d)   (*1 point*) Explain why Erie may want to purchase reinsurance even though it may reduce expected net income.

3. (*11 points*) You are being interviewed for a product manager position at Niagara Indemnity International.

To help you understand the company, you have been provided with Niagara's assessment of its exposure to market risk and credit risk.

Cumulative Probability of Loss

Loss in $Billions	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
Market Risk	87.0%	93.0%	95.0%	98.0%	99.2%	99.7%	99.9%	99.9%
Credit Risk	90.0%	94.6%	95.7%	97.2%	98.9%	99.2%	99.9%	99.9%

This position will be responsible for launching a new product targeted to U.S. bank clients, which will provide indemnification against operational risk losses.

(a) (*0.5 points*) Define operational risk.

(b) (*1 point*) The interviewer has asked for your opinion regarding the viability of Niagara offering this product for the U.S. banking market.

Provide arguments for and against Niagara offering the product.

(c) (*3 points*) Outline for the interviewer what you believe to be the most important considerations in structuring and underwriting such a product.

For the remainder of this question:

Assume that you have been hired by Niagara. Niagara has launched this product, called Pirlo, and has sold a policy under which it will indemnify the policy owner for operational risk losses above a $0.5 billion deductible.

A claim occurs when operational risk losses exceed the deductible. Niagara assesses the probability of a claim on this policy at 1% and models operational risk losses above the specified cutoff point using extreme value theory (EVT).

You are given:

$$F(y) = 1 - (1 + \xi y)^{-1/\xi} \qquad \text{if } \xi \neq 0$$
$$F(y) = 1 - e^{-y} \qquad\qquad \text{if } \xi = 0$$

where $y = (x - u)/\beta$ and u is the cutoff point set at the $0.5 billion deductible.

3. Continued

You are also given the following formula for the Gumbel copula:

$$C\left(\pi_1, \pi_2, \ldots, \pi_n\right) = e^{-\left[\sum_{i=1}^{n}\left(-\ln(\pi_i)\right)^{\alpha}\right]^{1/\alpha}}$$

(d)   (*6.5 points*)

    (i)   Explain the significance of ξ and whether using $\xi = 0$ would be appropriate in this context.

    (ii)   Using $\xi = 0.95$ and given a scale parameter of 20, determine the probability that Niagara will need to make an indemnification payment on this policy in excess of $0.75 billion.

        Show your work.

    (iii) Determine the probability that Niagara will experience a loss in excess of $0.75 billion for **at least one** of market risk, credit risk or Pirlo product risk, assuming these risks are independent.

        Show your work.

    (iv)   Determine the probability that Niagara will experience a loss in excess of $0.75 billion for **at least one** of market risk, credit risk or Pirlo product risk using a Gumbel copula with alpha parameter equal to 5.

        Show your work.

4. (*8 points*) You are employed with an investment bank and your responsibilities include managing two internal portfolios:

- US Fixed Income Strategies (USFIS) – a portfolio of government bonds.

- US Derivatives Income Strategies (USDIS) – a portfolio of specialized derivatives entered into with a variety of counterparties.

The sole aim of these portfolios is to generate profits using the bank's own monies.

You are given the following statistics for the USFIS portfolio:

$$\partial P / \partial i_S = 1.3$$
$$\partial P / \partial i_M = 2.7$$
$$\partial P / \partial i_L = 0.8$$

where P denotes the value of this portfolio and i_S, i_M, and i_L denote the three key rates (short, medium and long) that you have selected to represent the entirety of the yield curve.

(a)   (*1 point*) Compare and contrast the exposure to interest rate risk that the USFIS portfolio has relative to a typical pension plan.

(b)   (*1 point*) Describe Redington's immunization, and explain whether it can or cannot be used to manage the interest rate exposure of the USFIS portfolio. Justify your response.

(c)   (*1 point*) Compute the estimated change in the USFIS portfolio value for the alternate interest rate scenario in the table below:

Scenario	i_{short}	i_{medium}	i_{long}
Current / Baseline	5.0%	5.0%	5.0%
Alternate Scenario	4.5%	5.5%	6.0%

Show your work.

(d)   (*2 points*) Explain how the concept of key rate durations might be used to manage the volatility risk associated with an options portfolio.

4.   **Continued**

(e)   (*3 points*) You have informed the compliance department that substantially all of the USDIS portfolio is comprised of offsetting positions. The source of profit for this fund is derived from entering into specialized derivative contracts with a variety of less sophisticated counterparties (e.g. municipalities, small to medium sized commodity producers, etc.) who wish to hedge a given risk, and then purchasing an offsetting position with a major investment bank. As a result, the exposure to market risk for this portfolio is minimal.

