GI ADV Model Solutions
Fall 2018

1.   **Learning Objectives:**
   4.   The candidate will understand how to apply the fundamental techniques of
          reinsurance pricing.

Learning Outcomes:
(4a)   Calculate the price for a proportional treaty.

Source:
Basics of Reinsurance Pricing, Clark

Solution:
(a)   Calculate the technical ratio (loss ratio plus commission ratio) for 2017.

   The sliding scale commission is 10% (because $R \geq 60\%$). The profit commission
   is the premium (100%) – the loss ratio (65%) – the sliding scale commission
   (10%) – the expense margin (15%) = 10% to share. Amount returned is
   40%(10%) = 4%. The technical ratio is 65% + 10% + 4% = 79%.

(b)   Calculate the expected technical ratio for 2018.

   The loss ratio bands are 30-35 (probability 1/8), 35-55 (probability 4/8), and 55-
   70 (probability 3/8). Add the carryover of 5 to change the ranges to 35-40, 40-60,
   and 60-75. The averages are 37.5, 50, and 67.5. The commissions are 20, 15, and
   10. The profits in the three bands are 100 – 37.5 – 20 – 15 = 27.5, 100 – 50 – 15
   – 15 = 20, and 100 – 67.5 – 10 – 15 = 7.5. The 40% returned is 11, 8, and 3,
   respectively. The technical ratios are 32.5 + 20 + 11 = 63.5, 45 + 15 + 8 = 68,
   and 62.5 + 10 + 3 = 75.5. The weighted average is (1/8)63.5 + (4/8)68 +
   (3/8)75.5 = 70.25%.

(c)   Describe two complications with pricing the effect of carryforward provisions.

   - There is no obvious method for reducing the variance of the aggregate
     distribution.
   - It is difficult to take into account the uncertainty about whether the treaty will
     be renewed.

2.   **Learning Objectives:**
  5.   The candidate will understand methodologies for determining an underwriting profit margin.

  Learning Outcomes:
  (5d)   Allocate an underwriting profit margin (risk load) among different accounts.

  Source:
  An Application of Game Theory: Property Catastrophe Risk Load, Mango

  Solution:
  (a)   Calculate the risk load for the two contracts combined.

  The mean is $0.05(1500) + 0.03(2000) + 0.02(3000) = 195$.
  The variance is $0.05(1500)^2 + 0.03(2000)^2 + 0.02(3000)^2 - 195^2 = 374{,}475$.
  The risk load is $0.0001(374{,}475) = 37.45$.

  (b)   Calculate the renewal risk load for each contract using the Marginal Variance method.

  Contract X: Mean $= 0.05(500) + 0.03(2000) = 85$, Variance $= 0.05(500)^2 + 0.03(2000)^2 - 85^2 = 125{,}275$.
  Contract Y: Mean $= 0.05(1000) + 0.02(3000) = 110$, Variance $= 0.05(1000)^2 + 0.02(3000)^2 - 110^2 = 217{,}900$.
  Renewal risk load for X: $37.45 - 0.0001(217{,}900) = 15.66$.
  Renewal risk load for Y: $37.45 - 0.0001(125{,}275) = 24.92$.

  (c)   Calculate the renewal risk load for each contract using the Shapley method.

  The covariance to share is $374{,}475 - 125{,}275 - 217{,}900 = 31{,}300$. Dividing it equally allocates 15,650 to each contract. The renewal risk loads are:
  X: $0.0001(125{,}275 + 15{,}650) = 14.09$
  Y: $0.0001(217{,}900 + 15{,}650) = 23.36$

3. Learning Objectives:

5. The candidate will understand methodologies for determining an underwriting profit margin.

Learning Outcomes:

(5c) Calculate an underwriting profit margin using the risk adjusted discount technique.

Source:

Ratemaking: A Financial Economics Approach, D'Arcy and Dyer

Solution:

(a) Calculate the premium for this policy using the Risk Adjusted Discount Technique.

$$P = \frac{80}{0.98} + 20 + \frac{(P-20)(0.2)}{1.02} - \frac{80(0.2)}{0.98} + \frac{(50+P-20)(0.02)(0.2)}{1.02}$$

$$P = 81.633 + 20 + 0.196P - 3.922 - 16.327 + 0.118 + 0.004P$$

$$0.8P = 81.5$$

$$P = 101.875$$

(b) Describe three criticisms of the Risk Adjusted Discounted Technique as applied to insurance models.

