

# QFI ADV Model Solutions

## Fall 2018

### 1. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.
3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.

### Learning Outcomes:

- (2l) Understand and apply various approaches for managing credit risk in a portfolio setting.
- (3a) Understand the concept of liquidity risk and the threat it represents to financial intermediaries and markets.
- (3b) Measure and monitor liquidity risk, using various liquidity measurement tools and ratios.
- (3c) Understand the levels of liquidity available with various asset types, and the impact on a company's overall liquidity risk.

### Sources:

Quantitative Credit Portfolio Management, Ben-Dor, et. al., 2012, Ch. 5-6  
Handbook of Fixed Income Securities, Fabozzi, F.J., 8th Edition, 2012, Ch 66-67

### Commentary on Question:

*This question tests the candidates' understanding of mortgage defaults modeling and its application to the 2008 financial crisis.*

*Overall, candidates performed as expected.*

### Solution:

- (a) Determine whether each of Bonds A, B and C is compliant with the new policy.

### Commentary on Question:

*Candidates performed above average on this part. Marks were most commonly lost for not being able to identify which bonds are benchmark bonds. Successful candidates were able to correctly calculate the LCS of each bond, and concluded that a lower LCS is a better LCS. Partial credit was given if the correct conclusion was given based on an incorrect calculation.*

## 1. Continued

Bond A is a quoted benchmark bond since it has a trading percentile above 80% and is “quoted several times a month” – no factors are applied

Bond A LCS =  $(\$85 - \$80) / \$80 = 6.25\%$ , < 7%

Bond A is compliant with the new policy

Bond B is a quoted, but is not a benchmark bond since it does not have a trading percentile above 80% (p. 91) – so a non-benchmark factor applies (1.40)

Bond B LCS =  $(225 - 185) / 10000 \text{ bps} \times 14 \times 1.4 = 7.84\%$ , > 7%

Bond B is not compliant with the new policy

Bond C is non-quoted but since it is on the run, it is a benchmark bond - so only the non-quoted factor applies

Bond C LCS =  $(200 - 100) / 10000 \text{ bps} \times 3 \times 1.05 = 3.15\%$ , < 7%

Bond C is compliant with the new policy

- (b) Evaluate whether Bond E is likely to be compliant with the new policy.

### **Commentary on Question:**

*Candidates performed above average on this part. Successful candidates correctly concluded that Bond E was likely compliant based on the fact that Bond D was compliant, and Bond E favorably compared to Bond D among almost all characteristics that correlate with liquidity, and noting where they did not compare favorably. Partial credit was given for each data point compared, and a correct conclusion.*

*Higher trading volume indicates a lower LCS for Bond E.*

*Later issue date indicates a lower LCS for Bond E.*

*All else held equal, lower coupon payments indicate a higher duration, and therefore higher LCS for Bond E.*

*Lower OAS indicates a lower LCS for Bond E.*

*Larger amount outstanding indicates a lower LCS for Bond E.*

*Despite potentially having a higher duration, bond E is very likely to have a lower LCS than 7% and be appropriate for the portfolio.*

- (c)
- (i) Demonstrate which bond likely has the highest default risk.
  - (ii) Demonstrate which bond likely has the highest liquidity risk.

## 1. Continued

### Commentary on Question:

Candidates did as expected on this part. Successful candidates were able to calculate the liquidity component and credit default component for most bonds, and concluded that the bond with the highest default component or liquidity component, expressed in bps, had the highest default or liquidity risk. A common mistake was to either use the default component proportion as if it were expressed in bps, or apply the default component proportion to the yield rather than the OAS. Partial credit was given if the correct conclusion was given based on an incorrect calculation provided the candidate attempted to calculate the components in b.

Bond F:  $50\% \times (2.6\% - 2.2\%) = 20$  bps

Bond G:  $40\% \times (3.0\% - 2.2\%) = 32$  bps

Bond H:  $25\% \times (3.2\% - 2.2\%) = 25$  bps

Bond G likely has the highest default risk.

Bond F:  $(2.6\% - 2.2\%) * (100\% - 50\%) - 0$  bps = 20 bps

Bond G:  $(3.0\% - 2.2\%) * (100\% - 40\%) - 40$  bps = 8 bps

Bond H:  $(3.2\% - 2.2\%) * (100\% - 25\%) - 70$  bps = 5 bps

Bond F likely has the highest liquidity risk.

- (d) Calculate the number of credit default swap contracts necessary to fully hedge the default risk of the CIO's desired trade, including whether the contracts must be bought or sold.

### Commentary on Question:

Candidates performed below expectations on this part. Successful candidates identified the need to consider the bond's duration and market value when calculating the sensitivity of the bond to a one basis point change in credit spreads, before dividing by the DV01 of the credit default swap. Common mistakes were to hedge against a movement equal to the size of the entire default swap component instead of a one basis point movement, to calculate DV01 as a 100 bps movement instead of a 1 bp movement, or to use the Macaulay duration of 5 instead of effective or modified duration when calculating DV01.

Bond F MV =  $1/(1+0.026)^5 = 0.879555$

Bond F MV shifted 1 bp =  $1/(1+0.026+0.0001)^5 = 0.879127$

Bond F Effective Duration =  $(0.879555 - 0.879127)/((0.879555+0.879127)/2 * 0.0001) = 4.8673$

Alternatively, Bond F Modified Duration =  $5/(1+0.026) = 4.8733$

## 1. Continued

Bond F Spread DV01 = \$100 million \* 4.8733/10000 = \$48,730

Number of contracts required = 48730/5410 = 9.01 contracts (round to 9 contracts).

## 2. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:

- One and two-factor short rate models
- LIBOR market models

The candidate will understand approaches to volatility modeling.

### Learning Outcomes:

(1l) Define and explain the concept of volatility smile and some arguments for its existence.

(1m) Calculate the hedge ratio for a call option given the dependency of the Black-Scholes volatility on the underlying.

(1n) Compare and contrast “floating” and “sticky smiles.

### Sources:

Rebonato, R. – Chapter 6, p. 168-169, 175-180

### Commentary on Question:

*This question tests the candidates understanding of implied volatility as a function of the strike price. It explores the difference between a floating and a sticky volatility smile and the impact on the hedge ratio calculation for derivatives.*

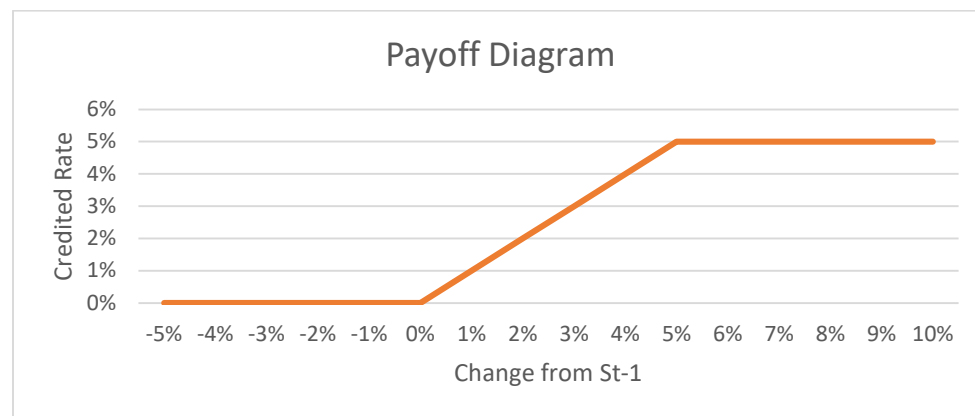
### Solution:

(a) Draw a payoff diagram for the credited rate profile.

### Commentary on Question:

*Candidates performed brilliantly on this section.*

*Successful candidates sketched where the curve is level and where it is trending upward, illustrating 0% and 5% on both the X and Y axes.*



## 2. Continued

- (b) Specify the embedded call option(s) underlying the product.

**Commentary on Question:**

*Candidates performed above average on this section. Successful candidates indicated the strike price,  $K$ , as being  $S_0$ ;  $S_0 - 1$  was also accepted as equivalent to  $S_0$ . Most candidates; however, failed to specify the moneyness of both options, the focus of the question was not to get the strike price correct but to provide more context. Many candidates did not specify that the options were “European”. The important aspect was identifying correctly which option was long and which was short with the correct strike prices for both.*

- 1-Long position in an at-the-money European Call Option with  $K = S_0$
- 2-Short position in an 5% out-of-the-money European Call Option with  $K = S_0 \times 1.05$

- (c) Explain the presence of the “volatility smile”.

**Commentary on Question:**

*Candidates performed as expected on this section. Many candidates incorrectly answered that the volatility is not constant for a given index level, where marks would have been awarded for indicating that it is the implied volatility that is not constant. Explaining that the implied volatility was obtained from the Black Scholes to the observed market price was expected from candidates to obtain full credit.*

- Using the Black-Scholes formula, the market defines the implied volatility as the volatility that gives the observed market value with  $T$ ,  $S$ ,  $K$  and  $r$  constant.
- That implied volatility obtained by back solving Black-Scholes is not constant for different strike price, even if for the same maturity.
- A plot of those implied volatilities, for options of the same maturities, typically produces a smile shape. Whereby implied volatilities are highest if the strike prices are away from the stock prices and lowest for at-the-money options.
- Higher demand for options with strike price away from stock prices can lead to higher implied volatilities.
- Out-of-the-money option may be priced higher to compensate for the risk of a crash.

- (d) Calculate the equity Delta of the product immediately after issue.

## 2. Continued

### Commentary on Question:

Candidates performed as expected on this section. The key point here was to

recognize that given the sticky volatility smile assumption  $\frac{\partial \sigma_{t,T}^{\text{implied}}}{\partial \sigma_{t,T}}$  is equal to 0 and thus  $\Delta_{\text{call}}$  is equal simply to  $N(d_1)$  for each call. Many candidates calculated correctly the  $\Delta_{\text{call}}$  for only one call. To answer the question, both  $\Delta_{\text{calls}}$  were needed. That product is made of two calls, long one at  $S_0$  and short the other at  $1.05S_0$ . Many unsuccessful candidates also failed to recognize the initial premium of \$10,000.