In response to your information, the compliance department has drafted the following disclosure statement:

"Given that this portfolio aims to enter into offsetting positions with various counterparties, the risk of experiencing a loss on this portfolio has been virtually eliminated."

Explain why you disagree with this statement by identifying and explaining two key risks that you believe this fund is exposed to.

5. (*10 points*) Lumber World (LWD) is a regional distributor of lumber to homebuilders. You are given the following information regarding LWD:

- LWD maintains deeper inventories than its competitors and therefore has a reputation for being the "one-stop" source for homebuilder materials.

- LWD has implemented a "Pay-As-You-Go" program, which offers discount pricing for customers choosing to pay for their lumber at the point of sale.

- LWD currently has no debt.

LWD has a market capitalization of $76 million and trades publicly on the over-the-counter (OTC) market. The outstanding stock of LWD is closely held and pays an annual dividend of 12%.

(a) (*1.5 points*) Describe the different kinds of liquidity risk that a firm, in general, may be exposed to.

(b) (*2 points*) Identify and explain a stress scenario which could create a liquidity crunch specifically for LWD.

LWD management wishes to obtain financing to expand into locations it currently does not serve. LWD is considering the following alternatives for obtaining $10 million in funding to support its expansion:

I.   Obtaining a line of credit

II.   Issuing publically registered debt securities which have an acceleration of principal provision in the event that LWD fails to maintain certain financial ratios

III.   Issuing new shares of LWD stock

IV.   Issuing private placement "bullet bonds" which are issued at a discount to par, do not pay periodic coupons, and require repayment of par value at maturity

V.   Issuing publically registered rate reset notes in which the coupon payment rate is a function of the current rating of the notes

VI.   Issuing a structured note which is collateralized by certain LWD receivables

5. Continued

Horseshoe Financial is evaluating whether to provide funding to LWD using one of the alternatives above. With respect to option I, Horseshoe's role would be to provide the line of credit. With respect to options II through VI, Horseshoe would become the holder of the security issued by LWD.

(c) (*2.5 points*) Describe the exposure to liquidity risk that each of the six financing alternatives creates for **Horseshoe**.

(d) (*4 points*) Assess each of the alternatives with respect to its impact on the liquidity risk exposure it creates for **LWD**, and rank them from least to greatest liquidity risk exposure. Justify your assessment.

6. (*9 points*) Tyrion has a portfolio of two commodities: wolf fur and lion mane. From experience Tyrion has found that RiskMetrics provides a pragmatic approach to modeling risk.

Variances are modeled using the exponentially weighted moving average (EWMA) process $h_t = \lambda h_{t-1} + (1-\lambda)r_{t-1}^2$ with a decay factor of 95%, where the time horizon is daily. Covariances are modeled using a similar process.

After a 5% increase in the market value of each commodity, the value of the portfolio and the conditional variances at the end of today are:

	Today's Market Value at Close	Today's Conditional Variance	Today's Conditional Covariance
Wolf fur	500	0.0225	0.0050
Lion mane	500	0.0100	

(a) (*3 points*) Calculate the 99% VaR for the portfolio as of today.

(b) (*2 points*) Calculate the expected conditional covariance of the portfolio to use in tomorrow's calculation.

(c) (*4 points*) Tyrion wants to increase one of his commodity positions by 1%. His goal is to increase his expected future profits while minimizing the change in VaR.

Determine whether Tyrion should increase the investment in wolf fur or lion mane. Justify your response.

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7.   (*7 points*) SLIC is interested in using VaR techniques to manage risk in its defined benefit pension plan (the "Plan"). Your analysis is focused on the Plan's two major asset classes: U.S. Bonds and U.S. Equity. You have gathered the following information to aid in your analysis:

	Proposed Allocation	Market Value ($000)	Index Mapping	Expected Return	Volatility	Correlation U.S. Bonds	Correlation U.S. Equity
U.S. Bonds	35%	157,835	Barclay's Aggregate	4.50%	3.7%	1.00	0.06
U.S. Equity	65%	293,122	S&P 500	6.50%	15.0%	0.06	1.00
Total	100%	450,957					

SLIC's risk appetite for the combined U.S. Bond and U.S. Equity portfolio is a volatility of 10%.

(a)   (*2 points*)

   (i)   Determine whether or not the proposed portfolio allocation results in a portfolio that satisfies SLIC's risk appetite objective.

   Show your work.

   (ii)   Using the proposed allocation, determine the portfolio VaR (in $000s) at a 95% confidence level.

   Show your work.

(b)   (*1 point*) Determine the risk budget (in $000s) for each of U.S. Bonds and U.S. Equity, consistent with your results in (a).

Show your work.