Commentary on Question:

Any three of the following are sufficient for full credit.

- The risk-free rate may not be the correct rate to use.
- It is not clear how to set the risk-adjusted rate.
- It is difficult to allocate equity to policies.
- It considers only one policy term, not renewal cycles.
- Actual taxes may not be at the corporate rate.
- Expenses may depend on the premium rate.

4. **Learning Objectives:**
1.   The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:
(1a)   Identify the assumptions underlying the chain ladder estimation method.

(1b)   Test for the validity of these assumptions.

(1c)   Identify alternative models that should be considered depending on the results of the tests.

Sources:
Measuring the Variability of Chain Ladder Reserve Estimates, Mack
Testing the Assumptions of Age-to-Age Factors, Venter

Solution:
(a)   Explain whether this implies that observed development factors within a given accident year are uncorrelated, independent, neither, or both.

The assumption does not imply that the observed development factors are independent. While it does imply that the expected development does not depend on past development, other aspects of the distribution (such as the variance) can depend on past development.

The assumption does imply that they are uncorrelated. Correlation relates to expectations, which do not depend on previous observed values.

(b)   State Mack's other two assumptions.

- The development in any given accident year is independent of the development in any other accident year.
- The variance of the claim amount in a given development year is proportional to the claim amount at the end of the previous development year. The constant can depend on the development year, but not on the accident year.

(c)   Demonstrate that the weighted average test statistic for the triangle below is −0.24.

Here $I = 7$ and note that $T_k = 1 - \dfrac{6(\text{sum of squared differences})}{(I-k)^3 - (I-k)}$.

4. Continued

k	2		3		4		5	
	s_{i2}	r_{i2}	s_{i3}	r_{i3}	s_{i4}	r_{i4}	s_{i5}	r_{i5}
	2	4	3	1	1	2	1	2
	4	3	2	3	2	3	2	1
	1	2	1	4	3	1		
	5	5	4	2				
	3	1						
Sum of squared differences	10		18		6		2	
T_k	0.500		-0.800		-0.500		-1.000	
$I-k-1$	4		3		2		1	

The weighted average is $[4(0.5) + 3(–0.8) + 2(–0.5) + 1(–1.0)]/10 = –0.24$.

(d) State the conclusion that should be drawn from the test. Justify your answer.

Commentary on Question:
Because 0.76 standard deviations could also be considered as an insignificant deviation from zero, it is also acceptable to conclude that the assumption is satisfied.

The standard deviation is $\sqrt{\dfrac{2}{5\times 4}} = 0.316$. The test statistic is $0.24/0.316 = 0.76$ standard deviations below zero. There is some evidence of a negative correlation and hence an alternative model should be considered.

(e) Describe two such alternative models.

Commentary on Question:
Any two of the following are sufficient for full credit.

- Linear with constant
- Factor times parameter
- Factor times parameter plus a calendar year effect
- Bornhuetter Ferguson
- Parameterized Bornhuetter Ferguson
- Cape Cod
- Additive

5.   **Learning Objectives:**
1.   The candidate will understand how to use basic loss development models to estimate the standard deviation of an estimator of unpaid claims.

Learning Outcomes:
(1a)   Identify the assumptions underlying the chain ladder estimation method.

(1e)   Apply a parametric model of loss development.

Sources:
LDF Curve Fitting and Stochastic Reserving: A Maximum Likelihood Approach, Clark
Measuring the Variability of Chain Ladder Reserve Estimates, Mack

Solution:
(a)   Explain why the accident year 2015 pattern showing a decrease from December 31, 2015 to December 31, 2016 does not violate this assumption.

Clark assumes that "the *expected* loss emergence moves from 0% to 100% in a strictly increasing pattern." However, the actual results can differ due to random fluctuations.

(b)   Demonstrate that the maximum likelihood estimate of ELR is 0.6291.