-Given that the volatility smile is sticky then  $\Delta_{\text{call}} = N(d_1)$

- $\Delta_{\text{product}} = 10000 \times (\Delta_{\text{call}}(K=100) - \Delta_{\text{call}}(K=105))$

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + .5\sigma_{t,T}^{\text{implied}}\right)\sigma_{t,T}^{\text{implied}}\sqrt{T}}{\sigma_{t,T}^{\text{implied}}\sqrt{T}}$$

-for  $\Delta_{\text{call}}(K=100)$

$$d_1 = \frac{\ln(100/100) + (.03 + .5 \times (.1812)^2)\sqrt{1}}{.1812 \times \sqrt{1}} = 0.256$$

$$N(d_1) = N(0.256) = 0.6026$$

-for  $\Delta_{\text{call}}(K=105)$

$$d_1 = \frac{\ln(100/105) + (.03 + .5 \times (.1629)^2)\sqrt{1}}{.1629 \times \sqrt{1}} = -0.0339$$

$$N(d_1) = N(-0.0339) = 0.488$$

$$\Delta_{\text{product}} = 10000 \times (0.6026 - 0.488) = 1,146$$

- (e) Explain why you need to change your calculations because of this suggestion.

### Commentary on Question:

Candidates performed below average on this section. Candidates were expected to explain what is the difference between a sticky smile and a floating smile. Most got the mathematical explanations right, correctly providing the formula for  $\Delta_{\text{call}}$ ; however, unsuccessful candidates failed to provide a satisfactory explanation of why they needed to update their calculations.

## 2. Continued

-If the smile is floating then  $\Delta_{\text{call}}$  is no more equal to simply to  $N(d_1)$ .  $\Delta_{\text{call}}$  is now:

$$\Delta_{\text{call}} = N(d_1) + \text{BlackVega}(S, K, \sigma_{\text{impl}}(S, K)) \times \frac{\delta \sigma_{\text{impl}}(S, K)}{\delta S}$$

-The implied volatility will change for a change in S too, not only for different K.

-If assumed a sticky volatility smile, the implied volatility doesn't change with a change in the stock price.

- (f) Recalculate the equity Delta of the product immediately after issue.

### Commentary on Question:

Candidates performed below expected on this section. This is an extension of part d), grading assumed that the candidates answer in d) was correct. Successful candidates recognized that  $\Delta_{\text{call}}$  is not simply equal to  $N(d_1)$  but equal to  $N(d_1) + \text{BlackVega} \times \frac{\delta \sigma_{\text{impl}}(S, K)}{\delta S}$ . Again, the product is made up of 2 calls, one long at  $K = S_0$  and one short at  $K = 1.05S_0$ . Successful candidates got the correct

formula and calculations for BlackVega and  $\frac{\partial \sigma_{t,T}^{\text{implied}}}{\partial \sigma_{St}}$  once they correctly identified the updated  $\Delta_{\text{call}}$ .

-Given that the volatility smile is floating then

$$\Delta_{\text{call}} = N(d_1) + \text{BlackVega} \times \frac{\delta \sigma_{\text{impl}}(S, K)}{\delta S}$$

Where

$$\text{BlackVega} = S_t \times \sqrt{T} \times N'(d_1) = S_t \times \sqrt{T} \times \frac{e^{-d_1^2/2}}{\sqrt{2\pi}}$$

And

$$\frac{\partial \sigma_{t,T}^{\text{implied}}}{\partial \sigma_{St}} \cong \frac{\sigma_{\text{impl}}(S_0 + \delta S, K) - \sigma_{\text{impl}}(S_0 - \delta S, K)}{2\delta S}$$

$$-\Delta_{\text{product}} = 10000 \times (\Delta_{\text{call}}(K=100) - \Delta_{\text{call}}(K=105))$$

-

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + (r + .5 \times \sigma_{t,T}^{\text{implied}} \times \sigma_{t,T}^{\text{implied}}) \times T}{\sigma_{t,T}^{\text{implied}} \sqrt{T}}$$

-for  $\Delta_{\text{call}}(K=100)$

$$d_1 = \frac{\ln(100/100) + (.03 + .5 \times (.1812)^2) \sqrt{1}}{.1812 \times \sqrt{1}} = 0.256$$

$$N(d_1) = N(0.256) = 0.6026$$

$$\text{BlackVega} = 100 \times \sqrt{1} \times e^{-.256^2/2} / \sqrt{2\pi} = 38.61$$

$$\frac{\partial \sigma_{t,T}^{\text{implied}}}{\partial \sigma_{St}} \cong \frac{20.57\% - 16.25\%}{10} = .00432$$



## 2. Continued

$$\Delta_{\text{call}} = 0.6026 + 38.61 \times 0.00432 = 0.7679$$

-for  $\Delta_{\text{call}} (K=105)$

$$d_1 = \frac{\ln(100/105) + (.03 + .5 \times (.1629)^2) \sqrt{1}}{.1629 \times \sqrt{1}} = -0.0339$$

$$N(d_1) = N(-0.0339) = 0.488$$

$$\text{BlackVega} = 100 \times \sqrt{1} \times e^{-(.0339)^2/2} / \sqrt{2\pi} = 39.92$$

$$\frac{\frac{\partial \sigma_{t,T}^{\text{implied}}}{\partial \sigma_{t,T}} \approx \frac{18.12\% - 15.50\%}{10} = 0.00262$$

$$\Delta_{\text{call}} = 0.488 + 39.92 \times 0.00262 = 0.59$$

$$\Delta_{\text{product}} = 10000 \times (0.7679 - 0.59) = 1,800$$

### 3. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

#### Learning Outcomes:

- (4h) Understand and apply various techniques of adjusting auto correlated returns for certain asset classes.

#### Sources:

Keith H. Black, Donald R. Chambers, and Hossein Kazemi CAIA Level II: Advanced Core Topics in Alternative Investments, 2nd Edition, Chapter 16

#### Commentary on Question:

*This question tests the candidates' understanding of a first-order autocorrelation model in the context of a real estate price index. Candidates should be able to recognize the unique characteristics of real estate and a real estate price index, that makes it more suitable to be modeled using autocorrelation models.*

#### Solution:

- (a) Describe two primary reasons why smooth series returns from the real estate market may not be easily unsmoothed by arbitrageurs.

#### Commentary on Question:

*The candidates performed as expected on this section. Successful candidates identified high transaction costs of the real estate markets as a deterrent to arbitrage. Some unsuccessful candidates described general barriers to arbitrage that were not specific to the real estate market.*

Reason 1: Real estate return series may not indicate true trading opportunities. In this case, appraisal-based data do not represent actual offers to buy or sell and are estimates only. Without a trading opportunity, arbitrageurs cannot unsmooth the return series.

Reason 2: Real estate assets have substantial transaction costs or other barriers to arbitrage. For example, in real estate, the time and transaction costs of buying and selling assets in order to exploit delayed pricing responses may be prohibitively expensive.

- (b) Assess why your choice of model is appropriate for a real estate price index.

#### Commentary on Question:

*The candidates performed as expected on this section. Some unsuccessful candidates did not draw the link between the model and a real estate price index.*

### 3. Continued

A first order autocorrelation model is appropriate, because the following characteristics of a real estate price index give rise to the autocorrelation effect.

1. A price index is based on the most recent transaction price or appraisal of each component. Because real estate markets are illiquid and appraisals are not done frequently, some of this price information will be stale.
2. An appraiser may exhibit the behavioral phenomenon known as anchoring. Thus the appraisal prices may only partially reflect new market information and partially be based on the prior period price.
3. Even current transaction prices in an efficient market may be selected such that they signal lagged price responses by market participants.
4. There is a potential delay between the setting of a price on a real estate transaction and the reporting of the transaction. A real estate price may be negotiated months before the transaction occurs, and the reported price of the transaction may become known to the appraiser or index on a delayed basis as well.

[ Note that only 2 characteristics are required for full credit. Other valid characteristics also received credit. ]

- (c) Estimate the parameter  $\rho$  using the model above.

#### **Commentary on Question: /**

*The candidates performed below average on this section. Many candidates recognized that the estimate for  $\rho$  is the correlation of the returns to the 1-period lagged returns. Some unsuccessful candidates tried to algebraically manipulate  $R_t^{\text{reported}} = (1 - \rho) R_t^{\text{true}} + \rho R_{t-1, \text{reported}}$  for the 5 values of  $t$  in order to isolate and solve for the true value of  $\rho$ , which was not possible to do so.*

Given the following data:

t	1	2	3	4	5
$R_{t, \text{reported}}$	0%	0%	7%	4%	3%
$R_{t-1, \text{reported}}$	0%	0%	0%	7%	4%

The estimate of  $\rho$  is given by the formula

$$\hat{\rho} = \frac{\widehat{\text{cov}}(R_{t, \text{reported}}, R_{t-1, \text{reported}})}{\sqrt{\widehat{\text{var}}(R_{t, \text{reported}}) \widehat{\text{var}}(R_{t-1, \text{reported}})}}$$

### 3. Continued

$$\begin{aligned}\widehat{Cov}(R_{t,reported}, R_{t-1,reported}) &= \frac{1}{5-1} [(-2.8\%)(-2.2\%) + (-2.8\%)(-2.2\%) \\ &\quad + (4.2\%)(-2.2\%) + (1.2\%)(4.8\%) + (0.2\%)(1.8\%)] \\ &= 0.023\%\end{aligned}$$

$$\begin{aligned}\widehat{Var}(R_{t,reported}) &= \frac{1}{5-1} [(-2.8\%)^2 + (-2.8\%)^2 + (4.2\%)^2 + (1.2\%)^2 \\ &\quad + (0.2\%)^2] \\ &= 0.087\%\end{aligned}$$

$$\text{Similarly, } \widehat{Var}(R_{t,reported}) = 0.102\%$$

$$\text{Therefore, } \hat{\rho} = \frac{0.023\%}{\sqrt{0.087\% \times 0.102\%}} = 0.244$$

An alternative solution is also correct if the 0% return for time 0 is not assumed and pre-pended.

- (d) Calculate the unsmoothed return at time 3, using the parameter  $\rho$  estimated in part (c).

**Commentary on Question:**

*Candidates performed above average on this section. Successful candidates recognized that the unsmoothed return at time 3 could be calculated using the reported returns at times 3 and 2, as well as an estimate for  $\rho$ , even if they were not successful in part c.*

$$R_{t,true} = \frac{(R_{t,reported} - \rho R_{t-1,reported})}{(1 - \rho)}$$

Using the estimate for  $\rho$  from part c,

$$\begin{aligned}R_{3,true} &= \frac{(R_{3,reported} - \rho R_{2,reported})}{(1 - \rho)} \\ &= \frac{7\% - 0.244 \times 0\%}{1 - 0.244} = 9.26\%\end{aligned}$$

- (e) Discuss a possible reason why the unsmoothed return you calculated in part (d) is different from the true unsmoothed return given in the table.