7. **Continued**

As indicated in the case study, SLIC's DB Plan U.S. equity investments are currently allocated equally to two U.S. Equity managers, Manager C and Manager D. Assume that the deviations for the U.S. Equity managers are independent of each other.

Maximizing the portfolio information ratio $\left(IR_p \right)$ subject to a fixed tracking error volatility (TEV) results in the following optimal asset allocation across managers:

$$x_i \omega_i = IR_i \left[\omega_p / IR_p \right]$$

(c)   (*3 points*) Determine the optimal allocation to each of the U.S. Equity managers and to the S&P 500 Index if a total U.S. Equity portfolio TEV of 4% is targeted.

Show your work.

(d)   (*1 point*) Determine the allocated principal and relative risk budget (in $000s) for each U.S. Equity manager under the optimal manager allocation, at a confidence level of 95%.

Show your work.

8. (*13 points*) You are reviewing the current asset allocation strategy for the AHA defined benefit plan ("the DB Plan"). The bond/equity split is a key driver of long-term performance for the DB Plan.

You will use a variation of the mean-variance optimization (MVO) framework in which the static risk-adjusted change in surplus (RACS) over a 1-year horizon is given by:

$$RACS_t = \frac{E_t\left[S_{t+1} - S_t\left(1 + R_f\right)\right]}{\sigma_t\left[S_{t+1}\right]},$$ where S_t is the plan's surplus at time t.

The assets are assumed to have an equity allocation percent of α and a bond allocation percent of $(1-\alpha)$.

Under the static version of the RACS model, surplus risk for a given funding ratio is minimized when the equity allocation α is:

$$\alpha = \frac{\left(1 - \frac{\beta L_t}{A_t}\right)\left(\sigma_B^2 - \rho\sigma_B\sigma_E\right)}{\sigma_E^2 + \sigma_B^2 - 2\rho\sigma_B\sigma_E}$$

Additionally, the minimum equity allocation needed to prevent a decline in surplus is:

$$\alpha = \frac{\mu_B\left(\frac{\beta L_t}{A_t} - 1\right) + \frac{L_t}{A_t}R_f\left(1 - \beta\right)}{\mu_E - \mu_B}$$

For your analysis, you will exclude the Cash and Real Estate asset classes. Therefore, you assume a current bond/equity split of approximately 32% / 68% (the "Current" allocation).

The DB Plan's Total Liabilities will be modeled using a levered Barclays Capital Aggregate 10+ Year Maturity Index as a proxy, whereby:

$$R_{L,t} - R_f = \beta\left(R_{LB,t} - R_f\right) + \varepsilon_t$$

The noise term ε_t has an expected value of 0% and a volatility of 2%.

Assume that the risk-free rate, R_f, is 1%.

8. Continued

You have gathered the following partial information for your analysis:

Table A

Index Data				Correlations			
				Equity	Barclay's Capital U.S. Bond Indices		
	Expected Returns	Annualized Volatility	Duration	S&P 500	Aggregate	Aggregate 10 + Year Maturity	Liability Index
S&P 500	6.5%	15.0%	0.0	1.00			
Aggregate	2.3%	3.7%	5.0	0.06	1.00		
Aggregate 10+ Year Maturity	3.0%	10.0%	14.2	−0.10	0.90	1.00	
Liability Index	???	13.1%	???	−0.06	0.85	0.95	1.00

Table B

	Allocation Weight	Market Value ($000)	Index
Bonds	32%	54,172	Aggregate
Equity	68%	115,117	S&P 500
Total Assets	100%	169,289	
Liabilities		192,949	Liability Index

(a)　(*2 points*)　Using the index mapping in Table B above, calculate the Sharpe ratio for each of:

    (i)　Bonds

    (ii)　Equity

    (iii)　Total Assets

    Show your work.

(b)　(*1 point*)　Provide your insights regarding the DB Plan's Current bond/equity split based on the Sharpe Ratio calculations above.

(c)　(*2 points*)　Identify and describe two shortcomings of using the Sharpe ratio approach for asset allocation decisions with regard to the DB Plan.

Question 8 is continued on the next page

8. **Continued**

(d) (*2 points*)

    (i)   Determine the static RACS framework's bond/equity split that minimizes surplus risk.

    (ii)   Determine the static RACS framework's bond/equity split that prevents a surplus decline.

    Show your work.

(e) (*3 points*)

    (i)   Interpret each of your static RACS analyses from (d).

    (ii)   Assess the appropriateness of the DB Plan's Current bond/equity split.

(f) (*3 points*) Define the M-CVaR Optimization framework, and identify two major advantages it would provide over the MVO framework in making asset allocation decisions for the DB Plan.

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END OF EXAMINATION

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