The numerator is the sum of observed claims: $6{,}200 + 5{,}500 + 3{,}000 = 14{,}700$. The denominator is the sum of the premium times growth functions. They are $10{,}000G(27) = 10{,}000(0.9391) = 9{,}391$, $8{,}500G(15) = 8{,}500(0.8938) = 7{,}597$, and $9{,}000G(4.5) = 9{,}000(0.7089) = 6{,}380$ for a total of $23{,}368$. Then, ELR $= 14{,}700/23{,}368 = 0.6291$.

(c)   Estimate the reserve for the three accident years combined.

Estimated reserve for a given accident year is premium x ELR x (1 – growth function).
2015: $10{,}000(0.6291)(1 - 0.9391) = 383$
2016: $8{,}500(0.6291)(1 - 0.8938) = 568$
2017: $9{,}000(0.6291)(1 - 0.7089) = 1{,}648$
The total is 2,599.

(d)   Estimate the expected payments that will be made during the last three months of 2017 for accident year 2016.

$8{,}500(ELR)[G(18) - G(15)] = 8{,}500(0.6291)(0.9103 - 0.8938) = 88$

5. **Continued**

(e)   Describe a situation where the Weibull is likely to be more appropriate.

The Weibull distribution generally provides a smaller tail factor than the loglogistic.

(f)   Explain why Mack's formulas are not appropriate in this situation.

Mack assumes that for each development age the expected value and variance are a constant multiple of past values with constant independent of accident year. This situation has different development ages by accident year and hence different factors are required.

6. **Learning Objectives:**
2.   The candidate will understand the considerations in selecting a risk margin for unpaid claims.

Learning Outcomes:
(2a)   Describe a risk margin analysis framework.

(2b)   Identify the sources of uncertainty underlying an estimate of unpaid claims.

(2c)   Describe methods to assess this uncertainty.

Source:
A Framework for Assessing Risk Margins, Marshall, et al.

Solution:
(a)   Describe what is meant by a bolt-on approach.

Commentary on Question:
The paper offers two definitions; either is acceptable.

- It is an approach that uses separate analyses for estimating the central estimate of the liabilities and for estimating the risk margins.
- It is any approach that does not use a single unified distribution of the distribution of future claim costs.

(b)   Explain why quantitative techniques alone are insufficient when determining risk margins and must be supplemented by qualitative techniques.

- Quantitative methods can be complex and require significant amounts of time, data, and cost.
- Quantitative analysis of historical data cannot capture all sources of future uncertainty.

(c)   Provide an example where qualitative techniques may be used in each of the following activities:

(i)   Preparing the claims portfolio for analysis

(ii)   Analyzing internal systemic risk

(iii)   Analyzing external systemic risk

(iv)   Determining correlation effects

6. **Continued**

        (i)   Selecting the risk classes to use or the granularity of classes

        (ii)   Developing a balanced scorecard

        (iii)   Identifying key systemic risks or ranking risk categories by expected impact

        (iv)   May need to set correlations without data or decide which correlations to model

7. **Learning Objectives:**
3.   The candidate will understand excess of loss coverages and retrospective rating.

Learning Outcomes:
(3e)   Explain Table M and Table L construction in graphical terms.

Source:
The Mathematics of Excess of Loss Coverages and Retrospective Rating – A Graphical Approach, Lee

Solution:
(a)   Define the following terms associated with Table L in a retrospective rating plan:

>   (i)   The entry ratio, r

>   (ii)   The Table L charge, $\phi*(r)$

>   (iii)   The Table L savings, $\psi*(r)$

>   (i)   The entry ratio is the multiple of a risk's expected loss or expected loss ratio.

>   (ii)   The Table L charge is the average amount by which a risk's actual limited loss exceeds r times its expected loss, divided by its expected loss. The loss elimination ratio is then added to this quantity.

>   (iii)   The Table L savings is the average amount by which a risk's actual limited loss falls short of r times its expected loss, divided by its expected loss

(b)   Complete the following Table L for this risk:

| r	$\phi*(r)$	$\psi*(r)$
0.00	1.00	0.00
0.20	0.80	0.00
0.40	0.60	0.00
0.60	0.44	0.04
0.80	0.32	0.12
1.00	0.24	0.24
1.20	0.20	0.40
1.40	0.20	0.60
1.60	0.20	0.80
1.80	0.20	1.00
2.00	0.20	1.20

7. **Continued**

Commentary on Question:
The table is completed above, with further explanation below.