**Commentary on Question:**

*The candidates performed poorly on this section. Many successful candidates recognized that the estimated  $\rho$  differed from the true  $\rho$ . Many unsuccessful candidates incorrectly suggested that the first-order autocorrelation model was incomplete or not correct, but the given data of true returns, reported returns, and true  $\rho$  can be used to verify that the model is correct and complete.*

### **3. Continued**

The unsmoothed return estimated in part (d) is different from the true return, because the estimated value of  $\rho$  is different from the true value. This estimation error is likely a result of the small sample size of 5. As the sample size is increased, the estimate of  $\rho$  will also improve.

#### 4. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

#### Learning Outcomes:

- (4a) Understand the concept of a factor model in the context of financial time series.
- (4b) Apply various techniques for analyzing factor models including Principal Component Analysis (PCA) and Statistical Factor Analysis.

#### Sources:

QFIA-125-16: Market Models: A Guide for Financial Data Analysis, Ch. 6, Principal Component Analysis

QFIA-119-14: Analysis of Financial Time Series, Tsay, 3rd edition, Ch. 9

#### Commentary on Question:

*This question focuses on testing several aspects of the candidates understanding of principal component analysis through listing the steps in a PCA, determining the needed number of components to explain a specific portion of the variation, and performing a simple calculation. Candidates performed well on the theoretical portion of the question and as expected on the overall calculation.*

#### Solution:

- (a) List all steps necessary to compute the related principal components.

#### Commentary on Question:

*The candidates performed above average on this question, often recognizing the need to create a correlation matrix and focusing on it's eigenvectors. Most candidates failed to note the variable normalization needed.*

After normalizing the stock prices to returns, compute the correlation matrix for the five series. The distinct eigenvectors, sorted from lowest to highest eigenvector, represent the principal components.

- (b) Determine the fewest number of principal components needed to explain at least 80% of the variation.

#### Commentary on Question:

*The candidates performed brilliantly on this question with almost all students achieving full credit.*

Sum of the lambda values is 0.002834, the total variation. After two components the cumulative sum represents 84% of the total variation, thus two components are needed.

## 4. Continued

- (c) Calculate the correlation between the returns of the first and fifth stocks using the first 3 principal components.

**Commentary on Question:**

*Candidates performed below average on this question, typically recognizing the inputs needed to complete the question but failing to calculate a final covariance required to be successful.*

The three component lambdas needed are given as 0.001853, 0.00053, and 0.000221. Weighting these together yields the individual variances of the first and fifth component and the subsequent correlation.

$$\begin{aligned}\text{Var}[X1] &= 0.000235, \text{Var}[X5]=0.00022 \\ \text{Cor}[X1, X5] &= \frac{\text{Cov}[X1, X5]}{\sqrt{\text{Var}[X1]*\text{Var}[X5]}} = 0.4025\end{aligned}$$

- (d) Describe how the principal components analysis on the 5 stocks between  $t_1$  and  $t_2$  can be used to generate price data for the first stock prior to  $t_1$  (assuming returns follow a stationary process).

**Commentary on Question:**

*Candidates performed as expected on this question, usually recognizing the two separate calculations needed and occasionally elaborating on the subsequent goodness of fit step.*

Perform a PCA on all components from  $t_1$  to  $t_2$ , then a second PCA before  $t_1$  on stocks two to five. Recreate the artificial history for stock one by weighting the PCA components and perform a regression versus the actual history.

## 5. Learning Objectives:

5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

### Learning Outcomes:

- (5a) Explain how behavioral characteristics of individuals or firms affect the investment or capital management process.
- (5b) Describe how behavioral finance explains the existence of some market anomalies.
- (5c) Identify and apply the concepts of behavioral finance with respect to individual investors, institutional investors, portfolio managers, fiduciaries and corporate managers.

### Sources:

QFIA-109-13: A Survey of Behavioral Finance, by Barberis & Thaler (pg. 1058-1061, 1063, 1064, 1069-1087)

### Commentary on Question:

*This question tests the concept of behavioral finance with respect to professional investors within a committee environment*

### Solution:

- (a) Critique the investment committee's choice of utility function  $v(X)$ .

#### Commentary on Question:

*The candidates performed as expected on this section. Most candidates were able to identify that the utility function is able to capture loss aversion but unable to reflect the concavity (convexity) over gains (losses). Candidates failed to achieve full marks if they did not identify that the function ignores the nonlinear probability transformation.*

The utility function is good because:

The utility function captures loss aversion, which empirical evidence suggests is the main factor driving economic behavior.

It measures utility over gains and losses rather than final wealth positions, which is characteristic of prospect theory and experimental results.

The 2.25 factor is credible as it is based on a broad-based experimental study.

The utility function is deficient because:

The function ignores the concavity (convexity) over gains (losses), which is an important component of prospect theory.

The function ignores the nonlinear probability transformation, i.e. small probabilities are overweighted.



## 5. Continued

The 2.25 factor is based on an empirical study of a population that may not be representative of the investment committee's utility profile.

- (b) Calculate for each of the two stocks the expected utility for both actions above, without assuming any mental accounting effects.

### **Commentary on Question:**

*The candidates performed below average on this section. Many candidates failed to recognize that the initial purchase price is irrelevant without mental accounting, and they calculated the utility function with mental accounting instead. Some candidates did not calculate the utility function for option I.*

Without any mental accounting, the initial purchase price is irrelevant and the decision is based solely on the expected future value of the stock.

For stock A,

The expected value of selling at the current price and investing in the risk-free asset is:

$$v(X) = V(52 * 0.05) = V(2.6) = 2.6$$

The expected value of holding for another year is:

$$v(X) = 0.5 * V(8) + 0.5 * V(-2) = 0.5 * 8 - 0.5 * 2.25 * 2 = 1.75$$

For stock B,

The expected value of selling at the current price and investing in the risk-free asset is:

$$v(X) = V(20 * 0.05) = 1$$

The expected value of holding for another year is:

$$v(X) = 0.5 * V(10) + 0.5 * V(-3) = 0.5 * 10 - 0.5 * 3 * 2.25 = 1.625$$

- (c) Calculate for each of the two stocks the expected utility for both actions, after taking into account the effect of mental accounting.

### **Commentary on Question:**

*The candidates performed below average on this section. Many candidates failed to recognize that current gain/loss is part of utility function when considering mental accounting, and they calculated the utility function without mental accounting instead. Some candidates did not calculate the utility function for option I.*

## 5. Continued

With mental accounting, the decision is based on both the current gain/loss on the stock and the expected future value of the stock.

For stock A,

The expected value of selling at the current price and investing in the risk-free asset is:

$$v(X) = V(2) + V(52 * 0.05) = V(2) + V(2.6) = 4.6$$

The expected value of holding for another year is:

$$v(X) = 0.5 * V(10) + 0.5 * V(0) = 0.5 * 10 + 0.5 * 0 = 5$$

For Stock B,

The expected value of selling at the current price and investing in the risk-free asset is:

$$v(X) = V(-5) + V(20 * 0.05) = V(-5) + V(1) = -10.25$$

The expected value of holding for another year is:

$$v(X) = 0.5 * V(5) + 0.5 * V(-8) = 0.5 * 5 - 0.5 * 8 * 2.25 = -6.5$$

- (d) Recommend for each of the two stocks whether to take action I or II and justify your answer.

### **Commentary on Question:**

*The candidates performed below average on this section. Most candidates were able to recommend hold stock B for another year; however, many failed to identify that XYZ company should avoid allowing mental accounting when making its investment decision and recommend sell stock A.*

Recommend XYZ Company sells Stock A and invests in the risk-free asset.

As a sophisticated institutional investor, XYZ Company should avoid allowing mental accounting to bias its investment decisions. Otherwise, this will likely lead to suboptimal performance of the portfolio, particularly over the long term.

For Stock B, the expected value of holding is higher than the expected value of selling for both with and without mental accounting taken into account, so recommend to hold on to Stock B for another year.

## 6. Learning Objectives:

6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
- Real Estate
  - Private Equity
  - Commodities
  - Hedge Funds
  - Managed Futures
  - Distressed Securities
  - Infrastructure

### Learning Outcomes:

- (6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.
- (6e) Demonstrate an understanding of infrastructure investments.

### Sources:

QFIA-111-13: Maginn & Tuttle, Managing Investment Portfolios, 3rd Ed. 2007, Ch. 8

QFIA-126-16: Infrastructure as an Asset Class

### Commentary on Question:

*This question tests the candidates' understanding of alternative assets and relevant allocation strategies.*

### Solution:

- (a) Describe four key features of alternative asset classes.

### Commentary on Question:

*The candidates performed above average on this part.*

*For full credit, we required the candidates to list the four key features of alternative assets as outlined in Maginn & Tuttle. Successful candidates were able to list at least some of the key features.*

Relative illiquidity, which tends to be associated with a return premium as compensation.

Diversifying potential relative to a portfolio of stocks and bonds.

## 6. Continued

High due diligence costs for the following reason: investment structures and strategies may be complex; evaluation may draw heavily on asset class, business-specific, or other expertise; reporting often lacks transparency.

Usually difficult performance appraisal because of the complexity of establishing valid benchmarks.

- (b) Explain four challenges of asset allocation involving infrastructure investments.

**Commentary on Question:**

*The candidates performed below average on this part. Many candidates rephrased key features of alternative assets from part a) in part b), which was insufficient to receive credit for this part. Many candidates listed specific risks (e.g. credit risk) associated with infrastructure investments, which was also insufficient to receive full credit. Other responses than the challenges below could also earn credit on this question. Successful candidates were those who listed and explained their responses.*

Lack of transparency and governance standards.

Lack of financial theory to back infrastructure as an asset class and empirical evidence suggests that infrastructure looks more like a sub asset class within a traditional investment vehicle.

Duration mismatch between the lifetime of the underlying assets and the lifetime of the investment vehicle (typically 10 years).

Investors can misunderstand the structure and features of infrastructure investments – investors look for stable, long-term income but end up with highly leveraged and high-risk funds.

- (c) Explain why the J-curve effect could dissuade ATL from adding infrastructure assets to its portfolio.

**Commentary on Question:**

*The candidates performed as expected on this part. Many candidates did not answer this part. Some unsuccessful candidates mistakenly explained the J-curve as legal or political risk. Successful candidates were expected to both explain the J-curve effect and explain how the J-curve applies to ATL's situation.*

## 6. Continued

The J-curve effect in this case refers to private equity-type infrastructure funds delivering negative returns in early years and investment gains in later years as the portfolio of companies mature.

Since ATL is concerned about short-term funding needs and hence short-term return, such infrastructure assets do not meet ATL's objective.

- (d) Recommend the optimal asset allocation for ATL's portfolio.

### **Commentary on Question:**

*The candidates performed above average on this part. Successful candidates were able to identify the allocations that meet ATL's constraints and recommend the optimal allocation. Some candidates did not recommend any of the allocations but rather specified a set of optimization constraints and objectives to blend the asset allocations in question. These candidates did not receive full credit for this part.*

The benefit payments over the next 12 months is expected to be  $\$200M \times 5.50\% = \$11M$ . Allocation C is eliminated, as there is insufficient liquidity to cover the liability cash flows.

All three allocations provide higher expected return than that of the liabilities.

Since both portfolios A and B satisfy the constraints, the portfolio with the highest Sharpe ratio is selected:

*Note that excess return relative to liabilities (i.e.  $\frac{\text{Expected return} - 4.1\%}{\text{Standard deviation}}$ ) is also acceptable.*

$$\text{Sharpe ratio}_A = \frac{4.25\% - 1.20\%}{8.52\%} = 0.3580$$

$$\text{Sharpe ratio}_B = \frac{4.45\% - 1.20\%}{8.61\%} = 0.3775$$

Allocation B should be selected as the new asset allocation for ATL's portfolio.

## 7. Learning Objectives:

3. Candidate will understand the nature, measurement and management of liquidity risk in financial institutions.
6. The candidate will understand and be able to describe the variety and assess the role of alternative assets in investment portfolios. The candidate will demonstrate an understanding of the distinguishing investment characteristics and potential contributions to investment portfolios of the following major alternative asset groups:
  - Real Estate
  - Private Equity
  - Commodities
  - Hedge Funds
  - Managed Futures
  - Distressed Securities
  - Infrastructure

### Learning Outcomes:

- (3d) Understand liability termination provisions such as book-value surrender and the impact on a company's overall liquidity risk.
- (3f) Apply liquidity scenario analysis with various time horizons.
- (3g) Understand and apply techniques to manage street liquidity risk.
- (3h) Create liquidity risk management plans and procedures, including addressing appropriate product design, investment guidelines, and reporting given a desired liquidity risk level.
- (6c) Demonstrate an understanding of the investment strategies and portfolio roles that are characteristic of each alternative investment.
- (6e) Demonstrate an understanding of infrastructure investments.

### Sources:

Liquidity Risk: Measurement and Management - A Practitioner's Guide to Global Best Practices, Matz, Leonard & Neu, Peter, 2006, Ch. 3, p.40, p.46

Liquidity Risk Management CRO Forum, Section 5.5.1, p.11-13

Liquidity Risk Management CRO Forum, Section 5.7 – 5.8, p.17-18

Liquidity Risk: Measurement and Management - A Practitioner's Guide to Global Best Practices, Matz, Leonard & Neu, Peter, 2006, Ch. 3, p.44-47

## 7. Continued

Infrastructure as an Asset Class, p.73, p.84, p.92-95

Commercial Real Estate Analysis & Investment, Chapter 12, p.283-284

### **Commentary on Question:**

*This question tests the candidates' understanding of liquidity risk management and the use of alternative investments.*

### **Solution:**

- (a) Critique the cited approach including recommending improvements for deficiencies.

### **Commentary on Question:**

*The candidates performed as expected on this section. Most candidates were able to identify some of the deficiencies and suggest improvements. Unsuccessful candidates did not cover enough areas, particularly the need to consider both assets and liabilities together, sufficiently to obtain the full credits.*

Agree with the policy that it's important to evaluate liquidity needs based on policyholder reaction as reflected in cash withdrawal levels, especially for this product because of the good cash value guarantee. The one-month horizon alone is not appropriate. Change in customer behavior and funding problems can build up over months or longer. A range of stress scenarios should be evaluated to understand short, intermediate and long duration events. Downgrades and changes in customer behavior are not the only liquidity stress sources. Other stress scenarios can include catastrophic claims and capital market liquidity impairment. Need to consider how both the asset and liabilities portfolio will be impacted. Need to examine cashflow behavior under a variety of different scenarios.

- (b) Describe what the Liquidity Policy should include with respect to liquidity adequacy and liquidity crisis planning.

### **Commentary on Question:**

*The candidates performed below average on this section. Candidates were expected to describe the high level areas that the Liquidity Policy should cover. Many candidates instead explained the action plans in a liquidity event in great details without discussing the policy itself.*

## 7. Continued

Liquidity adequacy:

- Should be described in the context of the company's risk philosophy and tolerances.
- The degree to which the company will expect to rely on external cash sources versus self funding liquidity needs should be clearly described.
- How frequently liquidity adequacy is to be measured.
- The policy should specify minimum standards that the company must meet to consider itself to be adequately protected from liquidity risk
- Cure periods prescribed if standards are not met.

Crisis planning:

- A liquidity policy should reflect the company's advance planning for times of liquidity stress.
- Plans should be developed that will guide the company's management actions before a crisis arrives.
- The policy should describe the designation of a liquidity crisis management team, along with defined roles and responsibilities;
- The design of appropriate internal and external lines of communication.

(c)

- (i) Evaluate the appropriateness of this approach, including consideration of the Black Swan problem.
- (ii) Describe one recommended improvement.

### **Commentary on Question:**

*The candidates performed as expected on this section. To obtain full points, candidates were expected to clearly explain the relationship between the calibrated parameters used in Monte Carlo modeling and historical data used for calibration in the context of the Black Swan issue. Most candidates were able to propose one appropriate improvement.*

Monte Carlo modeling requires a starting state and parameterization. Historical data doesn't include the extreme events (Black Swan Problem). It's unlikely that observed parameters based on historical data will reflect conditions during extreme liquidity events. Monte Carlo analysis provides information on both severity and probability.

We could use hypothetical data and assumptions in Monte Carlo, so that the modeling can be tailored by judgement. Alternatively, use deterministic scenario modeling to simulate shocks. This way we can model scenarios that never occurred or did not occur with sufficient frequency or severity in recent historical data.



## 7. Continued

- (d) Assess whether or not to add unlisted infrastructure investments to your portfolio for each of your manager's stated considerations.

**Commentary on Question:**

*The candidates performed below average on this section. Both Yes and No recommendations are valid and can obtain full points. To receive points, candidates were expected to provide a rationale for their decision. Unsuccessful candidates were those who simply stated whether the benefits exist without an explanation. Credit was given to appropriate statements about empirical results being mixed.*

Do not recommend adding the infrastructure investments.

Inflation protection: Some infrastructure companies actually hedge out inflation. Based on empirical studies, the correlation between listed infrastructure indices and inflation is low, and sometimes even negative. The results don't point to particular inflation-hedging features of infrastructure.

Diversification: Appraisal-based valuation of unlisted infrastructure and direct property tends to underestimate volatility and correlations with listed instruments and overestimate their diversification potential. Also, correlations can swing substantially over time. During financial crisis, correlations were rising resulting a loss of diversification when it's needed.

- (e) Critique the comment made by your colleague.

**Commentary on Question:**

*The candidates performed below average on this section. Most candidates can correctly critique some of the statements based on the characteristics of REIT, unsuccessful candidates failed to discuss liquidity any further in the context of an investment strategy.*

It's true that REIT reflects more up to date information. It's not true that it reflects the correct property valuation. It's debatable which one is more correct. The stock market may overreact to news leading to subsequent price corrections. The private market merely takes longer to reflect the same value implications.

Because we have a buy and hold strategy, knowing the short term market based prices is not a top concern and therefore REITs do not provide advantage to us. It's true that REITs are more liquid. However, REITs also do not compensate as much for illiquidity. If we can get liquidity from other sources, there is no need to switch because we could earn higher returns in our real estate portfolio.

## 8. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

### Learning Outcomes:

- (2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.
- (2l) Understand and apply various approaches for managing credit risk in a portfolio setting.

### Sources:

Introduction to Credit Risk Modeling, 2nd Edition, Ch 1 pg. 15 – 27

### Commentary on Question:

*This question tested a candidate's understanding of the concept of probability of default, loss given default, exposure at default, and expected loss. Overall, the candidates performed as expected on this question.*

### Solution:

- (a)
  - (i) Identify the contingent liabilities for each of these three borrowers.
  - (ii) Explain the possible random effects for the contingent liabilities.

### Commentary on Question:

*The candidates performed below average on this question. Candidates were able to calculate the contingent liabilities for each of the three borrowers; however, many candidates failed to understand and explain the random effects.*

- (i) The contingent liabilities are the guarantees or comparable credit constructs not for cash. They are 0 million, 15 million, 20 million for borrowers A, B, C respectively.
- (ii) An ideal approach to receive maximum points would be to provide several arguments covering the following topics regarding to possible random effects:
  - The contingent liabilities are subject to the optionality of usage of free parts of the credit line
  - The contingent liabilities do not necessarily lead to cash exposure.
  - A guarantee has no real exposure today but might coverage into exposure in the future.

## 8. Continued

- (b) Calculate the expected exposure at default (EAD) for each of the three borrowers.

**Commentary on Question:**

*The candidates performed as expected on this question. Most of candidates were able to identify the formula to calculate EAD and produced the correct numbers. Unsuccessful candidates failed to indicate the correct formula of EAD.*

- $(EAD) = Outstanding + E(EAD_{cash}) + E(EAD_{CL}) = Outstanding + Cash\ Credit\ Line\ Available \times DDF_{cash} + Contingent\ Liabilities \times DDF_{CL} \times CEEF$
- $E[EAD_A] = 10 + (30 - 10) * 50\% + 0 * 40\% * 60\% = 20\ million$
- $E[EAD_B] = 10 + (15 - 10) * 80\% + (30 - 15) * 60\% * 70\% = 20.3\ million$
- $E[EAD_C] = 10 + (10 - 10) * 95\% + (30 - 10) * 80\% * 80\% = 22.8\ million$

- (c) Explain why the EL formula is not realistic in real life.

**Commentary on Question:**

*The candidates performed above average on this question. A few candidates failed to identify the independence assumption or failed to provide examples in real life.*

An ideal approach to receive maximum points would be to provide several arguments covering the following topics:

- The formula is under an independence assumption that PD, EAD and LGD are independent.
- This independence assumption is not true since defaults and recoveries to some extent are influenced by the same underlying systematic risk drivers.
- In a recession, higher PDs can lead to higher LGDs which means that default rates and realized losses are positively correlated.