One approach is to calculate the values directly from the definition. The average limited loss ratio is $(20 + 30 + 40 + 50 + 60)/5 = 40$. The average loss ratio is then $40/(1 - 0.2) = 50$. For $r = 0.80$ (for example), the expected excess over $0.80(50) = 40$ is $(0 + 0 + 0 + 10 + 20)/5 = 6$. It is then divided by the average loss ratio to obtain $6/50 = 0.12$. Finally the loss elimination ratio is added to obtain the Table L charge of $0.12 + 0.20 = 0.32$. The Table L savings is $(20 + 10 + 0 + 0 + 0)/5 = 6$, divided by 50 to obtain 0.12.

A second approach is to construct a graph. The one below illustrates the calculation for $r = 0.80$. The cumulative probability increases by 0.2 at each of the entry ratios 20/50, 30/50, 40/50, 50/50, and 60/50. The charge is the area shaded with horizontal stripes, which is $0.08 + 0.04 = 0.12$ plus the loss elimination ratio, to give 0.32. The savings is the area shaded with vertical stripes, which is $0.04 + 0.08 = 0.12$.
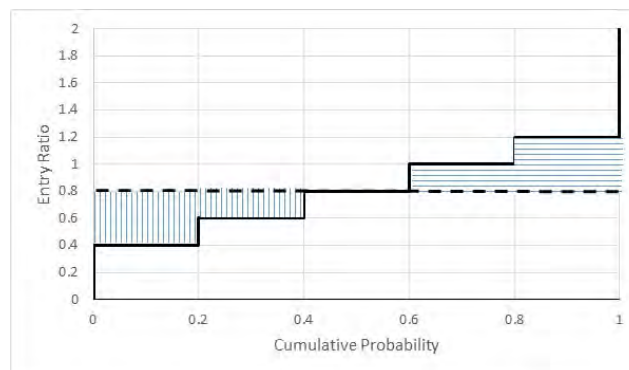
8.

Learning Objectives:
4.   The candidate will understand how to apply the fundamental techniques of reinsurance pricing.

Learning Outcome:
(4b)   Calculate the price for a property per risk excess treaty.

Source:
Basics of Reinsurance Pricing, Clark

Solution:
(a)   Estimate the experience rating loss cost as a percentage of the subject premium.

For each loss, values are first trended, then the layer is applied, and then they are developed:

$1,100(1.06)^4 = 1,389; 389 \times 1.0 = 389$
$2,200(1.06)^4 = 2,777; 1,777 \times 1.0 = 1,777$
$900(1.06)^3 = 1,072; 72 \times 1.2 = 86$
$1,400(1.06)^3 = 1,667; 667 \times 1.2 = 800$
$800(1.06)^2 = 899; 0 \times 1.5 = 0$
$1,800(1.06)^2 = 2,022; 1,022 \times 1.5 = 1,533$

Total is 4,585. The premium is 18,000. The rate is 4,585/18,000 = 0.255.

(b)   Define free cover.

Free cover occurs when no losses trend into the highest portion of the layer covered.

(c)   Calculate a suitable adjustment to the loss cost using these exposure factors to estimate the cost of free cover.

For the layer 2,000 xs 1,000 the top is 3,000 and the bottom is 1,000. The top factor is 60% and the bottom factor is 20%. The top exposure factor is 80% and the bottom exposure factor is 50%. The difference is 30%.

For the layer 2,000 xs 3,000 the top is 5,000 and the bottom is 3,000. The top factor is 100% and the bottom factor is 60%. The top exposure factor is 95% and the bottom exposure factor is 80%. The difference is 15% and the estimated cost of free cover is 25.5%(15%/30%) = 12.75%.

The total is 25.5% + 12.75% = 38.25%.

8.   **Continued**

(d)   Assess whether these exposure factors would be appropriate for pricing coverage on properties valued at 20 million.

Using these exposure factors would imply that the factors are scale invariant. This is reasonable for homeowners insurance, but not for commercial property.