- (d) Prove that the unexpected loss (UL) can be calculated as following:

$$UL = EAD \times \sqrt{V[LGD] \times PD + E[LGD]^2 \times PD(1 - PD)}$$

**Commentary on Question:**

*The candidates performed as expected on this question. Some candidates failed to provide step by step proofs with clear and correct justifications. A few candidates tried to justify the formula stated in the question by backwards engineering it rather than deriving an approach that aligns with first principles.*

## 8. Continued

- $UL = \sqrt{V[EAD \times LGD \times L]}$
- $UL^2 = V[EAD \times LGD \times L] = EAD^2 \times V[LGD \times L]$
- $V[LGD \times L] = E[LGD^2] \times E[L^2] - E[LGD]^2 \times E[L]^2$
- Because LGD and L are independent by assumption, and L is a Bernoulli variable, we have  $E[L^2] = E[L] = PD$ , hence  $V[LGD \times L] = E[LGD^2] \times PD - E[LGD]^2 \times PD^2$
- $V[LGD \times L] = E[LGD^2] \times PD - E[LGD]^2 \times PD^2 + PD \times E[LGD]^2 - PD \times E[LGD]^2$
- $V[LGD \times L] = PD \times V[LGD] - E[LGD]^2 \times PD(1 - PD)$
- [1 pt] Collecting the pieces together we have our proof.

(e) Calculate the unexpected loss of this portfolio

### Commentary on Question:

*The candidates performed below average on this question. Unsuccessful candidates failed to identify the correct formula to calculate the unexpected loss.*

- $UL_{PF}^2 = \sum_{i,j=1}^3 EAD_i \times EAD_j \times LGD_i \times LGD_j \times \rho_{ij} \sqrt{PD_i(1 - PD_i)PD_j(1 - PD_j)}$
- $UL_{PF}^2 = 100 * 0.05(1 - 0.05) + 100 * 0.1(1 - 0.1) + 100 * 0.2(1 - 0.2) + 100 * 2 * 0.5 * \sqrt{0.05(1 - 0.05) * 0.1(1 - 0.1)} + 100 * 2 * -0.5 * \sqrt{0.2(1 - 0.2) * 0.1(1 - 0.1)}$
- $UL_{PF} = \sqrt{24.28835} = 4.93 \text{ million}$

## 9. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

### Learning Outcomes:

- (2h) Demonstrate an understanding of credit default swaps (CDS) and the bond-CDS basis, including the use of CDS in portfolio and trading contexts.
- (2i) Demonstrate an understanding an understanding of CDS valuations

### Sources:

Fabozzi - Handbook of Fixed Income Securities,  
Ch67, p. 1574  
Ch67, p. 1586  
Ch67, p. 1588

### Commentary on Question:

*This question tests the candidates' understanding of credit default swaps, its uses, and valuation. Overall, candidates performed as expected on this question. In general, candidates were able to answer conceptual questions but had difficulty performing the tested calculations.*

### Solution:

- (a) Calculate the CDS basis for each case.

#### Commentary on Question:

*Candidates performed brilliantly on this part of the question. Most candidates were able to correctly identify and calculate the CDS basis for each case.*

CDS basis for case I =  $100 - 130 = -30$  bps

CDS basis for case II =  $100 - 85 = 15$ bps

- (b) Propose an arbitrage trading strategy for each case, showing all the steps required from inception to end.

#### Commentary on Question:

*Candidates performed above average on this part of the question. Many candidates were able to appropriately recommend an arbitrage trading strategy for each case and provided basic information on the thought process; however, many candidates did not receive full credit on this part because they did not fully explain why there was an arbitrage.*

## 9. Continued

For case I:

- (1) CDS par spread is lower than bond spread
- (2) Buy McLine bond
- (3) Buy CDS protection on McLine bond.
- (4) Because CDS coupon rate = CDS par spread, the upfront payment to enter into “buy protection” = \$0.
- (5) Before bond maturity, use bond coupon payment to pay LIBOR interest on the amount borrowed and the CDS coupon
- (6) At bond maturity or default, use proceeds from bond or CDS payoff to close the borrowing position.

For case II:

- (1) For case II, because CDS par spread is higher than bond spread, propose
- (2) Short the bond (borrow the bond and sell it in the market)
- (3) Sell CDS protection on Gene Company bond.
- (4) Because CDS coupon rate = CDS par spread, the upfront payment to enter into “sell protection” = \$0.
- (5) Before bond maturity, use the interest earned in (11) and the CDS coupon payment to pay bond coupon owed on (9)
- (6) At bond maturity or default, close the lending position and use the proceeds to buy back bond or payoff CDS claim.

- (c) Outline the reasons that your proposed strategy may not work.

**Commentary on Question:**

*Candidates performed below average on this part of the question. Most candidates were able to cite one reason for only one of the arbitrage strategies but not both. To receive full credit, candidates should have included reasons that both sides of the strategy would not work.*

- (1) May not be able to borrow at LIBOR for the required term = bond maturity
- (2) May not be able to short the bond
- (3) Ignores CDS delivery option that could work against the hedge fund
- (4) Ignores transaction cost and market technical effect.

- (d) Calculate the accrued coupon payment as of the trade date.

**Commentary on Question:**

*Candidates performed below average on this part of the question. Most candidates were able to identify the period which the accrued coupon covers. Candidates lost marks for not recalling that a bond calculation uses a 360 day calendar year.*

## 9. Continued

Since CDS date falls on 3/20 and the trade date is 6/14/2018, the accrued coupon covers the period of 3/20/2018 to 6/14/2018 for 86 days (92-6).

Accrued coupon = notional \* coupon rate \* # days since last coupon date  
= (86/360) \* 1% \* 5,000,000 = \$11,944

- (e) Estimate the upfront payment to enter the contract as of the trade date.

### Commentary on Question:

*Candidates performed poorly on this part of the question. Candidates were not able to identify the correct formula or key inputs to the formula.*

$$A(\bar{S}, T) \approx \frac{1 - \exp\left(-\left(W + \frac{\bar{S}}{(1-R)}\right)T\right)}{W + \frac{\bar{S}}{(1-R)}} \times \frac{365}{360}$$

Where  $W$  = 3-year swap rate = 3.5%,  $S$  = quoted spread = 0.5%,  $R$  = recovery rate = 40%,  $T$  = time to contract maturity = 3.016 (3 years + 6/360)

$[(1 - \exp[-(3.5\% + (0.5\%/ (1-40\%))] * 3.016)] / (3.5\% + (0.5\%/ (1-40\%))) * (365/360)$   
= 2.86649

Upfront = 5,000,000 \* (0.5% - 1%) \* 2.86649 - 11,944 = -\$83,606

## 10. Learning Objectives:

7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

### Learning Outcomes:

- (7a) Identify and evaluate the impact of embedded options in liabilities, specifically variable annuities guaranteed riders (GMAB, GMDB, GMWB and GMIB).
- (7b) Demonstrate understanding of risks associated with guarantee riders including: market, insurance, policyholder behavior, basis, credit, regulatory and accounting.

### Sources:

QFIA-116-13, The impact of stochastic volatility on pricing, hedging, and hedge efficiency of withdrawal benefit guarantees in variable annuities

### Commentary on Question:

*This question tests the candidates' understanding of various features and associated risks relating to Variable Annuity Guarantee products.*

### Solution:

- (a) Explain which of these two features is “richer”.

#### Commentary on Question:

*The candidates performed below average on this section. Most candidates successfully identified that the Remaining Withdrawal Benefit Base is richer; however, a meaningful explanation was omitted by unsuccessful candidates.*

The Remaining WBB Ratchet is “richer” than the Lookback Ratchet. The Remaining WBB is more likely to trigger a ratchet because fund performance only needs to exceed policy charges but not withdrawals in order to trigger a ratchet.

- (b) Demonstrate your answer to (a) numerically.

#### Commentary on Question:

*The candidates performed poorly on this section. In order to get full points, the candidates needed to correctly calculate the guaranteed withdrawal for both features to demonstrate that the remaining WBB is richer. Some candidates were able to calculate  $W_1$  for Lookback Ratchet (and partial points were given), successful candidates demonstrated an understanding of the calculation for the Remaining WBB Ratchet.*



## 10. Continued

$$W_0^g = X_{WL} * WBB_0 = X_{WL} * P = 10\% * 50,000 = 5,000$$

$$AV_0^+ = \max(0, AV_0^- - W_0) = \max(0, 50,000 - 5,000) = 45,000$$

$$AV_1^- = AV_0^+ * S_1/S_0 * \exp(-\Phi^{adm} - \Phi^{adm}) = 45,000 * 110\% * \exp(-1.3\%) = 48,861$$

For Lookback Ratchet,

$$WBB_0^+ = WBB_0 = 50,000, \text{ and } WBB_1^- = \max(WBB_0^+, AV_1^-)$$

$$\text{So } WBB_1^- = \max(50,000, 48,861) = 50,000$$

$$W_1^g = X_{WL} * WBB_1^- = 10\% * 50,000 = 5,000$$

$W_1^g = W_0^g$ , so no increase in guaranteed withdrawal in this scenario for Lookback Ratchet.

For Remaining BBB Ratchet,

$$WBB_0^+ = \max(0, WBB_0 - W_0) = 45,000$$

$$W_1^g = W_0^g + X_{WL} * \max(0, AV_1^- - WBB_0^+) = 5,000 + 10\% * (48,861 - 45,000) = 5,386$$

$W_1^g > W_0^g$ , so there is an increase in guaranteed withdrawal in this scenario for Remaining BBB Ratchet.

Above numerical analysis demonstrates that the Remaining BBB Ratchet is “richer”.

- (c)
- (i) Compare the probability distribution of the trigger time for the guarantee of the 50% Performance Bonus (PB) feature versus the No Ratchet (NR) feature reviewed in the Stochastic Volatility paper by Kling, Ruez, and Russ.
  - (ii) Explain the shape of the probability distribution of the trigger time for the Performance Bonus in relation to the bonus feature.

### Commentary on Question:

*The candidates performed below average on this section. The candidates performed better on part i) than part ii). Most candidates were able to get one statement at least for part i); however, only successful candidates presented more than one statement. For part ii), the key is to identify that the bonus is given by 50% of the difference between the account value and the remaining WBB, and after certain years the account value is paid out. Unsuccessful candidates failed to identify that the account value decreases as a result of the bonus payment.*

## 10. Continued

- (i) In order to receive full points, the candidates must identify at least two of the following statements.
- (1) In the early durations, the probability of PB feature is increasing rapidly and hither than NR feature.
  - (2) Later triggers for PB feature do not occur at all compared to NR feature.
  - (3) The trigger probability for PB is earlier than NR.
  - (4) NR feature has higher probability of not triggering until maturity.

Some candidates drew graphs for probability distributions, who received full points as well.

- (ii) The reason of the shape can be explained as follow. The PB feature is given by 50% of the difference between the current account value and the remaining withdrawal benefit base. However, the benefit base is annually reduced by the initially guaranteed withdrawal amount and therefore reaches 0 after 26 years. Thus, after around 20 years, almost half of the account value is paid out as bonus every year. This leads to a tremendously decreasing account value in later years. Therefore, there is less uncertainty with respect to the trigger time on the insurer's side.

- (d) Explain the purpose of the equivalent local martingale measure (EMM).

### **Commentary on Question:**

*The candidates performed below average on this section. Unsuccessful candidates failed to identify that EMM is required to price the assets and hedging strategies, under the Heston model.*

Within Heston model, EMM is to determine the fair values of assets used in the hedging strategies and of the guarantees to be hedged. i.e. for pricing, we have to transform the real-world measure  $P$  into an equivalent martingale measure  $Q$ .

- (e)
- (i) Determine if EMM exists for each of the following market prices of volatility risk  $\lambda$ , assuming underlying model is the Heston Model.  
$$\lambda_1 = 5, \lambda_2 = 0, \text{ and } \lambda_3 = -10,$$
  - (ii) Determine which  $\lambda$  has the highest corresponding fair guaranteed withdrawal rate (among the  $\lambda$  values for which EMM exists).

## 10. Continued

### **Commentary on Question:**

*The candidates performed poorly on this section. Unsuccessful candidates failed to identify the correct criteria for EMM to exist.*

- (i) The EMM exists if the inequality  $-\kappa/\sigma_\gamma \leq \lambda$  is fulfilled.  $\lambda_3 = -10$  does not meet the requirement as  $-\kappa/\sigma_\gamma = -4.75/0.55 = -8.64$ , and the inequality is not fulfilled.
- (ii)  $\lambda_1$  has the highest fair guaranteed withdrawal benefits because higher values of lambda correspond to lower volatility and higher mean reversion speed, therefore lower value of the corresponding guarantee, or equivalently increase the fair guaranteed withdrawal rate for a given guarantee value.

## 11. Learning Objectives:

7. The candidate will understand various investment related considerations with regard to liability manufacturing and management.

### Learning Outcomes:

- (7a) Identify and evaluate the impact of embedded options in liabilities, specifically variable annuities guaranteed riders (GMAB, GMDB, GMWB and GMIB).
- (7b) Demonstrate understanding of risks associated with guarantee riders including: market, insurance, policyholder behavior, basis, credit, regulatory and accounting.
- (7c) Demonstrate understanding risk management and dynamic hedging for existing GMXB and its embedded options – including:
  - (i) Hedgeable components including equity, interest rate, volatility and cross Greeks
  - (ii) Partially Hedgeable or Unhedgeable components include policyholder behavior, mortality and lapse, basis risk, counterparty exposure, foreign bonds and equities, correlation and operation failures
  - (iii) Static vs. dynamic hedging
- (7d) Demonstrate understanding of Target Volatility funds and their impact on option costs.
- (7e) Demonstrate an understanding of how differences between models of markets and actual market and policy-holder behaviors affect the risks associated with equity linked guarantees.

### Sources:

QFIA-115-13: Stochastic Modeling

QFIA-120-15: Guarantees and target volatility fund

On the Importance of hedging dynamic lapse

### Commentary on Question:

*This question intends to test candidates' understanding of investment return calculations, and its application to dynamic lapse rate setting. In addition, the question covers certain advantages and disadvantages of specific stochastic modelling.*

*Overall candidates did not perform well on this question, with candidates demonstrating an understanding of parts a and b, but less so on parts c and d.*

### Solution:

- (a) Calculate the effective lapse rate applicable in year 6, assuming the policy does not lapse during the first 5 years.

## 11. Continued

### **Commentary on Question:**

*Candidates who attempted this section performed as expected. Successful candidates demonstrated their understanding of real-life like investment return calculation, recognizing that the calculation was not difficult but requiring some repetitive steps to be done. Some unsuccessful candidates calculated the account value and GLWB ITM%, but did not continue to calculate the effective lapse rate.*

First, we calculate the account value at the end of year 5:

$$\begin{aligned} AV_5 &= 0.5 \times (100,000 \times (1 - 1.2\%)^5 \times 1.12 \times 1.1 \times 0.8 \times 1.04 \times 1.05) \\ &\quad + 0.5 \times (100,000 \times (1 - 1.2\%)^5 \times 1.04^5) \\ &= 107,930.74 \end{aligned}$$

Then we calculate the GLWB ITM%:

$$\text{GLWB ITM\%} = 124,120.35 / 107,930.74 = 1.15 > 1.1$$

$$\begin{aligned} \text{Therefore the dynamic lapse multiplier} &= \max(8\%, 1 - 80\% \times (1.15 - 1.1)) \\ &= 0.96 \end{aligned}$$

$$\text{And the effective lapse rate} = 20\% \times 0.96 = 19.2\%$$

- (b) Describe how a target volatility mechanism works.

### **Commentary on Question:**

*Candidates performed as expected on this section. This part test candidates' understanding of the concept of target volatility fund, and mechanically how to dynamically rebalance the equity allocation to achieve the target volatility level. Unsuccessful candidates described the mechanism by repeating the name rather than giving a more in-depth description.*

The target volatility fund is that the investment fund underlying the Variable Annuity product is dynamically rebalanced so as to achieve a certain target level of volatility, thereby reducing the cost of guarantees written on this fund.

To achieve target volatility, the equity weight is typically determined as  $\min(\sigma_{\text{target}} / \sigma_{\text{equity}}, 100\%)$ .

## 11. Continued

- (c) Calculate the mean age of the historical data.

**Commentary on Question:**

*This part tests candidates' memory on the formula to calculate the average age of the historical data used to estimate volatility.*

*The Candidates performed below average on this section. Unsuccessful candidates failed to recognize the appropriate formula, while others appeared others were confused with the rebalancing timing.*

Given the rebalancing frequency is weekly,  $\Delta t = 5$  business days, and there are 252 business days in a year. (full credits also given if candidates used 7 calendar days or  $1/52$  year).

Mean age =  $5 / 252 / (1 - 0.9) = 0.2$  years (full credits also given if correct corresponding formulas was used and obtained 10 weeks or 50/70 days).

- (d) Critique your Chief Actuary's opinion.

**Commentary on Question:**

*This part tests candidates' understanding of SVJD model feature and the underlying implication on modelled volatility.*

*The Candidates performed below average on this section. Unsuccessful candidates did not answer the question but rather discussed how higher account values would lead to greater dynamic lapse experience, whilst a valid argument, it was not a critique of the Chief Actuary's opinion.*

The Chief Actuary's statement is not correct.

Large jumps will cause underestimation of the model-implied volatility of the equity fund, this shall lead to increase in equity allocation.

The cost of guarantee will be higher if jump element is included.

## 12. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
  - One and two-factor short rate models
  - LIBOR market modelsThe candidate will understand approaches to volatility modeling.

### Learning Outcomes:

- (1h) Understand and explain the features of the G2++ model, including: The motivation for more than one factor, calibration approaches, the pricing of bonds and options, and the model's relationship to the two-factor Hull-White model.

### Sources:

Brigo, D and Mercurio F, Interest Rate Models – Theory and Practice, 2nd Edition, Section 3.3, 3.6, 4.1, 4.2

### Commentary on Question:

This question tests the *candidates understanding of the G2++ model and their ability to identify the advantages and disadvantages of using this model over a one factor interest rate model.*

### Solution:

- (a) Discuss possible issues in calibrating the model.

### Commentary on Question:

*Candidates performed below expected on this part. Most candidates were able to identify the interaction between interest rates and volatilities. Unsuccessful candidates did not explain why the interaction might cause issues in calibration.*

The model after the request needs to be fitted for both the interest rates and volatilities. Even though an exact calibration is a desired feature, perfect fitting of the volatility structure can be dangerous.

This is due to the fact that:

1. Not all volatilities quoted in the markets are significant. When the particular market sector is less liquid, the associated quotes may be neither informative or reliable.
  2. Future implied volatility structures are likely to be unrealistic in the sense that they do not conform to any typical market shapes.
- (b)
    - (i) Discuss how to obtain the implied volatility of a cap.
    - (ii) Explain how to calibrate your model to cap-volatility market data.

## 12. Continued

### **Commentary on Question:**

*Candidates performed as expected on this part. Unsuccessful candidates failed to provide details on how to calibrate the model.*

- (i) The market cap volatility is simply defined as the parameter  $\sigma$  that must be plugged into the Black's cap formula to obtain the correct market cap price. Therefore, the cap volatility is obtained by inverting the market price through Black's formula.
  - (ii) The parameters of the model are set to the values such that the market cap volatilities are as close to the model volatilities as possible. The calibration is performed by minimizing the squared differences between the model and market volatilities.
- (c) Explain advantages of two-factor models over one-factor models.

### **Commentary on Question:**

*Candidates performed well on this part. Successful candidates were able to identify the advantages of the two factor model.*

The major weakness of the one factor model is that the evolution of the entire interest rate curve is characterized by the evolution of the short rate  $r$  only. The instantaneous rates of all maturities are perfectly correlated. A shock to the initial interest rate is propagated rigidly through all maturities. A two-factor model allows decorrelation among rates of different maturities.

- (d) Identify two possible situations where one-factor models are as useful as two-factor models.

### **Commentary on Question:**

*Candidates performed as expected on this part. Unsuccessful candidates only discussed one situation.*

1. products to be priced do not depend on the correlation between different maturities.
  2. Approximations can be acceptable when rates that jointly influence the payoff are close at every instant and almost always perfectly correlated.
- (e) Compare and contrast the two interest rate scenario models: G2++ versus CIR2++.



## 12. Continued

### **Commentary on Question:**

*Candidates performed above expectations on this part. Successful candidates identified and described the differences and similarities between the two models. Unsuccessful candidates often identified the differences; however, did not identify the similarities.*

Both models are two factor models that allow for more realistic correlation patterns. The instantaneous correlation between the two factors can be defined explicitly in the joint dynamics. G2++ model is based on the Gaussian distribution and the CIR2++ model is based on CIR processes. Both models are capable of fitting the initial term structure.

The CIR2++ model can maintain positive through reasonable restriction on the parameters. The distributions of the short rate are the sum of two independent noncentral chi-squared random variables which has fatter tails comparing to the Gaussian model in G2++.

G2++ model is more analytically tractable and easier to implement. When the correlation factor is less than zero, the model can produce a humped volatility curve, which is in agreement with market observations. G2++ could produce negative rates due to the Gaussian distribution.

- (f) Provide your observations on these results and a recommendation to your manager regarding the model to be used and the calibration method.

### **Commentary on Question:**

*Candidate performed as expected on this part. Candidates were able to identify the calibration benchmarks, but unsuccessful candidates did not provide suitable recommendations based on their observations.*

Result 1 is calibrated to the cap volatility data. The  $\rho$  is very close to -1, which implied that G2++ model tends to degenerate into a one-factor model. This is due to the fact that cap volatility does not require the correlation of rates between many maturities.

Result 2 is calibrated to the swap volatility data. In this case, correlations between different maturities need to be considered. This is observed as the calibrated  $\rho$  is far from -1.

### **Recommendation:**

To price a cap as indicated in the question, Result 1 provides an one factor model and is sufficient for the purpose. With  $\rho$  close to -1, the effort to use a two-factor model outweighs the benefits that it brings.

However, if the goal is to price a swap, then Result 2 must be used.

### 13. Learning Objectives:

1. The candidate will understand the standard yield curve models, including:
  - One and two-factor short rate models
  - LIBOR market modelsThe candidate will understand approaches to volatility modeling.
2. The candidate will understand and be able to apply a variety of credit risk theories and models.
4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.
5. The candidate will understand the behavior characteristics of individuals and firms and be able to identify and apply concepts of behavioral finance.

### Learning Outcomes:

- (1b) Understand and explain the terms Time Homogeneous Models, Affine Term Structure Models and Affine Coefficient models and explain their significance in the context of short rate interest models.
- (1c) Explain the dynamics of and motivation for the Hull-White extension of the Vasicek model.
- (1d) Explain the features of the Black-Karasinski model.
- (2j) Demonstrate an understanding of mortgage default models in the valuation of MBS.
- (4b) Apply various techniques for analyzing factor models including Principal Component Analysis (PCA) and Statistical Factor Analysis.
- (5a) Explain how behavioral characteristics of individuals or firms affect the investment or capital management process.

### Sources:

QFIA-100-13 - Modeling Mortgage Defaults.pdf (page 9)

QFIA-108-13 -Behavioral Finance and Investment Committee Decision Making

QFIA-119-14 - Tsay, Analysis of Financial Time Series 3rd Ed Ch 9 (page 484)

QFIA-125-16 Principal Component Analysis (page 145)

Sec 1 – Brigo and Mercurio 2nd Ed Ch 3 (Pages 57, 63-64 ,68-70, 83-84)

## 13. Continued

### **Commentary on Question:**

*This question tests the candidate's understanding of Behavioral Finance concepts, mortgage default models, Principal Component Analysis and yield curve models.*

### **Solution:**

- (a) Identify and describe the sociological conditions of this committee that could impact its decision making.

### **Commentary on Question:**

*The candidates performed above average on this section. Most candidates did well in identifying and describing many of the sociological conditions of this committee that could impact its decision making. Unsuccessful candidates only identified 1 or 2 of the conditions.*

A committee, rather than structure it similar to a crowd, which tend to be homogenous (lacking in diversity),

Members of the committee are from the same investment team which lacks diversity in the decision-making process.

The agenda allocates more time on less relevant topics, such as equity investing, rather than focusing on the committee's intended purpose, which is to establish model governance for the models used by Mortgage Bank A.

All of the members of the committee report to John. There is pressure to conform to any recommendations or decisions made by John.

*Other responses were also given credit. The above list would earn full credit and covers the most popular responses that were given credit.*

- (b) Explain the relationship between borrower's credit score, debt-to-income (DTI) ratio and borrower's default rates.

### **Commentary on Question:**

*The candidates performed brilliantly on this section. Unsuccessful candidates either gave incomplete responses or failed to speak to both credit score and DTI components in their response.*

A borrower's credit score is often used as an indicator of his creditworthiness. The lower the credit score, the more likelihood that a borrower will default

### 13. Continued

The higher the DTI ratio, the greater the proportion of the borrower's income that goes towards making debt payments, and hence the greater the strain a trigger event puts on the borrower's ability to continue making mortgage payments

*Other responses were also given credit. The above list covers the most popular responses that were given credit.*

(c) Demonstrate that

$$(i) \quad k_1 = -\sqrt{\frac{1}{(4-2\sqrt{2})}} = 0.92388 \text{ (as appears in exam)}$$

$$k_1 = -\sqrt{\frac{1}{(4-2\sqrt{2})}} = -0.92388 \text{ (as originally intended)}$$

(ii) Variance of  $y_1 = \lambda_1$

(iii) Covariance of  $y_1$  and  $y_2 = 0$

#### **Commentary on Question:**

*For this section there was a typographical error in the published exam book regarding the demonstration for  $k_1$ . Candidates were given credit so that no candidate was disadvantaged for either making the requested demonstration as it appears in the exam book or for making the demonstration as was intended*

*Candidates performed below average on this section. Candidates were given partial credit for each correctly provided demonstration; however, many unsuccessful candidates either i) did not adequately provide some or all of this section's required demonstrations or ii) did not attempt the demonstration(s) at all.*

$$w_1' w_1 = 1$$

$$k_1^2 ((-\sqrt{2} + 1)^2 + 1^2 + 0^2) = 1$$

$$k_1 = -\sqrt{\frac{1}{(4-2\sqrt{2})}} = -0.92388 \text{ as } k_1 < 0$$

### 13. Continued

$$\begin{aligned} \text{Var}(y_1) &= w_1' \sum_i w_i = k_1^2 x (-\sqrt{2} + 1 \quad 1 \quad 0) \begin{pmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} -\sqrt{2} + 1 \\ 1 \\ 0 \end{pmatrix} \\ &= \frac{4+2\sqrt{2}}{4-2\sqrt{2}} = 5.8284 = \lambda_1 \end{aligned}$$

$$\begin{aligned} \text{Cov}(y_1, y_2) &= \text{Cov}(k_3 r_1 - k_1 r_2, r_3) \\ &= k_3 \text{Cov}(r_1, r_3) - k_1 \text{Cov}(r_2, r_3) \\ &= k_3 \times 0 - k_1 \times 0 \\ &= 0 \end{aligned}$$

(d) Describe:

- (i) the shortcomings of performing PCA on data that has a covariance matrix like Q;
- (ii) how to overcome those shortcomings; and
- (iii) any general requirements that the data should meet for PCA to be successful.

**Commentary on Question:**

*Candidates performed below average on this section. Many candidates did not articulate the concerns regarding the covariance matrix, nor did they identify the requirement that the data be at least weakly stationary.*

The covariance matrix Q has variances and covariances that are greater than 1 which means the data is not standardized (normalized).

If the data is not standardized, the variable with the largest variance will tend to dominate the first principal component.

The data should be standardized by subtracting the variable mean and dividing by the variable standard deviation.

The data should be at least weakly stationary. If the data is not stationary, then a transformation such as first differences or a logarithm may be necessary.

*Other relevant responses were also given credit.*

### 13. Continued

- (e) Assess, for each of Models 1 and 2, whether there exist conditions on  $b(t, r(t))$  and  $\sigma(t, r(t))$  such that it displays an affine term structure.

**Commentary on Question:**

*Candidates performed above average on this section. Many candidates commented just on the class that each interest rate model belongs to and used that to draw a conclusion regarding whether an affine term structure is displayed by the model. To achieve full credit required providing an analytic demonstration.*

By Ito's lemma,

$$df = (f'(x)\mu(t) + f''(x)\frac{\sigma^2}{2})dt + \sigma f'(x) dW(t)$$

Under Model 1,

$$dr(t) = r(t)[\theta(t) + \sigma^2/2 - \alpha \ln r(t)]dt + \sigma r(t)dW(t)$$

$r(t)[\theta(t) + \sigma^2/2 - \alpha \ln r(t)]$  is not in the form of  $\lambda(t) r(t) + \eta(t)$

for suitable deterministic functions of  $\lambda, \eta$

$b(t, r(t))$  is not an affine function and there exist no condition on  $b(t, r(t))$  and  $\sigma(t, r(t))$  such that the Model 1 displays an affine term structure.

Model 2 displays an affine term structure as the coefficients  $b$  and  $\sigma^2$  for Hull and White Model are affine functions and are of the form

$$b(t, x) = \lambda(t)x + \eta(t), \quad \sigma^2(t, x) = \gamma(t)x + \delta(t)$$

for suitable deterministic functions of  $\lambda, \eta, \gamma$  and  $\delta$ .

- (f) Assess, for each of Models 1 and 2, whether they satisfy the other two model selection criteria.

**Commentary on Question:**

*Candidates performed brilliantly on this section. Unsuccessful candidates either failed to attempt the section, mischaracterized model characteristics or only provided information on one of the models.*

### **13. Continued**

There is no explosion of bank account under Model 2. Since Model 1 implies a lognormal distribution for short rate, it shares the explosion problem.

Model 2 is analytically tractable while Model 1 is not analytically tractable.

## 14. Learning Objectives:

4. The candidate will understand important quantitative techniques relating to financial time series, performance measurement, performance attribution and stochastic modeling.

### Learning Outcomes:

- (4c) Describe and assess performance measurement methodologies for assets portfolios.
- (4f) Calculate and interpret performance attribution metrics for a given asset, portfolio.
- (4g) Explain the limitations of attribution techniques

### Sources:

Fabozzi, F.J., Handbook of Fixed Income Securities, 8th Edition, Ch. 69-70.

### Commentary on Question:

*This question tests the candidates understanding of performance attribution with a focus on fully analytical performance attribution models.*

### Solution:

- (a) Describe each step of a performance attribution algorithm.

#### Commentary on Question:

*This part tests the candidate's understanding of the steps used in calculating the output of a performance attribution model.*

*Candidates did poorly on this part of the question. Unsuccessful candidates failed to discuss the algorithm described in the text. Partial credit was given to answers that described an algorithm that pertained to performance attribution. Candidates who described criteria used to select a performance attribution model received no credit.*

#### Step One:

##### Return Splitting

The total return of the portfolio is split into the linear contributions of factors.

#### Step Two:

##### Factor Return Attribution

Factors are categorized as common or allocated.

Outperformance from common factors is explained using bottom up aggregation.

Outperformance from allocated factors is explained using either the absolute allocation method or the relative allocation method.



## 14. Continued

Step Three:

Recursive application

Sector management terms from top-down allocations can be decomposed using the above algorithm recursively.

- (b) Discuss the reasons why NOP Life might want to make this change to its attribution model.

**Commentary on Question:**

*The part tests the candidate's ability to decide what type of performance attribution model should be used in place of another.*

*Candidates performed poorly on this part of the question. This part of the question was conceptually more challenging and unsuccessful candidates stated points that while true, were not relevant to why a company would want to change their performance attribution model to a top level model that treats spread duration as a common factor.*

If there is a decision at the portfolio level to over/underweight the spread duration relative to the benchmark, the top-level model is a better fit than the sector-level model.

Helps portfolio managers reveal risk exposures that they have been implicitly taking and helps them improve their management process.

- (c) Show how the Portfolio Outperformance Details Report for NOP's fully analytical sector-level model above changes if outperformance due to spread duration is calculated using the relative allocation algorithm.

**Commentary on Question:**

*This part of the question tests the candidate's ability to apply various aspects of a performance attribution model.*

*Candidates performed poorly on this part of the question. Candidates understood how to calculate performance attribution due to spread duration; however, unsuccessful candidates failed to show how the Details Report would change through demonstrating that most values remained unchanged with the exception of the asset allocation which was reduced by the outperformance of the spread duration.*

## 14. Continued

Spread Duration Mismatch =  $-(OASD^P - OASD^B) * (\Delta OAS^B) = -(5.1 - 4.9) * (13 \text{ bps}) = -2.6 \text{ bps}$

Yield Curve, Implied Volatility, Mortgage, and Residual do not change because they are calculated by bottom up aggregation.

Security Selection does not change because the spread duration calculation is the same whether spread duration outperformance is calculated using the relative or absolute algorithm.

Asset allocation changes from the absolute formula to the relative formula. However, the change in asset allocation is going to be equal to the negative of spread duration mismatch. Asset Allocation =  $-3.4 - (-2.6) = -0.8$ .

<b>Portfolio Outperformance Details Report (bp)</b>	
Yield Curve	33.2
Implied Volatility	0.2
Spread Duration Mismatch	-2.6
Asset Allocation	-0.8
Security Selection	-5.0
Mortgage	-2.0
Residual	2.0
<b>Total</b>	<b>25.0</b>

## 15. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

### Learning Outcomes:

- (2b) Demonstrate an understanding of the basic concepts of credit risk modeling such as probability of default, loss given default, exposure at default, and expected loss.
- (2l) Understand and apply various approaches for managing credit risk in a portfolio setting.
- (2m) Understand the rationale, markets and risks of structured finance.

### Sources:

Managing Credit Risk: The Great Challenge for Global Financial Markets, Ch 24

### Commentary on Question:

*This question is a good example of the actual application and analysis of concepts and issues learned in the reading material, which should be expected in an "Advanced" level examination. This question attempts to test the candidate's ability to analyze RBC formulas, RBC arbitrage and securitization.*

### Solution:

- (a) Outline the most relevant considerations of the current and proposed RBC factors as they impact the ERM, Operations and Valuation teams.

### Commentary on Question:

*The candidates performed poorly on this section. The majority of the candidates, who answered this section noted that investment in lower rated assets, for a higher yield, could be achieved without incurring additional capital charges. But few were able to realize the relationship between the higher yield, the 250% target RBC ratio and VAR. Even fewer candidates could see the opportunity to take lower quality assets and securitize them.*

The proposed factors indicate that we can invest in lower rated assets for a higher yield without increasing the capital charges that we are currently being charged. If we don't change our allocation we will incur a lower RBC on the lower rated assets.

This offers us the opportunity to increase our yield, but if we allocate too many investments to lower quality assets our required capital could increase too much. We will have to work with valuation to make sure we stay under 250% RBC ratio and see how close we can get to 5.25% desired yield.

## 15. Continued

Even then if we increase our lower rated assets we have to be concerned that our VAR will increase beyond the ERM threshold.

We will be charged twice the current capital requirements on NAIC 1B-1G. Should we increase our exposure to NAIC 1A countered by an increase in exposure to the lower rated classes such as 3 and 4? Is the 0.25% pick up in yield worth it for almost 3x the capital charge?

There is a 30% savings of capital right now on what we have in 1A so in theory we can increase our exposure to other classes to increase yield and still have room in our RBC.

There are opportunities to take the lower quality assets and securitize them particularly if they can obtain the highest risk rating.

- (b) Recommend how your company might change its investments from its current allocation of fixed income assets based on the RBC proposed factors in respect to regulatory capital arbitrage.

**Commentary on Question:**

*The candidates performed below average on this section. The majority of the candidates indicated use of capital arbitrage through higher yielding assets coupled with a lower capital charge. Fewer mentioned use of securitization.*

Securitization can play an important role in regulatory capital arbitrage because loans even if provided to relatively high-quality borrowers incur a significant required capital charge and companies are not able to earn a high enough risk adjusted return to warrant investment.

However, through the use of securitization companies can invest in loans or other securities that would allow them to obtain a higher yield rate and incur a lower capital charge.

- (c) Explain how the proposed RBC factors encourage, discourage, or do not change the incentives for the use of securitization for the Company.

**Commentary on Question:**

*The candidates performed poorly on this section. Successful candidates recognized that without doing optimization work it is hard to tell which way the incentives to use securitization will go, and it is likely a company specific problem. Unsuccessful candidates only had one or two short sentences, which could not suffice as an adequate answer to a 2-exam point section. Note: the question asks to "Explain" which requires more than a few descriptive words.*

## 15. Continued

The motivation for securitization exists in the current and proposed frameworks. The proposed framework may provide less incentive because the NAIC 3 and NAIC 4 classes will incur a lower capital charge compared to the current framework.

However, all companies will be optimizing their strategies to earn the highest yield while maximizing their RBC ratio and keeping within a risk tolerance. The new formula will apply to all companies so the demand for the yields available on higher RBC assets will still exist of course. The highest rated assets will incur an even lower capital charge than before so the incentive to structure NAIC 4 or NAIC 3 assets and receive a rating of NAIC 1A (assuming possible) will be similar to the demand in the current formula perhaps even greater.

For structures that don't receive a 1A rating the arbitrage for RBC purposes would be less as the proposed formula does already have lower capital charges so the gain can't be as great as it was before by structuring assets.

Optimization of all the constraints could project an incentive to securitize assets under either RBC structure. ABC has not done any optimization work as yet so it is hard to say if they have any more incentive to invest in structured securities or less so I think there will be a similar demand as there is with current RBC structure.

If the assets that have been structured can receive 1A-1G classifications for RBC purposes with yields that are greater than other assets in 1A-1G there could be a strong desire to securitize as much as possible----although other constraints may come into play such as the size of the available asset pool of the strategy. The interaction of more securitized assets with VAR would be interesting to monitor but it seems possible certain structures may be beneficial to VAR as structured assets will have higher yields and the cash flows desired could be determined with the assistance of the VAR model to optimize the desired assets in consideration of their cash flows and risks.

## 16. Learning Objectives:

2. The candidate will understand and be able to apply a variety of credit risk theories and models.

### Learning Outcomes:

- (2a) Demonstrate an understanding of events and causes of the 2008 global credit crisis.
- (2e) Demonstrate an understanding of the term structure of default probability.
- (2j) Demonstrate an understanding of mortgage default models in the valuation of MBS.

### Sources:

Modeling of Mortgage Defaults, by Hayre et al., 2008.

### Commentary on Question:

*This question tests the candidates' understanding of mortgage defaults modeling and its application to the 2008 financial crisis.*

### Solution:

- (a) State two necessary conditions for a mortgage default to have occurred.

#### Commentary on Question:

*Candidates performed brilliantly on this part. Partial credit was given whenever a candidate had only one of the necessary conditions listed below.*

A default has occurred if:

- The loan has been liquidated (= prepaid, with the previous state being either delinquent or foreclosed or real-estate-owned).
- The servicer has incurred a loss.

*Alternatively, candidates could get full credit by answering from the point of view of the borrower, as follows:*

- The borrower is unable to make the mortgage payments, usually because of some trigger event.
- The borrower is unable to refinance or sell the property without a loss.

- (b) Calculate the probability that the investor has negative equity on their property on January 1<sup>st</sup>, 2018.

## 16. Continued

### **Commentary on Question:**

*Candidates performed above average on this part. A common mistake was to identify the SDE given as Geometric Brownian Motion and to use a version of Merton's asset value model to compute the desired probability. Partial credit was awarded in cases where the mean and/or variance were incorrect, or for calculations that yielded a high probability of negative equity.*

Compute  $\Pr[r(8) < -20\%]$ , where  $r(8)$  is normally distributed, with:

- mean  $E[r(8)] = 8\mu = 16.8\%$ , and
- variance  $\text{Var}[r(8)] = 8\sigma^2 = 0.98\%$ , hence standard deviation = 9.9%.

We have  $\Pr[r(8) < -20\%] = \Pr[Z < (-20\% - 16.8\%) / 9.9\%] = \Pr[Z < -3.72] = 0.01\%$ .

- (c) Identify a challenge to predictive models such as the one above.

### **Commentary on Question:**

*Candidates did as expected on this part. Successful candidates sought to give a high-level critique of predictive models, while answers identifying weaknesses of the specific model (with regards to model parameters and their calibration, distributional assumptions, or factors not incorporated in the model) were given full credit only if they were well-argued.*

The profile of the borrowers in the future, and the mix of mortgage products will be quite different from that of the recent past. This presents a challenge to predictive models, which tend to use the past as a guide to future behavior.

- (d) Evaluate your colleague's claim.

### **Commentary on Question:**

*Candidates did as expected on this parts. Points were deducted for seeing an increase in second liens instead of second lien defaults, and for not making the connection with the 2008 mortgage credit crisis.*

Second-lien mortgage defaults tend to show up earlier and are steeper than first-lien mortgage defaults under stress conditions for mortgage credit, which is what took place right before the 2008 financial crisis.

*Alternatively:*

Defaults on second-lien mortgages shot up before defaults on first-liens showed signs of stress in the lead up to the 2008 mortgage credit crisis. The graph shows a similar trend for 2017.

## 16. Continued

- (e) Describe three borrowing and loan characteristics that are key factors in assessing default risk.

**Commentary on Question:**

*Candidates did above average on this part. Partial credit was given for listing factors without an explanation as to what they mean and how they are connected to the assessment of default risk.*

- Credit scores have predictive power when used to model mortgage defaults, although not as high as originally thought.
- Debt-to-income ratios show the strain a trigger event (such as a job loss or payment shock) may put on the borrower's ability to keep making payments.
- Lacking loan documentation introduces uncertainty with respect to the borrower's income and ability to handle mortgage payments.

*Any of the above three characteristics can be replaced by:*

- Occupancy status is an important factor because mortgages on owner-occupied properties are less likely to go into default.
- The loan purpose (purchase vs. refinancing) determines whether the property value is based on the sale price or an appraisal, and the borrower's creditworthiness.
- LTV ratios and HPA, as well as the occurrence of inflated appraisals or existence of silent second liens, reduce the equity left in the property and increase the likelihood of the borrowed defaulting.
- A high spread-at-origination (SATO), which is the difference between the coupon on the loan and the prevailing mortgage rate, is a characteristic of high-risk borrowers.